A MONTE CARLO SAMPLING STRATEGY FOR THE AUTOMATED OPERATIONAL MODAL ANALYSIS OF ROAD BRIDGES

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Key words: Bridge Monitoring, Structural Health Monitoring, Environmental and Operational Variability, Automated Operational Modal Analysis.

Abstract. Automated Operational Modal Analysis (AOMA) is a highly convenient technique to identify the modal properties of a target system, based only on its measured output and without human supervision. In particular, AOMA is very useful for permanent and continuous bridge monitoring, as it would otherwise be impractical to perform input-output dynamic testing on such large and complex structures or to manually process the acquisitions on a daily basis.

Nevertheless, its implementation requires a fairly articulated algorithm, made up of several steps. Some of them have been well-optimised throughout the years thanks to contributions by many researchers. Other aspects, however, are still open to improvements.

Specifically, the standard AOMA procedure operates on the so-called stabilisation diagram, i.e. a complete set of identified dynamic properties for different model orders. Traditionally, the model order n is increased from an initial (and arbitrary) minimum, n_{min}, up to a similarly arbitrary maximum n_{max}, with a constant step and no omissions.

However, feeding the AOMA algorithm with all the models included in the $[n_{min},n_{max}]$ range is here proved to not be the most efficient course of action. Instead, a Monte Carlo Sampling strategy is proposed, randomly picking a set of models with order $n \in [n_{min}, n_{max}]$. This is verified on an experimental dataset, the Z24 bridge, to provide comparable results in terms of accuracy and at a lower computational cost.

1 INTRODUCTION

For the System Identification (SI) and vibration-based Structural Health Monitoring (SHM) of large bridges, flyovers, and viaducts, traditional experimental modal analysis (EMA) methods are almost always unfeasible due to logistical and/or practical constraints. In these cases where input-output identification is not doable, Operational Modal Analysis (OMA) is an output-only alternative to the classic EMA framework.

Conceptually, in the OMA framework, recordings from Ambient Vibration (AV) tests are assumed as obtained with a white Gaussian noise (WGN) input [1]. With this assumption, it is possible to test large and massive civil structures and infrastructures without the need for

extensive systems to provide a controlled input force. For further details, the core concepts of OMA are well documented and detailed in [2] and the textbook of [3].

Automated OMA (or AOMA for short) [4] represents the logical extension of this procedure. In this case, not only the need for a human technician to perform the on-site test is removed. Even the need for a human expert to process and evaluate the collected data is bypassed, thanks to Machine Learning (ML) multi-stage clustering approaches. This is essential when large quantities of data have to be analysed, for instance with a permanent continuous monitoring system [5], [6]. Importantly, applications of AOMA are not limited exclusively to damage detection and SHM [7]. The identified modal parameter can also be used e.g. for finite element model calibration and model updating. Some examples of applications specifically tailored for different bridge typologies can be found in [8].

1.1 The current state of AOMA procedures

Any AOMA procedure involves processing raw vibration time histories, jointly using signal processing and ML-based data analysis algorithms to extract modal parameters. The process involves several steps, including data preprocessing, identification of the modal parameters, sifting of the identified parameters, and finally estimation of cluster-wide values. Some well-described examples can be found in [9] and [10].

Overall, this process flow has been used and tested throughout many years and it still stands, mostly unaltered, since its early conception. Nevertheless, many technical aspects of the current AOMA procedure are still non-optimised and leave room for further ameliorations. The one aspect addressed in this work concerns the set of model orders to be included in the so-called *stabilisation diagram*.

In brief, the Covariance-driven Stochastic Subspace Identification (SSI-Cov)[11], [12], adopted here for System Identification, requires the user to preliminarily define the number of modes to be identified. This input parameter is known as the model order n.

Since the number of relevant vibration modes of the target system is not known a priori, the strategy is to run the SSI-Cov identification process multiple times, starting from a minimum value (n_{min}) and moving upward to a maximum model order (n_{max}).

On the one hand, using a short range limited to the lowest model orders would miss many higher order and/or weakly excited modes. However, on the other hand, when running an overcomplicated model on noise-affected data, misidentifications are inevitably included in the results. That is to say, spurious identifications (due to overfitting over noisy data) are expected to appear at higher model orders. Thus, n_{max} should be large enough to allow physical but weak modes (e.g. the bridge lateral modes) to be discerned. Conversely, n_{min} should be low enough to create a consistent base of stable poles to help discern physically-meaningful modes from spurious (mathematical) ones. In fact, since all the model orders in between $n_{min} \leq n \leq n_{max}$ are included, these are used in the sifting procedure. The rationale is that physical modes will be found in any identification (or almost any identification), while the spurious modes should be more randomly distributed.

Thus, as mentioned in the previous section, the setting of both n_{min} and n_{max} is critical but hard to automate. Generally, a case-specific study is needed, often resorting to human-made

trial and error or complex data-driven parameter setting.

In this work, an alternative is proposed, based on a Monte Carlo (MC) Sampling strategy. This enables selecting only a fraction of the whole array of model orders included in the $n_{min} \leq n \leq n_{max}$ range. The main improvement of this proposed approach with respect to the conventional procedure is that, as it will be shown, it can produce results comparable to the whole $[n_{min}, n_{max}]$ range with many fewer input data.

2 METHODOLOGY

The proposed approach revolves around the standard AOMA procedure, using an MC Sampling strategy instead of the whole range of model orders. To this aim, the algorithm described in [13] and applied to masonry arch bridges in [14] has been used as both the base of the AOMA version tested here and as a benchmark. The algorithm uses a hierarchical clustering approach and evaluates the cluster-wide modal parameters as the mean of the elements included inside the same cluster. Hereinafter, this benchmark algorithm will be referred to as the 'full-range deterministic' approach, as it includes the complete range of model orders.

For direct comparability, all the steps related to the sifting of the SSI identifications are kept identical for the deterministic and the MC Sampling-based variants of the algorithm. Thus, also due to space limitations, these will not be recalled here. The interested reader can refer to [13] for a complete description of the hard and soft validation criteria (HVC and SVC) and the other technical details.

The general idea of the proposed MC Sampling-based AOMA is to sample the parameter input space (here, the model order $n \in [n_{min}, n_{max}]$), perform the identification with the sampled input, and get the corresponding sampled output. The underlying concept is to use the randomness of repeated sampling to obtain results that might be deterministic in principle – here in this case, the equivalent modal parameters corresponding to the same procedure run on the complete $n_{min} \leq n \leq n_{max}$ range.

This procedure is statistically valid as long as all samples belong to the same distribution. That is exactly what is expected, as physically meaningful modes should be obtainable as the mean of the corresponding clusters (that is to say, distributions) for all model orders.

Operatively, the Matlab function `randperm()` was used to generate a random permutation of an arbitrary quantity K of integer numbers, without repeating elements, uniformly distributed between n_{min} and n_{max}. Such arrays are also known as *K-permutations without replacement*. Importantly, as no number produced by a mathematical operation can truly be random, actually *pseudo*random integers numbers are generated, using the Mersenne Twister generator [15] for a given seed. To verify the statistical validity of the results, four different seeds have been considered for the (pseudo)random number generation in MatLab: `rng(1234)`, `rng(3456)`, `rng(5678)`, and `rng(7890)`.

3 CASE STUDY

The experimental data from the Z24 road bridge benchmark (Figure 1) have been used to validate the proposed approach [16]. This is a well-known case study, extensively studied in

the scientific literature. The Z24 bridge was a classical post-tensioned reinforced concrete box-girder bridge; it was located in Switzerland, linking the villages of Koppigen and Utzenstorf and overpassing the A1 highway tract between Bern and Zurich. Before demolition, a controlled experiment was conducted, inserting progressively damage into the infrastructure by means of lowering one of the two central piers. A detailed description of the infrastructure and the experimental campaign conducted on it can be found in [17]–[19]. Recordings of the dynamic response of the bridge under ambient vibrations were taken at hourly intervals before and after the introduction of damage. For this work specifically, Acquisition 43A08 is considered. This corresponds to the recordings at 08:00 on 30 August 1998, i.e. the last day after the complete lowering of the pier (95 mm) before the failure of the concrete hinge [17].
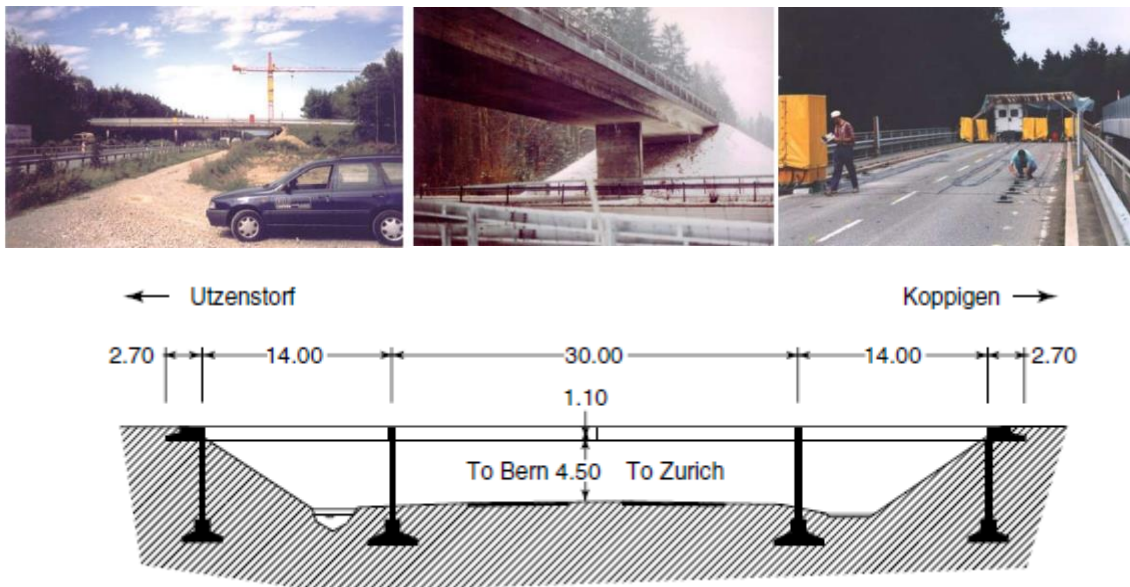

Figure 1: Top: The Z24 bridge (pictures retrieved from [20] and [21]). Bottom: schematics of the side view of the same (retrieved from [17]; distances in metres). The damage was inserted in the second pier from the Koppigen side.

3 RESULTS

In Figure 2, an example of a subset of pseudorandomly selected model orders is portrayed in comparison to the Traditional Stabilisation Diagram (TSD), as obtained with the deterministic approach and including the complete set of model orders. In this study, this ranged from $n_{min} = 10$ to $n_{max} = 160$. These upper and lower bound have been defined for comparability with the results reported in Table 3 of [22], which corresponds to the application of the deterministic hierarchical algorithm to the same signal and with the same settings and parameters. Please note that in this case, $M = 76$ model orders were used, since n increases in steps of 2 (this is due to the model order corresponding to the number of complex conjugate poles). Since a single natural frequency corresponds to any pair of poles, $n_{min} = 10$ returns five identifications, and so on.

The aforementioned values will serve as the targets and are, in this order, $f_1 = 3.89$ Hz, $f_2 =$

4.74 Hz, $f_3 = 9.89$ Hz, $f_4 = 10.39$ Hz, $f_5 = 12.34$ Hz, and $f_6 = 13.47$ Hz. The six identified modes correspond to, in the same order as before, the first bending mode (f_1), the first lateral mode (f_2), the first mixed bending-torsion mode (f_3), the second mixed bending-torsion mode (f_4), the second bending mode (f_5), and the third bending mode (f_6).
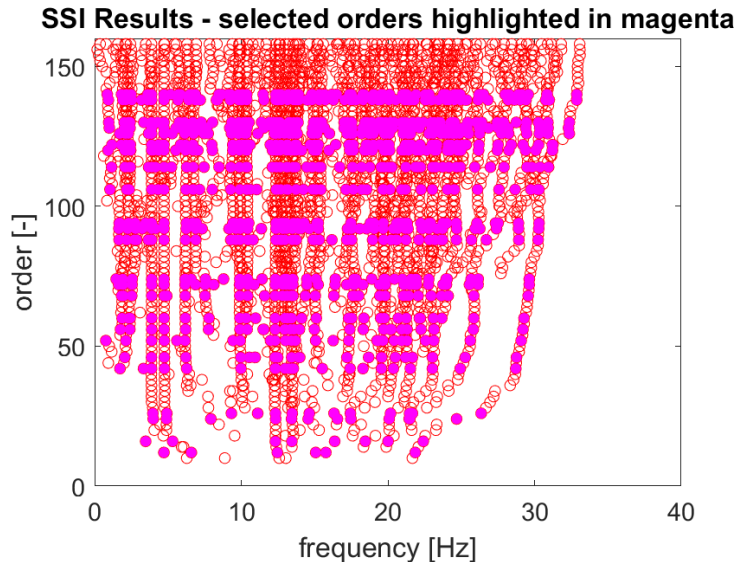
Figure 2: An example of MC Sampled Stabilisation Diagram superimposed to a Traditional Stabilisation Diagram. $M = 25$ model orders (highlighted in magenta) are pseudorandomly selected, out of the 76 ones defined in the full range between $n_{min} = 10$ and $n_{max} = 160$ (red circles).

In Figure 3, the results are also compared to the estimates obtained with three reduced-range deterministic alternatives. Indeed, to guarantee the fairest comparison, these three different options have been considered:

- 'low range', defined by starting from the fixed $n_{min} = 10$ and increasing n_{max} according to the number of samples considered ($n_{max} = n_{min} + 2(M-1)$, e.g. $n_{max} = 58$ for $M = 25$ samples);

- 'high range', defined by finishing at the fixed $n_{max} = 160$ and decreasing n_{min} according to the number of samples considered ($n_{min} = n_{max} - 2(M-1)$, e.g. $n_{min} = 22$ for $M = 70$ samples);

- 'mid range', defined by considering a range of model orders centred around $n_{mid} = 84$ ($n_{min} = n_{mid} - M + 2$, $n_{max} = n_{mid} + M$, e.g. $n_{min} = 36$ and $n_{max} = 134$ for $M = 50$ samples).

The rationale is that, as previously mentioned, low model orders are expected to perform relatively poorly for weakly excited modes, such as f_2, while higher model orders might underperform due to the inclusion of some spurious modes after the automated clustering operation.

For completeness, two benchmarks from the authors of the original dataset are reported in Figure 3 alongside the target values: (a) from [4] and (b) from [17]. While it is unspecified in

the respective sources to which recordings they specifically belong, it is safe to assume that both identifications refer to the same damage state as the one investigated here.

To assess the capabilities of the MC Sampling-based approach to returning accurate results with fewer samples, 10 cases have been considered, by reducing the number of model orders from $M = 70$ to 25 in steps of 5. The same amounts of samples have been then used for the reduced-range deterministic approach as well; i.e., it was considered $K = M$ for all cases.

Since, as mentioned, the pseudorandom number generation was run four times with different seeds, the mean value μ is reported as a blue circle, alongside with the interval $\mu \pm \sigma$, represented by the horizontal bar of the same colour. This indicates the variability of the results for different seeds.
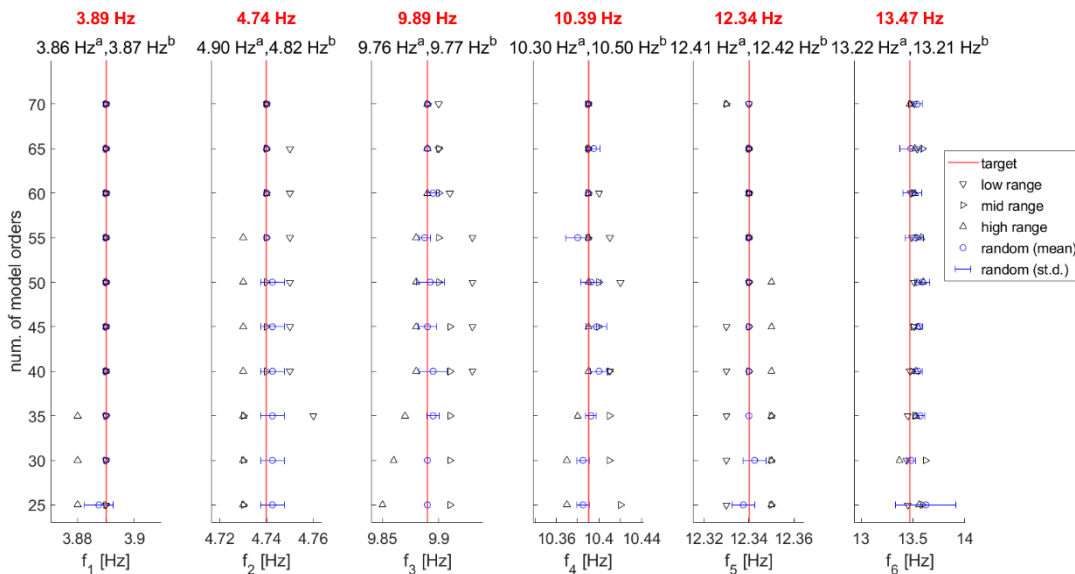
Figure 3: Estimated natural frequencies for the MC Sampling AOMA procedure, in comparison with the range-reduced deterministic alternatives and the target values (full-range deterministic AOMA).

4 DISCUSSIONS

From the error bars depicted in Figure 3, it is evident that the variability of the results for different seeds is quite contained, except for the higher modes and the smallest sets of samples. This good repeatability of the results indirectly proves the validity of the assumption made at the beginning, which is to say, that all samples belong to the same distribution.

Table 1 quantitatively analyses the results qualitatively shown in Figure 3. Specifically, for each natural frequency and each number M of samples, the best-performing alternative is reported ("PSEUDO" stands for pseudorandom, while "LOW", "MID", and "HIGH" indicates the three reduced-range deterministic alternatives). The Table also reports the absolute error between the best-performing identification and the target value (rounded up to the second decimal digit). When two or more alternatives performed similarly, these are all indicated equally.

It can be clearly seen that, obviously, the first mode is the easiest to identify; all options,

except for the high range deterministic variant at a very low number of model orders, perform equally well. In that case, also the MC Sampling-based approach can be skewed when, by chance, a larger number of higher model orders are included.

Table 1: Best performing option for each natural frequency and number of samples.

	f_1	f_2	f_3	f_4	f_5	f_6
$K = M = 70$	"PSEUDO, LOW, MID, HIGH"	"PSEUDO, LOW, MID, HIGH"	"PSEUDO, MID, HIGH"	"PSEUDO, LOW, MID, HIGH"	"PSEUDO, LOW"	"HIGH"
	<0.01 Hz	<0.01 Hz	<0.01 Hz	<0.01 Hz	<0.01 Hz	<0.01 Hz
65	"PSEUDO, LOW, MID, HIGH"	"PSEUDO, MID, HIGH"	"PSEUDO, HIGH"	"LOW, MID, HIGH"	"PSEUDO, LOW, MID, HIGH"	"PSEUDO"
	<0.01 Hz	<0.01 Hz	<0.01 Hz	<0.01 Hz	<0.01 Hz	0.01 Hz
60	"PSEUDO, LOW, MID, HIGH"	"PSEUDO, MID, HIGH"	"HIGH"	"PSEUDO, MID, HIGH"	"PSEUDO, LOW, MID, HIGH"	"LOW"
	<0.01 Hz	<0.01 Hz	<0.01 Hz	<0.01 Hz	<0.01 Hz	0.01 Hz
55	"PSEUDO, LOW, MID, HIGH"	"PSEUDO, MID"	"PSEUDO"	"MID, HIGH"	"PSEUDO, LOW, MID, HIGH"	"LOW"
	<0.01 Hz	<0.01 Hz	<0.01 Hz	<0.01 Hz	<0.01 Hz	0.02 Hz
50	"PSEUDO, LOW, MID, HIGH"	"PSEUDO, MID"	"PSEUDO"	"PSEUDO, HIGH"	"PSEUDO, LOW, MID"	"LOW"
	<0.01 Hz	<0.01 Hz	<0.01 Hz	<0.01 Hz	<0.01 Hz	0.04 Hz
45	"PSEUDO, LOW, MID, HIGH"	"PSEUDO, MID"	"PSEUDO"	"HIGH"	"PSEUDO, MID"	"MID"
	<0.01 Hz	<0.01 Hz	<0.01 Hz	<0.01 Hz	<0.01 Hz	0.03 Hz
40	"PSEUDO, LOW, MID, HIGH"	"PSEUDO, MID"	"PSEUDO, HIGH"	"HIGH"	"PSEUDO, MID"	"LOW"
	<0.01 Hz	<0.01 Hz	0.01 Hz	<0.01 Hz	<0.01 Hz	<0.01 Hz
35	"PSEUDO, LOW, MID"	"PSEUDO"	"PSEUDO"	"PSEUDO"	"PSEUDO"	"LOW"
	<0.01 Hz	<0.01 Hz	0.01 Hz	<0.01 Hz	<0.01 Hz	0.02 Hz
30	"PSEUDO, LOW, MID"	"PSEUDO"	"PSEUDO"	"PSEUDO"	"PSEUDO"	"PSEUDO"
	<0.01 Hz	<0.01 Hz	<0.01 Hz	<0.01 Hz	<0.01 Hz	0.01 Hz
25	"PSEUDO, LOW, MID"	"PSEUDO"	"PSEUDO"	"PSEUDO"	"PSEUDO"	"LOW"
	<0.01 Hz	<0.01 Hz	<0.01 Hz	<0.01 Hz	<0.01 Hz	0.02 Hz

Among all the six modes inspected, the second one is the most interesting. Here, it is evident

how the high range option performs worse at a relatively low number of model orders, for the same reasons as before (too few stable modes to properly discern spurious and physical modes). On the other hand, the low range option performs even worse for the same very short ranges (the identifications for $M = 25$ and 30 samples are, respectively, 4.78 Hz and 4.77 Hz, too far from the target to be even visible in Figure 3). The results improve for larger sets but reach sufficiently good accuracy only for $M = 70$ samples, which is the last step before the full range (76 model orders). Instead, the pseudorandom MC Sampling approach ensures that the estimated value is never too far from the actual target, even for this very weakly excited lateral mode and for a reduced set of only $M = 25$ samples, thus outperforming all the other options. The results are, however, relatively quite variable, with a standard deviation up to $\sigma = \pm 0.05$ Hz.

On the other hand, even when the proposed MC Sampling AOMA does not outperform its equivalent (i.e. reduced-range) deterministic counterparts, it never deviates excessively from the expected outcome. Only in one instance, for the highest mode (f_6) and the smallest set of model orders ($M = 25$), the absolute difference is larger than 1 Hz. Conversely, in three out of six modes (f_1, f_2, and f_3) the error is < 0.01 Hz for any number of samples, even considering only $M = 25$ model orders (less than a third of the complete set).

5   CONCLUSIONS

Automated Operational Modal Analysis is well-known as an effective technique for the vibration-based inspection and continuous Structural Health Monitoring of bridges and viaducts. Nevertheless, many aspects of the standard procedure are still open to further improvements, both to enhance the accuracy of the results and to optimise the computational efficiency of the algorithm.

In this work, a Monte Carlo Sampling strategy has been proposed to avoid the use of a large number of model orders to discern spurious identifications from physically meaningful ones.

In particular, it has been shown that:

- Even for weakly excited modes, using a very small set of M model orders (pseudo)randomly selected from a large $[n_{min}, n_{max}]$ range outperforms a continuous set of similar size. This has been verified regardless of whether $[n_{min}, n_{min} + M]$, $[n_{max} - M, n_{max}]$, or any other range continuously defined in between these two extremes is used.

- The MC Sampling strategy can reach the same accuracy as the complete full-range $[n_{min}, n_{max}]$ set with far fewer samples.

The main insight is, therefore, that including all model orders between $[n_{min}, n_{max}]$ is too redundant and not very efficient. This classic procedure is also prone to errors induced by the subjective preconceptions of the user, who can under or overestimate these two extreme values. Instead, it is here proposed to prefer a large range, using very low n_{min} and/or very high n_{max}, while reducing the amount M of samples taken from it.

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