

A COMPARISON STUDY BETWEEN ISOGEOMETRIC ANALYSIS AND FINITE ELEMENT ANALYSIS FOR NONLINEAR INELASTIC DYNAMIC PROBLEMS WITH GEOMISO DNL SOFTWARE

PANAGIOTIS L. KARAKITSIOS¹, IOANNIS A. PRENTZAS^{2*}, ATHANASIOS I.
LEONTARIS³ AND ALEXANDROS S. PAPAKONSTANTINO⁴

¹ Geomiso Company
Themistokleous 19, Farsala
pkarakitsios@geomiso.com, <https://www.geomiso.com>

² Geomiso Company
Themistokleous 19, Farsala
geomiso.prentzas@gmail.com, <https://www.geomiso.com>

³ Geomiso Company
Themistokleous 19, Farsala
aleontaris@geomiso.com, <https://www.geomiso.com>

⁴ Geomiso Company
Themistokleous 19, Farsala
geomiso.papakonstantinou@gmail.com, <https://www.geomiso.com>

Key words: *Isogeometric analysis, FEA, Computational structural dynamics, Inelastic analysis, Splines, Cloud*

Abstract. *The new Geomiso DNL software is proposed to facilitate the use of isogeometric analysis for nonlinear inelastic dynamic applications. This hybrid software solution combines isogeometric analysis and 3D design with advanced spline techniques, such as NURBS and T-splines. Its dual nature satisfies the rising industrial need for unification of the fields of computer-aided design (CAD) and computer-aided analysis (CAE), as it eliminates geometric errors by merging geometry design with mesh generation into a single procedure. This paper presents sample nonlinear applications in structural dynamics. Geomiso DNL is seen to handle these situations remarkably well, as the numerical examples exhibit significantly improved accuracy of the results, and reduced computational cost, when compared with finite element software packages. It is argued that Geomiso DNL is a new, more efficient, alternative to FEA software packages. This is the first time ever such a cloud-based program has been developed.*



Co-financed by Greece and the European Union

1 INTRODUCTION

In this paper we introduce Geomiso DNL, a new both on-premises and cloud-based software, which combines isogeometric analysis and 3D design with NURBS and T-splines. Geomiso DNL is a viable alternative to finite element software packages with several advantages, as it provides nonlinear inelastic dynamic isogeometric analysis. The isogeometric method satisfies the rising industrial need for unification of the fields of computer-aided design and computer-aided analysis. Modern T-splines overcome limitations inherent to NURBS, permit local refinement, constitute a solution to the gap problem between patches, ensure higher-order continuity across patches, and provide great superiority of modeling irregular geometries with hole features. There is linear independence for a topologically restricted subset of T-splines, the analysis-suitable T-splines. Moreover, thanks to the high-regularity properties of its basis functions, IGA shows a better accuracy per-degree-of-freedom and an enhanced robustness with respect to standard FEA. Therefore, IGA has attracted growing interest in both scientific community and industry.

Geomiso DNL is not just a plug-in, but a complete software solution, which enables engineers to simulate dynamic phenomena, whose impact on products and structures in real-world environments can be more efficiently evaluated. This new software fully integrates the industrial design of any geometry with its computational real-time testing by facilitating the geometry modeling within analysis. A key feature is that this hybrid program provides parameterized geometries in the design, as it weaves the mesh generation process within CAD, while its comprehensible modern graphical user interface offers an innovative way to preserve the exact geometry at all refinement levels in contrast with FEA. Moreover, this software merges geometric design with mesh generation into a single procedure by creating, with its hybrid graphical user interface, 3D models as tensor product grids. This hybrid software, used for both design and analysis, has many features in common with both FEA software and design programs. The isogeometric method, in combination with material non-linearity and dynamic analysis, has attracted increasing attention, as a result of the industrial need for high product quality, coupled with increasingly stringent safety.

Applications to 2D and 3D dynamic problems are demonstrated with a comparison between Geomiso DNL and finite element software packages. We compare the matrix assembly and solver time, as well as the accuracy of the numerical results, such as displacement, strain, and stress fields, for real-world and industrial applications arising in structural dynamics. We also perform parametric tests on the effects of polynomial order of shape functions and the number of patches, elements, control points, and quadrature points. This program appears to be preferable to FEA programs, as it represents major improvements, such as higher accuracy, robustness, and stability level, combined with significantly shortened computational cost.

This paper is organized as follows. In Section 2 we exhibit an overview on IGA with NURBS and T-splines. Section 3 refers to nonlinear inelastic dynamic isogeometric analysis, while section 4 presents the user interface of Geomiso DNL. A comparison between Geomiso DNL and FEA programs is made in Section 5 for a rectangular parallelepiped beam, a two dimensional plate with a circular hole, and a transient analysis on a rectangular plate, while conclusions are drawn in Section 6.



Co-financed by Greece and the European Union

2 A BRIEF OVERVIEW ON ISOGEOMETRIC ANALYSIS

IGA was introduced by Hughes et al. [1] and since then it has attracted a lot of attention for solving boundary value problems as a result of using the same shape functions for both describing the domain geometry and building the numerical approximation of the solution. The idea is to build a geometry model and directly use the functions describing the geometry in analysis, rather than develop a finite element mesh approximating the geometry.

2.1 NURBS

NURBS technologies, which allow conic sections to be represented exactly, have been used in CAD for decades due to their mathematical properties. IGA has brought them into the setting of analysis, leading to more accurate results in comparison with standard FEA based on Lagrange polynomials [1]. Parameter space is important as all calculations take place in it, while index space plays an auxiliary role. NURBS geometries inherit properties, such as partition of unity, non-negativity, boundary-curve interpolation, continuity, and compact support for their basis functions. Due to their higher inter-element continuity, the overlapping is greater in comparison with FEA. On the contrary to interpolatory shape functions in FEA, the basis functions in IGA are not interpolatory. NURBS are built from B-splines and unlike in FEA, the B-spline parameter space is local to patches rather than elements. Patch is a subdomain, within which element types and material models are assumed to be uniform.

The tensor product structure of a single patch makes its representation poorly suited, while using a single patch, the mapping might produce severe mesh distortion, which is unavoidable. The effect of mesh distortion on the performance of IGA for solid mechanics could be overcome using higher-order basis functions, which are able to somewhat alleviate the impact of the distortions. In almost all practical circumstances to represent real world geometries and industrial applications, it will be necessary to describe domain with multiple NURBS patches, especially when the geometry is sophisticated. Most of the industrial components has complex shape, such as hole features, circular shape or sharp curves, consequently multi-patch geometries are a robust solution for the model creation. This possibility overcomes the feature removal procedure, which is common practice in creating FEA models leading to incorrect stress concentrations.

The model is decomposed into patches, which are subdomains with the same material and geometry type. Patches have their own parameter space. In the geometry modeling, parts of a product are modeled separately using a single patch. The patches should match geometrically and parametrically on the internal faces where they meet. This means that refinement of one patch must necessarily propagate from that patch to the next. It is difficult to join multiple surfaces in a single, smooth, watertight model, unless the patches have the same parameterization. Trimming techniques provide a promising alternative for representing complex NURBS domains. The connection's result should be that the patches are joined as though they were one. A serious problem inherent in NURBS is that it is mathematically impossible for a trimmed NURBS to accurately represent the intersection of two NURBS surfaces without introducing gaps in the model. The inability of NURBS to make a watertight connection is their major drawback.



Co-financed by Greece and the European Union

2.2 T-splines

T-splines are a generalization of NURBS representation for surfaces and solids with their main feature being to allow a row of control points to terminate before reaching the boundary. NURBS were until lately the main shape functions used in isogeometric analysis [1]. One might consider NURBS as a special case of T-splines. They inherit the basic properties of NURBS, means partition of unity, non-negativity, compact support, affine covariance, and boundary-curve interpolation. There is linear independence for a topologically restricted subset of them, the analysis-suitable T-splines, in which T-junctions are defined properly, and blending functions are smooth. Their main advantages over NURBS are that they exhibit local refinement properties and ensure watertight connection between patches [3]. Cubic T-splines can accurately design any geometry and make the problematic and often impossible merging of patches feasible. T-splines show great superiority of modelling sophisticated geometries, especially when the model is irregular with hole features. It is not unlikely that T-splines are capable of representing complex shapes with only one single T-mesh.

On the contrary to NURBS, index space is the important one for defining junctions, anchors, and local knot vectors, which are assigned to every blending function, with each one having an association with the topology of the whole patchwork of the object, while parameter space plays an auxiliary role. T-splines correct the deficiencies of NURBS in that they permit local refinement and coarsening, and a solution to the gap problem, albeit not totally free of superfluous control points. Knot insertion can be locally accomplished by performing it into the basis functions whose local knot vectors will be altered by the presence of a new control point. This difference enables T-splines to permit T-junctions, while for the same refinement shorter knot lines are generated. Cartesian space represents the real geometry of the analyzed structure. Index space of a T-mesh is a rectangular tiling of a region in \mathbb{R}^2 or \mathbb{R}^3 , such that each edge of every rectangle has positive integer value and vertices connecting three edges, referred to as T-junctions. The kind of available T-junctions are limited in order the T-mesh to be analysis suitable and more specifically only T and cross junctions are accepted. Each anchor will be used to infer local knot vectors, which will define the respective blending function.

The main obstacle in IGA using T-splines is how to robustly construct surface and volumetric T-spline models. A discrete procedure should be followed. Firstly, the user should define the T-mesh with the appropriate junctions. The continuity of a T-spline in physical space is defined by the blending functions in the parameter space. This was a straightforward statement in the NURBS setting, but the local T-spline construction demands detailed consideration, especially as it is based on numerical quadrature. The continuity of the blending functions is determined from its local knot vector, while it influences the quadrature, therefore continuity reduction lines in T-spline surfaces or continuity reduction faces in T-spline volumes are necessary. For example, for 2D problems, the continuity reduction lines divide the T-mesh into rectangular regions over which the blending functions are smooth. Thus, quadrature can be performed over these regions using classical quadrature rules. These requirements of using T-splines in IGA are not met in the design process. In particular, there is no restriction for the junctions' type, means L, I and point junctions are allowed, as the linear independence of the blending functions is not a requirement [3].



Co-financed by Greece and the European Union

3 NONLINEAR INELASTIC DYNAMIC ANALYSIS

3.1 Nonlinear dynamic analysis

Newmark solver is used for dynamic problems. Newmark's method is an implicit method, which is based on discretising the differential equation of motion using a finite difference approximation. The equation of equilibrium is applied at $t+\Delta t$, while it allows larger time steps. Its major assumption refers to how the acceleration varies between two consecutive time steps, such as constant or linear acceleration method. Corrective Newton-Raphson iterations are performed within every single time step of the dynamic solver. An energy-based convergence criterion is used to ensure that both displacements and forces are in equilibrium.

3.2 Material nonlinearity in IGA

This paper focuses on the advantages of IGA in describing nonlinear behavior of materials, which is commonplace in many applications, such as metal forming, and vehicle crash test. Material nonlinearity represents the case when the constitutive law, means the stress-strain relation, is not linear. Once yield occurs, the material will deform plastically. The constitutive law provides the mathematical relationships that describe the material and govern the nonlinear analysis. A fundamental observation comparing elastic and inelastic analysis is that the total stress in elastic solutions can be evaluated from the total strain alone, whereas the total stress in inelastic response calculations also depends on the stress and strain history [4].

To describe a particular case of nonlinear material behavior in solid mechanics, a suitable model must be adopted. The nonlinear nature of a structural analysis problem emerges from the relationship between the applied external loads and the displacements, which are no longer analogous. This nonlinearity arises from either the geometry of the structure, where large deformations impose the equilibrium equations to be solved in the deformed configuration, or the material itself, in case of inelastic materials. In each case, the stiffness matrix can be expressed as a function of the displacements; $\mathbf{K}=f(\mathbf{u})$. Nonlinear problems in mechanics are solved with incremental iterative algorithms based on the Newton-Raphson Method, such as Force Control Algorithm, in which the external load \mathbf{P} is applied directly with load increments $\Delta\mathbf{P}$, or Displacement Control Algorithm, in which an effective load is applied in such a way, so that a specific degree of freedom acquires constant displacement increment.

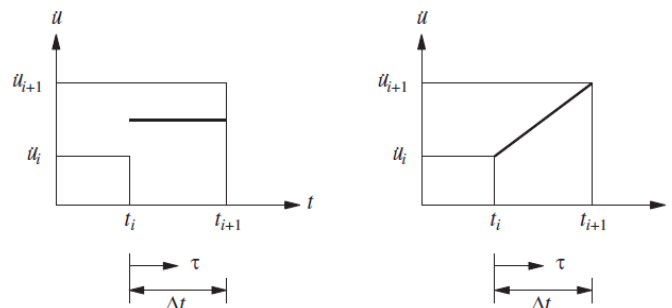
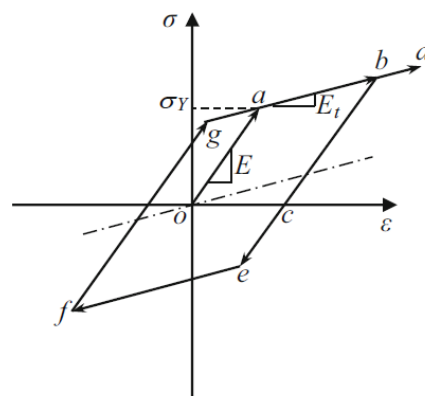


Figure 1: Constant and linear acceleration method for the dynamic solver. Each method describes how the acceleration varies between two consecutive time steps.

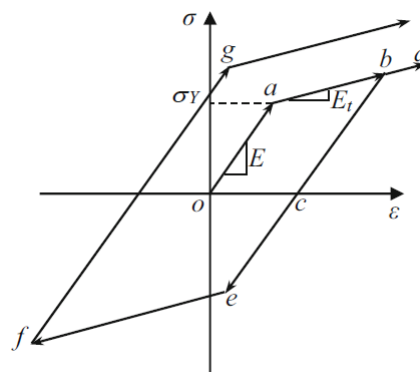
3.3 Hardening models

Several different hardening models have been proposed for determining the yield stress in cyclic loading situations. Two of the most used models, are known as the kinematic and the isotropic hardening model.

The kinematic hardening model assumes that the elastic range, means twice the initial yield stress, remains constant. In the isotropic hardening model, the magnitude of yield stress for the reversed loading is equal to that of the previous yield stress. Note that the materials discussed always receive as input the strain ε and give as output the stress σ ($\sigma = f(\varepsilon)$). Although there is no fundamental restriction to have models of the $\varepsilon = f(\sigma)$ form, the structure of FEA software makes the use of such materials considerably and unnecessarily more complicated. Many practical materials show a combined effect of isotropic and kinematic hardenings. In such a case, the yield stress initially increases due to plastic hardening, but it decreases when the direction of strain changes. This phenomenon is related to the dislocation structure in the cold worked metal.



Kinematic hardening



Isotropic hardening

Figure 2: Hardening models for elastoplasticity, (a) Kinematic hardening, depending yield stress in the opposite direction on the last reverse point, (b) Isotropic hardening, same yield stress in both directions.

4 THE DUAL NATURE OF GEOMISO DNL

The recently developed Geomiso DNL software is not just a plug-in, but a new software having a dual CAD/CAE nature. Users can define, through its modern graphical user interface, the geometry, mesh, refinement, material, quadrature, constraints, loading, and analysis parameters, in order to calculate the stiffness matrix, the pseudo-displacements, as well as the contour plots of displacement, strain, and stress fields. The geometrical data include control variables, polynomial orders, number of univariate basis functions and knot value vectors for each of the three parametric directions (ξ , η , ζ).

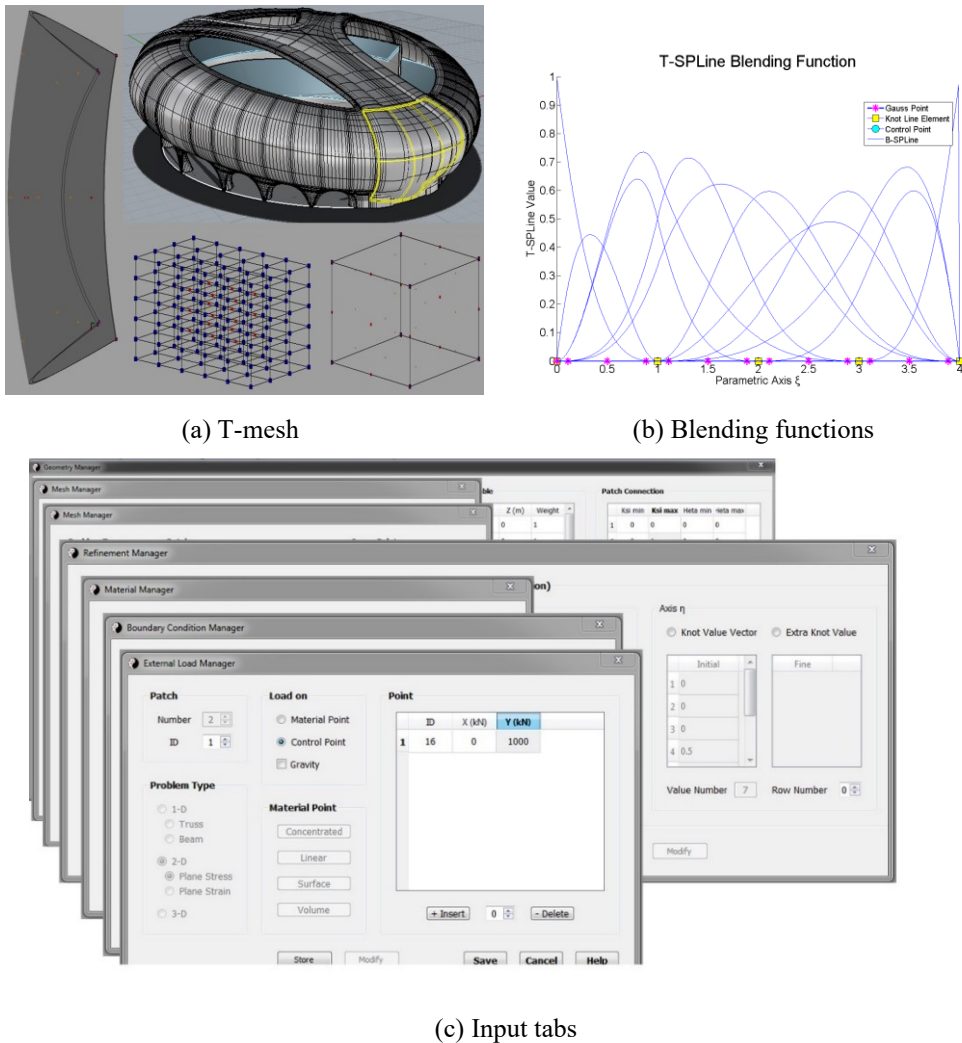
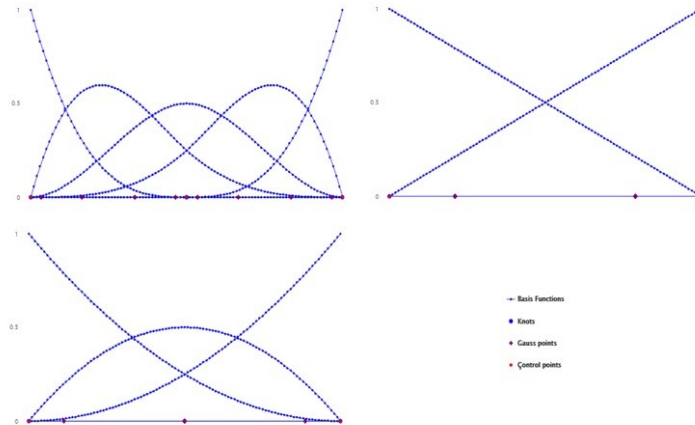


Figure 3: Geomiso DNL graphical user interface. (a) The T-mesh for a part of stadium’s roof (3D problem) using a single patch is depicted. The model is shown in index, parameter and physical space. Control points are shown in red, while knots in blue. (b) T-spline blending functions for ξ parametric axis. Gauss points are shown in purple. (c) Hybrid input tabs. User define the initial coarse exact mesh of the geometry, which is used for both design and analysis. Geomiso DNL help all users leverage its simulation capabilities, while increasing speed and productivity for the entire product development process.

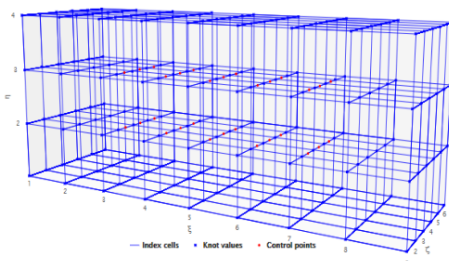
5 APPLICATIONS

5.1 Rectangular beam

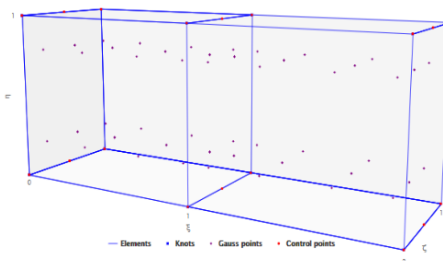
Figure 4 depicts a steel ($E=200.000$ MPa, $\nu=0.3$) beam in index, parameter, and physical space.



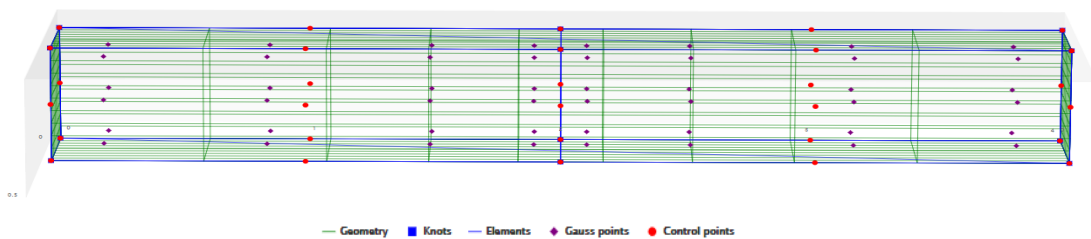
(a) Basis functions



(b) Index space



(c) Parameter space



(d) Physical space

Figure 4: A 3D NURBS model of a rectangular beam, designed in Geomiso DNL.

- (a) The basis functions are depicted as blue lines, with control points in red and Gauss points in grey.
- (b) Index cells are presented as blue lines, while knots as blue points. (c) Parameter space.
- (d) Undeformed configuration in physical space. Control points are shown in red, knots in blue., and Gauss points in purple.

5.2 Plate with a circular hole

A plate with a circular hole with thickness equal to 0.2 m, width 10 m, and height 10 m, is depicted in **Figure 5**.

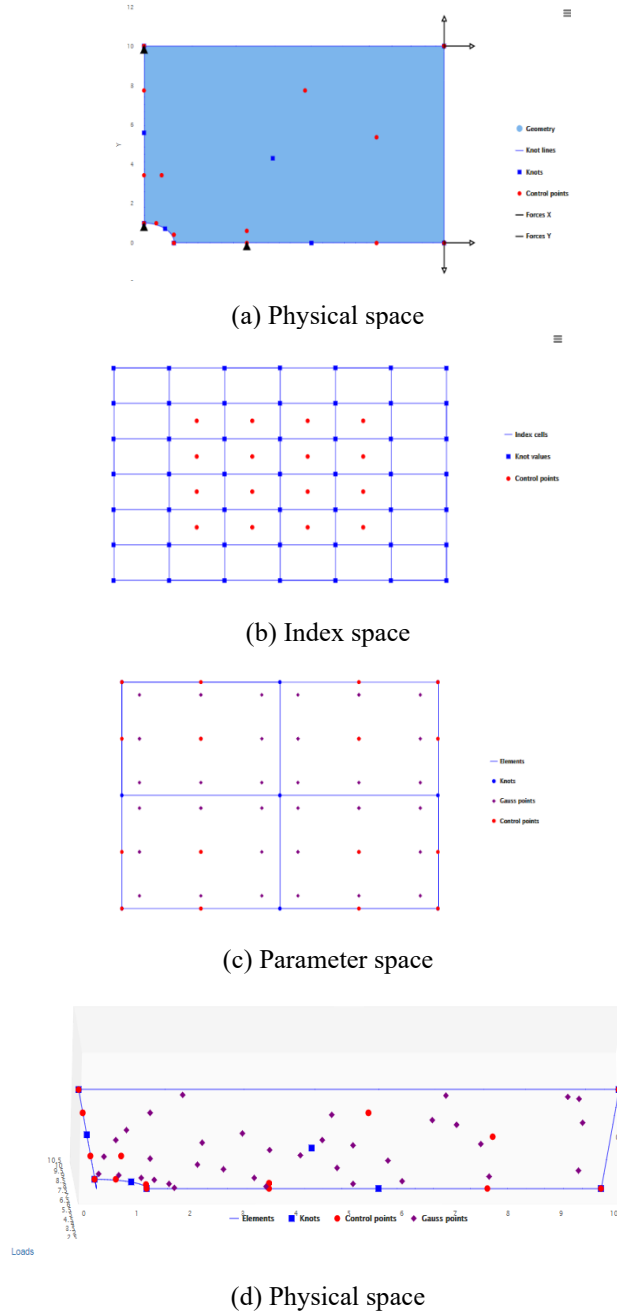
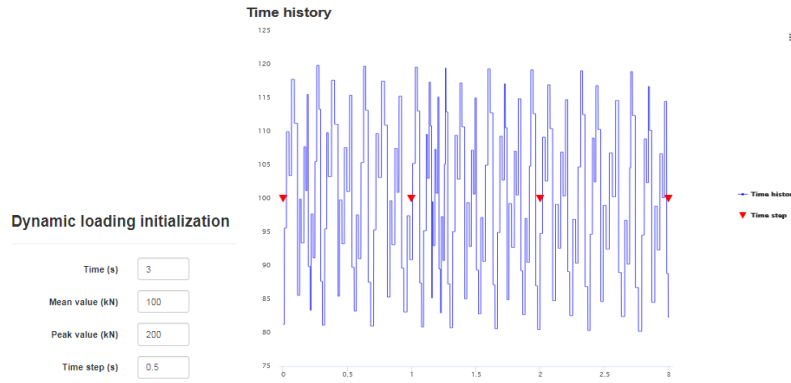


Figure 5: A 2D NURBS model of a circular plate with a hole, designed in Geomiso DNL.

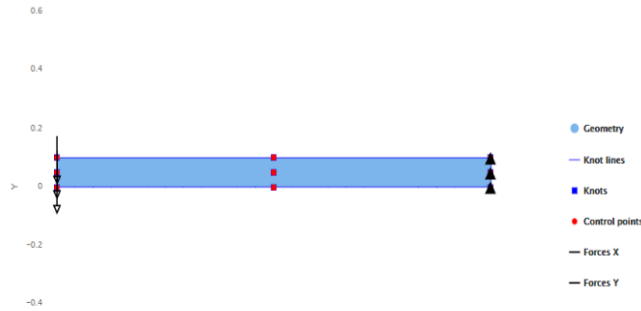
- (a) Physical space. Control points are shown in red, while knots in blue.
- (b) Index space. (c) Parameter space with elements, knots, Gauss points, and control points.
- (d) Undeformed configuration in physical space. Gauss points are shown in purple.

5.3 Transient analysis on a rectangular plate

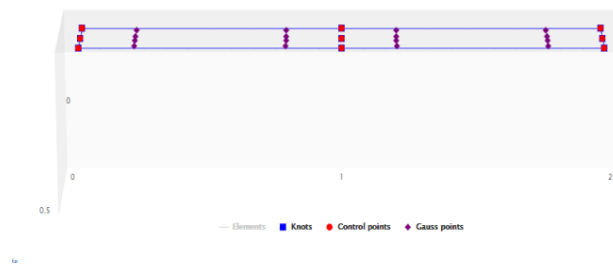
The time history in **Figure 6a** is used as loading pattern for the rectangular plate, which is shown in **Figure 6b**. It is applied on the free edge of the plate in Y axis with negative direction, while the other edge is fixed.



(a) Time history for the transient analysis



(b) Physical space



(c) Knot, control point, Gauss points in physical space

Figure 6: A 2D NURBS model of a rectangular plate, designed in Geomiso DNL.

- (a) Time history pattern for the transient analysis. The total loading duration time is equal to 3s. The mean value is equal to 100 kN, while the peak is 200 kN. The time step is chosen to be equal to 0.5s.
- (b) Geometry, control points, knots, knot lines, constraints, and loading in physical space.
- (c) Undeformed configuration of the plate in the physical space. Control points are shown in red, knots in blue, and Gauss points in purple.

The deformed shape of the plate at every time step of the loading pattern, is depicted in Figure 7.

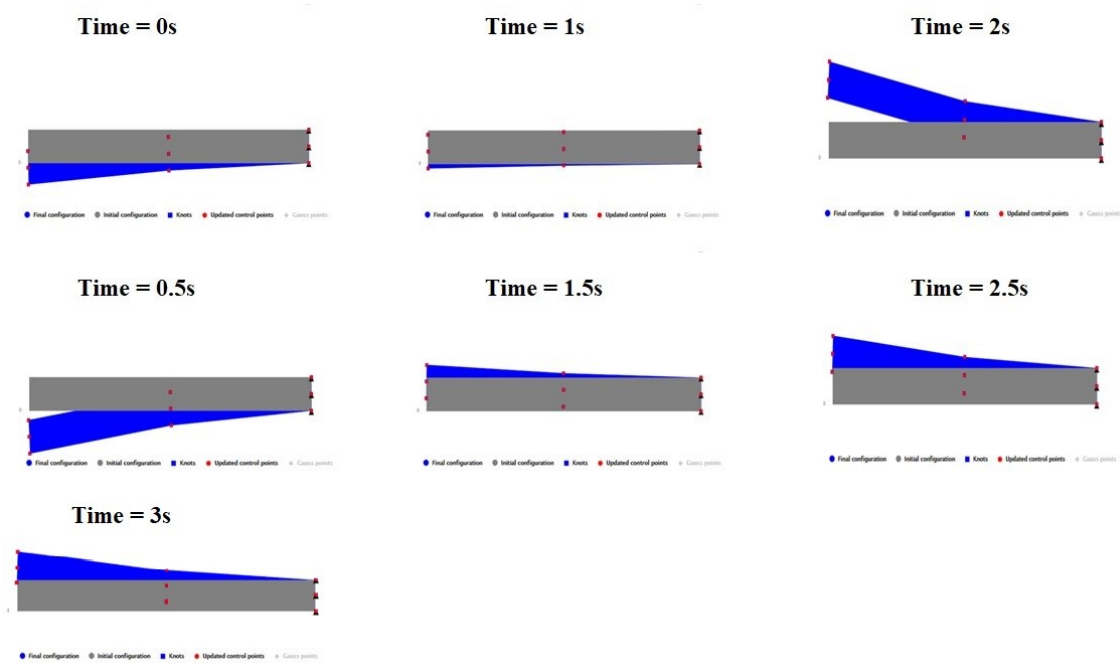


Figure 7: Deformed configuration of the rectangular plate at different time steps. The final configuration is depicted in blue, while the initial in gray. Knots are presented as blue squares and the updated control points as red circles. There are seven deformed shapes, as the total duration of the time history is 3s and the time step is 0.5s.

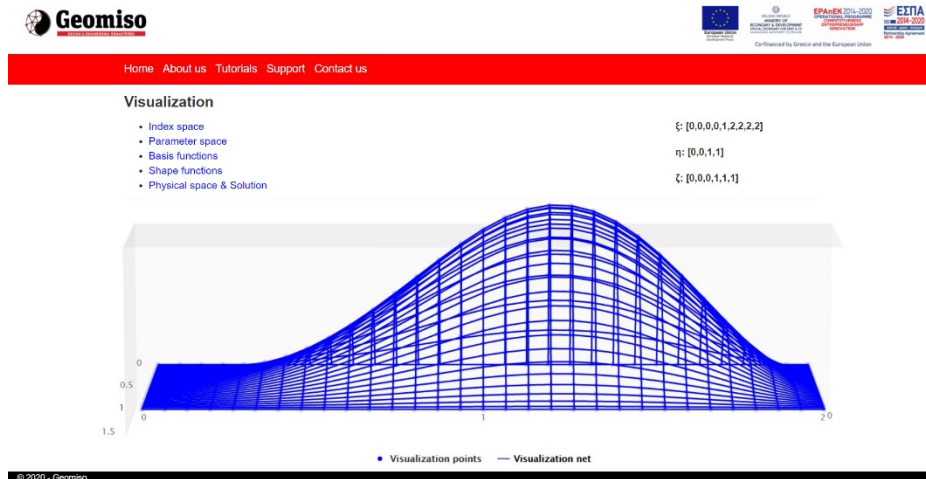


Figure 8: The world's first cloud-based simulation platform for IGA, www.geomiso.cloud. This online software solution can design and analyze structures, and products, with demanding loading, geometries, or material properties in a cloud environment. As soon as the simulation is complete, users can access the results on the platform or download them locally.

6 CONCLUSIONS

- The new Geomiso DNL software is a hybrid software solution, which combines isogeometric analysis and 3D design with NURBS and T-splines, for applications to nonlinear inelastic dynamic problems, as it facilitates the geometry modeling within analysis, exactly preserves geometry at all refinement levels ensuring that its detailed features can be retained without excessive mesh refinement, offers superior quality numerical results, and allows significant time savings by avoiding time-consuming repetitions in creating new models for newly imported modified geometries. This is the first time ever such a hybrid software has been developed.
- Applications to 2D and 3D dynamic problems in structural mechanics have proved successful. The matrix assembly, solver time, as well as the accuracy of the displacement, strain and stress fields is particularly noteworthy. Geomiso DNL represents remarkable improvements over traditional finite element programs thanks to its dual nature, as higher accuracy and stability level in analysis is accomplished with considerably shortened computational cost.

7 ACKNOWLEDGEMENTS

This research has been co-financed by the European Union and Greek national funds through the Operational Program Competitiveness, Entrepreneurship, and Innovation, under the call RESEARCH – CREATE – INNOVATE (project code: T2EDK-00338).



Co-financed by Greece and the European Union

8 REFERENCES

- [1] Hughes, T.J.R., Cottrell, J.A. and Bazilevs, Y. *Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement*. Comput. Methods Appl. Mech. Engrg. (2005) 194:4135–4195.
- [2] Karatarakis, A., Karakitsios, P. and Papadrakakis, M. *GPU accelerated computation of the isogeometric analysis stiffness matrix*. Comput. Methods Appl. Mech. Engrg. (2014) 269:334-355.
- [3] Bazilevs, Y., Calo, V.M., Cottrell, J.A., Evans, J.A, Hughes, T.J.R., Lipton, S., Scott, M.A. and Sederberg, T.W. *Isogeometric analysis using T-splines*. Comput. Methods Appl. Mech. Engrg (2010) 199:229-263.
- [4] Elguedj, T., Bazilevs, Y., Calo, V.M. and Hughes, T.J.R. *\bar{B} and \bar{F} projection methods for nearly incompressible linear and non-linear elasticity and plasticity using higher-order NURBS elements*. Comput. Methods Appl. Mech. Engrg. (2008) 197:2732-2762.



Co-financed by Greece and the European Union