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ON THE PSEUDOMATERIAL APPROACH FOR THE ANALYSIS OF TRANSIENT FORMING PROCESSES

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SUMMARY

A fixed-mesh method for the analysis of transient forming processes is presented. The mesh covers material regions and zones through which the material may flow. These last zones are identified by a pseudomaterial with relatively small physical parameters. During time processing, the interface between both materials is followed by an arbitrary Lagrangian mesh. This technique appears to be suitable for the treatment of moving surfaces with sharp corners. A particular boundary condition for the Navier–Stokes equations is also introduced in order to model a porous wall.

KEY WORDS pseudomaterial; forming processes; porous wall; moving surfaces

INTRODUCTION

Forming processes are currently solved by an Eulerian flow formulation in which the material is treated as a non-Newtonian fluid.[1,2] This is a good approach when plastic strains are large enough so that elastic strains are negligible. When transient processes need to be modelled, changes in the geometry and moving free surface have to be considered. The most intuitive procedure to solve this problem is the Updated Lagrangian method where the mesh for a given position is updated at each time step in terms of the instantaneous velocity field.[2] This method has the disadvantage that the mesh becomes distorted during the analysis and some forms of remeshing are required. Furthermore, complex configuration with the creation of new free boundary surfaces are not possible. An alternative approach is the so-called Arbitrary Lagrangian–Eulerian method in which the mesh is deformed in terms of an arbitrary velocity independently of the flow velocity except at the boundary. This method is very versatile but requires some experience by the user to be successful.[3]

A simple and original approach to solve this problem is the so-called pseudoconcentration method proposed by Thompson.[4,5] This is a fixed-mesh approach in which the mesh is defined in such a way that it covers both the material and the region through which the material will flow. In regions where the material is flowing, appropriate physical variables (viscosity, density, weight, etc.) are used. In those regions where the material has not yet penetrated, an artificial low value of the physical parameters is used. Furthermore, the method assigns a new variable (the pseudoconcentration) throughout the mesh in such a manner that its value indicates the presence or absence of material. This concentration is updated at each time step by setting its material derivative equal to zero.

The main advantage of the pseudoconcentration method is the use of a fixed mesh. This implies economy, no distorted elements and easy treatment of complex free surface geometries

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with the possibility of creation of new free surfaces. Furthermore, the definition of the pseudoconcentration to convex free surfaces has the advantage of simplicity, and it does not require user's manipulations during the analysis. The robustness and conservative character of this method have been proved in several works.[6,7]

Nevertheless, in some particular forming problems, the pseudoconcentration method gives unacceptable results and *ad hoc* user's manipulations are needed to overcome these difficulties. In particular, this situation appears when the free surface presents corners (a classical problem in metal-forming processes). Corners represent a singularity in the definition of a smooth concentration function.

Even with re-smoothing techniques, the treatment of corners with a single concentration function implies that corners become round or numerical instability appears. This drawback leads to erroneous results. Corners can be treated with two different pseudoconcentration functions, one of each for each side of the corner, but this solution is not applicable when several corners are present or in 3D problems.

Another classical problem connected with fixed-mesh methods appears in the treatment of injection in closed moulds. Owing to the fact that the incompressibility condition is imposed in the whole domain, the pseudomaterial must escape or become compressed when the mould is filled. This difficulty appears also in some contact problems when internal bubbles of pseudomaterial remain confined between the material and the mould walls. Holes on the boundary or partial elimination of the boundary condition have been proposed to overcome this difficulty.[8]

In this paper we use a fixed mesh with a fictitious material (pseudomaterial) as in the original method proposed by Thompson,[4,5] but we use an independent Lagrangian mesh to follow the free surface.

Thus, a fixed finite element mesh is initially defined covering both regions, one region with the material and the other one containing the pseudomaterial. On the interface, a surface mesh which may or not coincide with the fixed mesh will be defined. The surface mesh is defined in such a way that it can identify points which are outside the surface as pseudomaterial, and inside the surface as real material.

After each time step the co-ordinates of the surface mesh are updated with the instantaneous velocity field evaluated on this surface. In this way, we preserve the advantage of fixed-mesh methods in that the finite element mesh never becomes distorted. Besides, we have also the advantage of Lagrangian methods concerning the representation of moving corners. The flow problem is exactly the same as in the pseudoconcentration method but the evaluation of the new concentration function is changed, at each time step, by computing the flow velocity on the interface. This is a non-expensive step which must be carried out just on the interface nodes. In 3D problems, the method seems to be very competitive in comparison with standard pseudoconcentration evaluations in which a solution of a non-symmetrical system of equations is necessary on the 3D domain.

The pseudo-material escaping problem in closed moulds is treated by introducing a new boundary condition in the Navier–Stokes equations: the porous boundary condition (PBC). On this boundary, the flow may filter through using a Darcy type law, i.e. the normal velocity is proportional to the pressure gradient, and inversely proportional to the viscosity. In this manner, when the boundary is in contact with the pseudomaterial, the viscosity has a small value and the fluid filters through the wall with a velocity proportional to the flow pressure. When the viscosity grows, i.e. the material contacts the mould walls, the boundary becomes nearly impermeable. This is a more realistic boundary condition and it is in accordance with the physics of the problem.

GOVERNING EQUATIONS

For simplicity we consider here the case of an incompressible viscous creeping flow neglecting the effect of inertial acceleration terms. This situation is typical of many forming problems where the assumption of small elastic strains leads to an analogy between the equations describing the metal deformation process and those of the flow of an incompressible non-Newtonian creeping fluid.[1,2] In addition a quasistatic approach will be chosen where the transient problem is integrated in time as a series of stationary steps. With these assumptions the momentum equations can be simply written as

$$\frac{\partial \sigma_{ij}}{\partial x_j} + \rho b_i = 0 \text{ in } \Omega \tag{1}$$

in which $i = 1, \ldots, n_{\text{dim}}$, $j = 1, \ldots, n_{\text{dim}}$ $(1 \leqslant n_{\text{dim}} \leqslant 3)$, ρb_i are the body forces, and the stress tensor σ_{ij} is given by

$$\sigma_{ij} = -p\delta_{ij} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{2}$$

The unknown functions are the velocity field u_i and the pressure p. The physical parameters are the body force ρb_i and the dynamic viscosity μ, which can be taken as a function of the strain rate, the yield stress, the temperature and the stresses.[1]

The basic formulation of the problem (1) is completed with the incompressibility condition

$$\frac{\partial u_i}{\partial x_i} = 0 \text{ in } \Omega \tag{3}$$

and boundary conditions

$$\sigma_{ij}\mathbf{n}_j = \overline{T_i} \text{ on } \Gamma_{\mathrm{T}}$$

$$\mathbf{u}_i = \overline{\mathbf{u}_i} \text{ on } \Gamma_u$$

where $\mathbf{n}_j$ is the unit outward normal vector to the surface Γ_{T}, $\overline{T_i}$ is the specified surface traction on Γ_{T} and $\overline{\mathbf{u}_i}$ the specified velocity on Γ_u.

PSEUDOMATERIAL METHOD WITH LAGRANGIAN FREE SURFACE UPDATE

Equations (1) and (3) are applied to both parts of the domain: the material part in which the 'true' viscosity and gravitational forces are defined and the pseudomaterial one, through which the material will flow. In this last part, a small value of the viscosity and body forces are used, normally corresponding to the physical properties of air.

The whole domain is divided by a fixed finite-element mesh in which standard interpolation functions may be used. In this paper, isoparametric quadrilateral elements, with biquadratic velocity fields and bilinear discontinuous pressure, were employed. An iterative penalty method to properly satisfy the incompressibility condition was used.[9] In this context, a condensation at element level to eliminate the nodal pressure variables was performed.

Furthermore, an Arbitrary Surface Mesh is defined on the free surface material (Figure 1). This ASM may not necessary match the fixed mesh. An arbitrary number of nodes and elements may be used in this definition for the purpose of locating the moving surface. This ASM must be defined in such a way that a unit normal vector to this surface ($\vec{\mathbf{n}}$) identifies the material and pseudomaterial regions. The physical properties are evaluated at each Gauss integration point according to the sign of the ASM normal vector which determines the

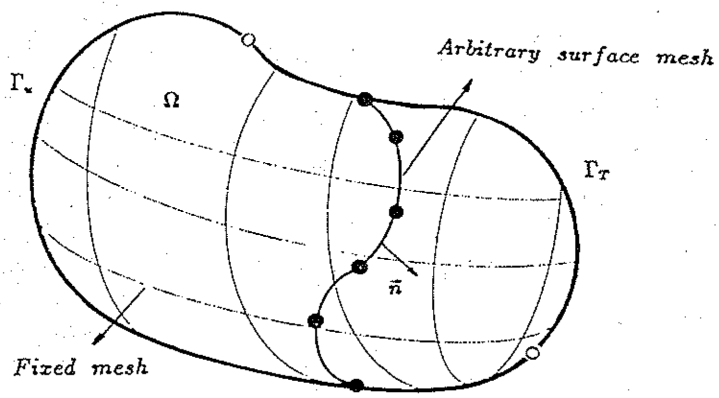

Figure 1.

existence of pseudomaterial at such point. A geometrical search has been implemented for evaluating this location. Once the flow problem is solved, the position of the ASM is updated by a Lagrangian step:

$$x_i^{n+1} = x_i^n + u_{\mathrm{ASM}_i}^{n+1}\,\Delta t \tag{4}$$

in which Δt is the time step, and x_i^n is the position of the ASM node number i at time t_n. The velocities u_{ASM_i} at each node on the ASM are evaluated accordingly to its last position using the standard interpolation functions and nodal velocities of the finite element containing such an ASM node. Therefore, the ASM nodes velocities do not appear directly in the FEM equations.

POROUS BOUNDARY CONDITIONS

A problem connected with the use of fixed-mesh algorithms appears due to the fact that the incompressibility condition is imposed in the whole domain. The pseudomaterial must escape or is compressed in some regions, for instance in the walls of closed moulds. This is a real physical problem which is solved, in practice, using porous moulds through which the pseudomaterial may filter.

Numerically, this problem has been solved using holes in the same specific places, through which the pseudomaterial may flow.[8]

A more realistic boundary condition, in accordance with the physics of the problem, can be introduced to the Navier–Stokes equations. Thus, in those boundaries in which a porous material exists (named Γ_k), we assume that the velocity field is governed by Darcy's law:[10,11]

$$v_n = \frac{k}{\mu}\,(p - p_0) \text{ on } \Gamma_k \tag{5}$$

in which v_n is the velocity normal to the boundary, k the permeability of the porous boundary, μ the viscosity and p_0 an external reference pressure. When the material gets into contact with the boundary, the viscosity grows, and the boundary becomes almost impermeable.

Equation (5) states a linear restriction between the velocity and the pressure field. This may be introduced in a weak form on the boundary as

$$\int_{\Gamma_k} \varphi \left[v_n - \frac{k}{\mu} (p - p_0) \right] \, \mathrm{d}\Gamma = 0 \tag{6}$$

where φ are appropriate weight functions, or, in a local way at each node,

$$v_n^i = \frac{k}{\mu} (p^i - p_0) \tag{7}$$

in which v_n^i and p^i are the normal velocity and the pressure at node i.

Considering v_n and p discretized on the boundary Γ_k by standard finite element shape functions ϕ_i and ψ_i:

$$v_n = \mathbf{\Phi v}_n$$
$$p = \mathbf{\Psi p}$$

in which $\mathbf{v}_n$ and $\mathbf{p}$ are the local values of the normal velocities and pressure, equation (6) becomes (choosing $\varphi_i = \phi_i$)

$$\int_{\Gamma_k} \mathbf{\Phi}^{\mathrm{T}} \mathbf{\Phi} \, \mathrm{d}\Gamma \mathbf{v}_n = \int_{\Gamma_k} \mathbf{\Phi}^{\mathrm{T}} \frac{k}{\mu} \mathbf{\Psi} \, \mathrm{d}\Gamma \mathbf{p} - \int_{\Gamma_k} \mathbf{\Phi}^{\mathrm{T}} \frac{k}{\mu} p_0 \, \mathrm{d}\Gamma$$

or

$$\mathbf{v}_n = \mathbf{Np} - \mathbf{g} \tag{8}$$

with

$$\begin{aligned} \mathbf{N} &= \mathbf{Q}^{-1} \int_{\Gamma_k} \mathbf{\Phi}^{\mathrm{T}} \frac{k}{\mu} \mathbf{\Psi} \, \mathrm{d}\Gamma \\ \mathbf{Q} &= \int_{\Gamma_k} \mathbf{\Phi}^{\mathrm{T}} \mathbf{\Phi} \, \mathrm{d}\Gamma \\ \mathbf{g} &= \mathbf{Q}^{-1} \int_{\Gamma_k} \mathbf{\Phi}^{\mathrm{T}} \frac{k}{\mu} p_0 \, \mathrm{d}\Gamma \end{aligned} \tag{9}$$

Equation (8) represents the linear restrictions between $\mathbf{v}_n$ and $\mathbf{p}$ which must be introduced in the global system.

The spatial finite element discretization of equations (1) and (3) using a standard penalty approach[8,12] can be written as:

$$\begin{aligned} \mathbf{Kv} - \mathbf{H}^{\mathrm{T}}\mathbf{p} &= \mathbf{f} \\ \mathbf{Hv} + \varepsilon \mathbf{Mp} &= \mathbf{0} \end{aligned} \tag{10}$$

All matrices appearing in system (10) are standard in the context of the finite element method (see Reference 12).

The local velocity vector $\mathbf{v}$ is now split into two parts:

$$\mathbf{v} = \begin{bmatrix} \mathbf{v}_n \\ \mathbf{v}_l \end{bmatrix} \tag{11}$$

where local axes must be defined at those nodes in which one of the global axes does not coincide with the direction of the outward normal velocity $\mathbf{v}_n$. Combining (10) and (11) it gives:

$$\begin{aligned} \mathbf{K}_{nn}\mathbf{v}_n + \mathbf{K}_{nl}\mathbf{v}_l - \mathbf{H}_{pn}^{\mathrm{T}}\mathbf{p} &= \mathbf{f}_n \\ \mathbf{K}_{ln}\mathbf{v}_n + \mathbf{K}_{ll}\mathbf{v}_l - \mathbf{H}_{pl}^{\mathrm{T}}\mathbf{p} &= \mathbf{f}_l \\ \mathbf{H}_{pn}\mathbf{v}_n + \mathbf{H}_{pl}\mathbf{v}_l + \varepsilon\mathbf{M}\mathbf{p} &= \mathbf{0} \end{aligned} \tag{12}$$

Using (8) and eliminating the first equations corresponding to the $\mathbf{v}_n$ terms gives:

$$\begin{aligned} \mathbf{K}_{ll}\mathbf{v}_l - (\mathbf{H}_{pl}^{\mathrm{T}} - \mathbf{K}_{ln}\mathbf{N})\mathbf{p} &= \mathbf{f}_l + \mathbf{K}_{ln}\mathbf{g} \\ \mathbf{H}_{pl}\mathbf{v}_l + (\mathbf{H}_{pn}\mathbf{N} + \varepsilon\mathbf{M})\mathbf{p} &= \mathbf{H}_{pn}\mathbf{g} \end{aligned} \tag{13}$$

Once system (13) is solved, vector $\mathbf{v}_n$ is evaluated using (8).

Special care must be taken in solving (13) by a penalty method as indicated above. As is well known the penalty parameter ε must tend to zero to reasonably satisfy the incompressible condition. For a small value of the ratio k/μ, vector $\mathbf{g}$ tends to zero but the matrix $\mathbf{H}_{pn}\mathbf{N}$ in the second equation of system (13) is of the same order of $\varepsilon\mathbf{M}$. This situation can introduce significant errors in the incompressibility condition. To overcome this difficulty, a smaller value of ε may be introduced which can lead to ill-conditioning or alternatively an iterative penalty method may be used as explained in Reference 9.

In the iterative penalty method, system (10) is solved iteratively, writing

$$\begin{aligned} \mathbf{K}\mathbf{v}_i - \mathbf{H}^{\mathrm{T}}\mathbf{p}_i &= \mathbf{f} \\ \mathbf{H}\mathbf{v} + \varepsilon\mathbf{M}\mathbf{p}^i &= \varepsilon\mathbf{M}\mathbf{p}^{i-1} \end{aligned} \tag{14}$$

In this way, the penalty incompressibility condition is adjusted iteratively and larger values of ε are allowed.

NUMERICAL RESULTS

The objective of the first two examples is to test the validity of the porous boundary condition. The material properties are shown in Table I and the external reference pressure is taken equal to zero.

The first example is a uniform flow inside a closed mould with a porous wall in the outflow boundary as indicated in Figure 2(a). Increasing the value of the permeability coefficient k, we must obtain a decreasing resistance to the flow through the wall. As the flow is incompressible, we must always obtain the income velocity at the exit.

Erroneous results in the velocity field for small values of the parameter k are obtained using a simple penalty method to impose the incompressibility condition. This is shown in Figure 2(b) in which the velocity along the x-direction should be constant in order to satisfy the incompressibility condition. On the other hand, the use of an iterative penalty method adjusts the penalty parameters automatically and, even for smaller values of permeability, the incompressibility condition is satisfied and the correct constant velocity field is obtained as shown in Figure 2(c).

Table I. Material properties

Dynamic viscosity	: $\mu = 1000 \cdot 0$
Density	: $\rho = 10 \cdot 0$

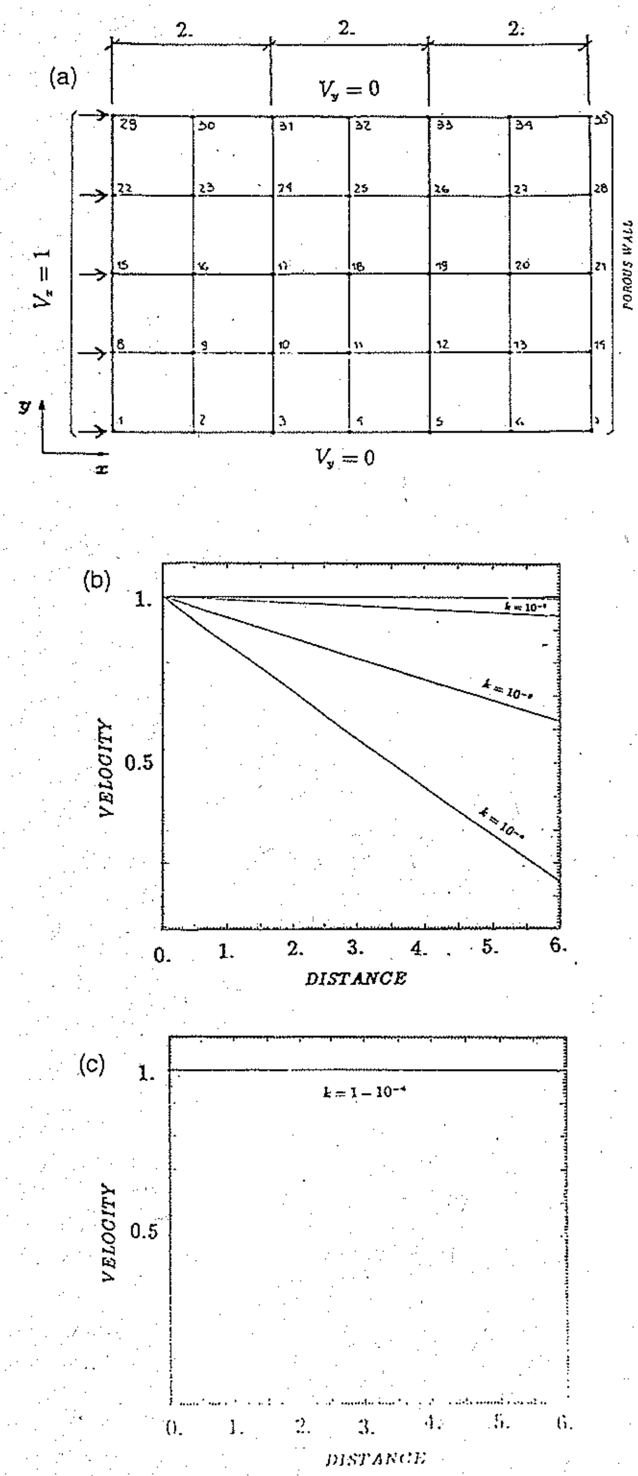

Figure 2. Uniform flow inside a closed mould: (a) finite element mesh and boundary conditions; (b) velocity along the x-direction using standard penalty method; (c) velocity along the x-direction using an iterative penalty method

The second example is a 2D extrusion problem. A porous wall was imposed in the vertical wall at the exit in order to test the behaviour of the material and the pseudomaterial in contact with the wall. The finite element mesh and boundary conditions are displayed in Figure 3(a). The problem was solved using the flow properties shown in Table I.

Figures 3(b)–(d) show the flow lines after the contact of the material with the porous wall for different values of the permeability coefficient varying from the impermeable case to a perfectly permeable case.

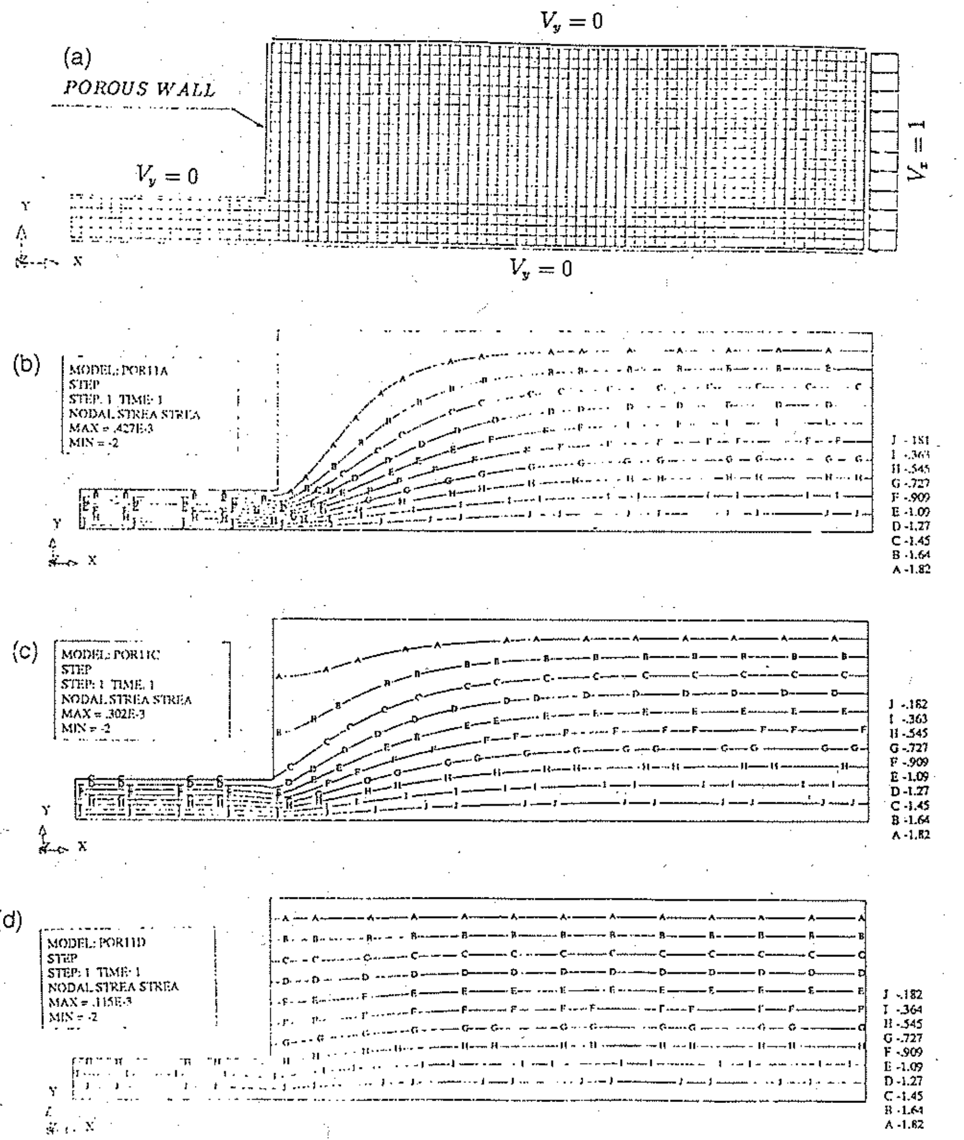

Figure 3. 2D extrusion: (a) finite element mesh and boundary conditions; (b) streamlines for $k = 0 \cdot 0$; (c) streamlines for $k = 0 \cdot 01$; (d) streamlines for $k = 0 \cdot 1$

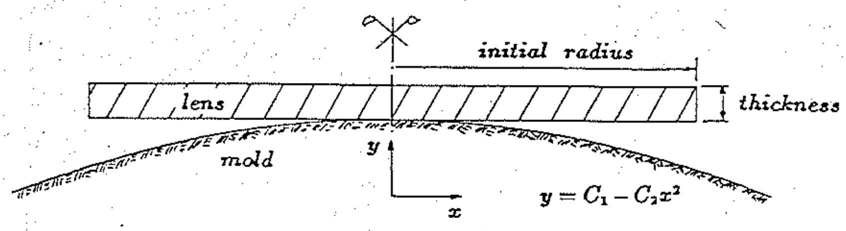

Figure 4.

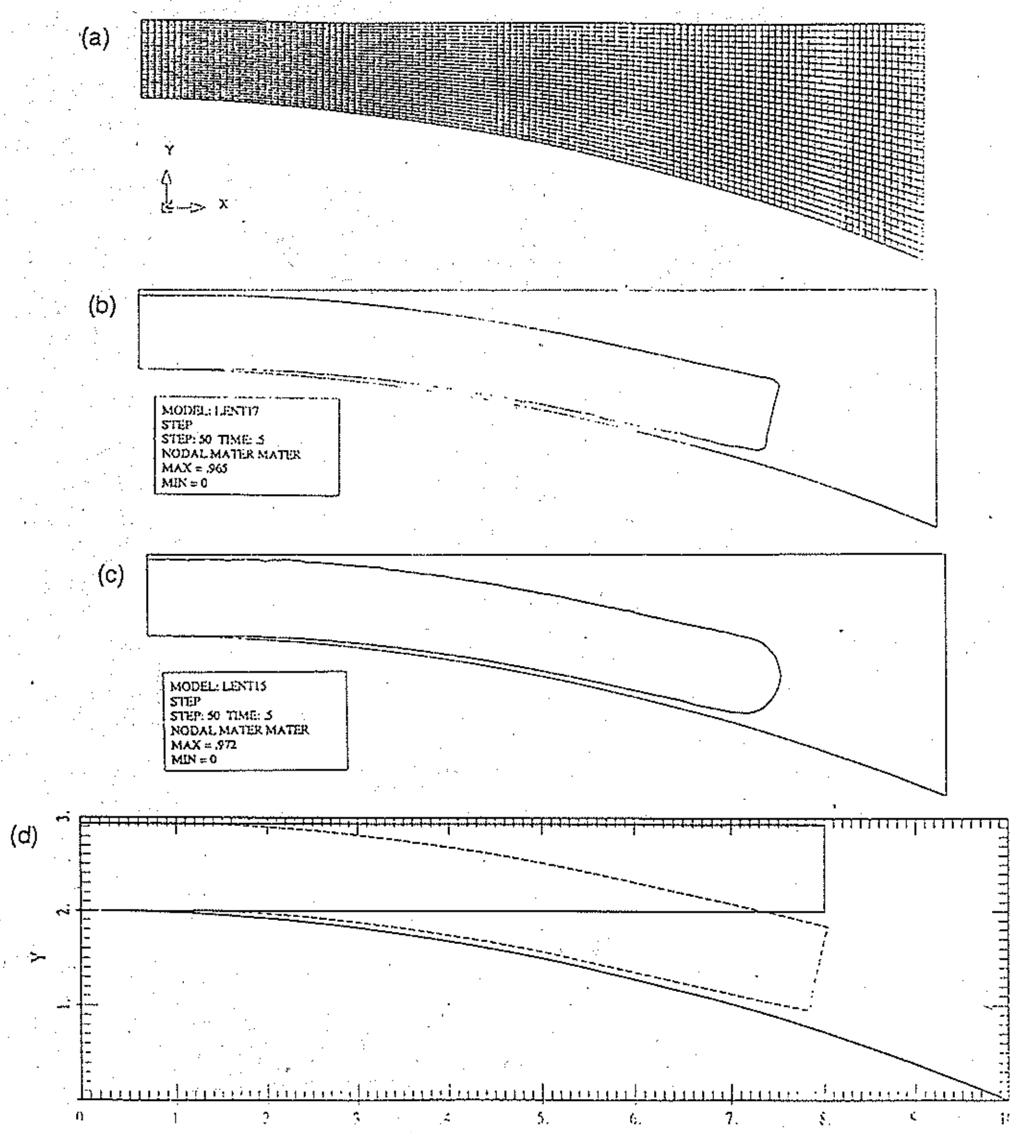

Figure 5. Lens slumping: (a) finite element mesh; (b) final lens position with an unsmoothed pseudoconcentration method; (c) final lens position with a smoothed pseudoconcentration method; (d) initial (continuous line) and final (dashed line) lens positions with Lagrangian free surface update

Table II. Properties

Material dynamic viscosity	: $\mu = 1000 \cdot 0$
Material density	: $\rho = 1 \cdot 0$
Pseudomaterial dynamic viscosity	: $\mu = 0 \cdot 001$
Pseudomaterial density	: $\rho = 0 \cdot 001$
Gravity	: $g = 10 \cdot 0$
Lens geometry	
Initial thickness	: 0·93
Initial radius	: 8·0

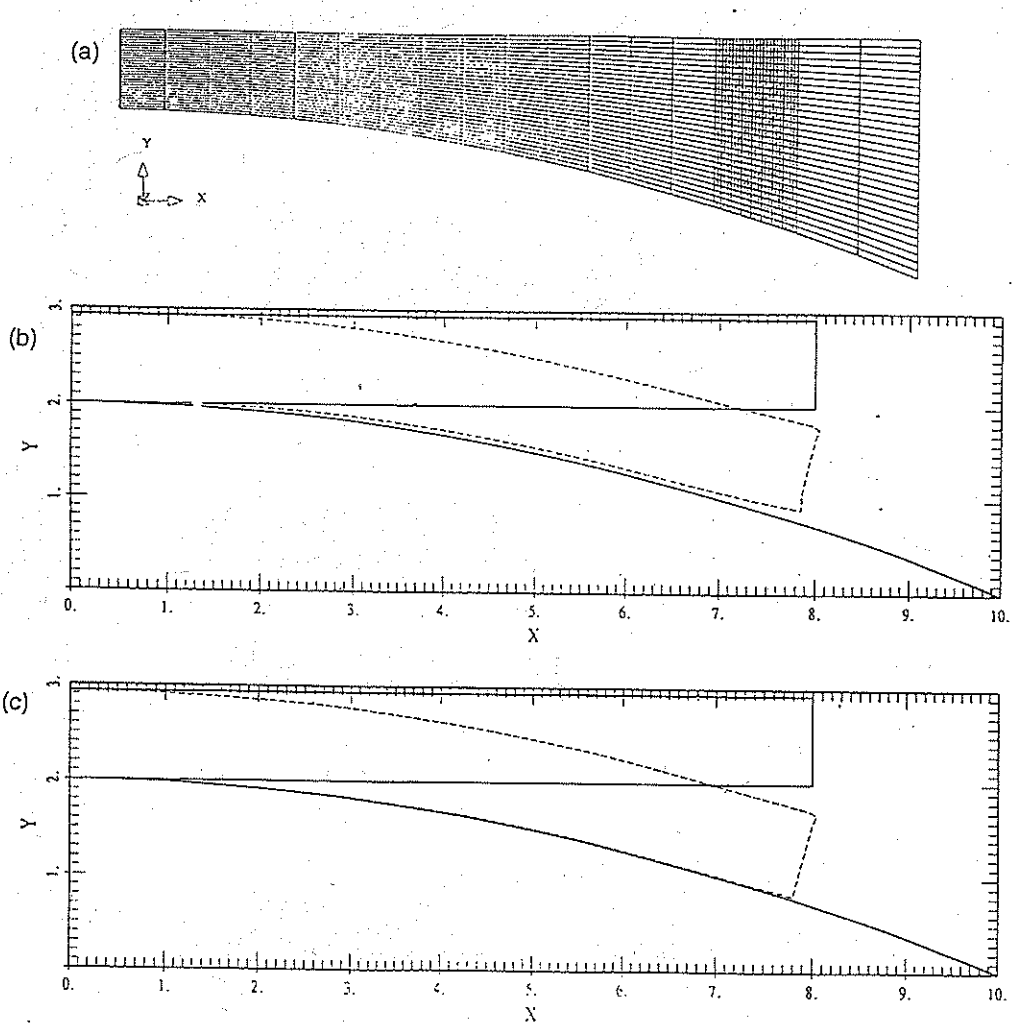

Figure 6. Lens slumping: (a) enhanced finite element mesh; (b) final lens position with Lagrangian free surface update using standard boundary conditions; (c) final lens position with Lagrangian free surface update using porous boundary condition

The third example is an industrial process in which the advantage of the method proposed in this paper can clearly be noted. The problem is the slumping forming of a lens. The lens, initially plane, is heated until plasticity occurs and takes the shape of a prescribed fixed mould by gravity (Figure 4).

The problem was solved using two different pseudomaterial methods. Firstly with the standard pseudo-concentration technique[4–8] and then using the Lagrangian free surface update proposed in this paper.

The material physical properties and other general characteristics of the problem are shown in Table II (using SI units).

The first mesh used is plotted in Figure 5(a). Four quadrilaterals represent a quadratic finite element of nine nodes.

Figures 5(b) and 5(c) show the final lens positions using the pseudoconcentration method, and Figure 5(d) the final position obtained with the present method. In the case of Figure 5(c) a smoothing technique for the pseudoconcentration variable has been employed. This is a classical technique used in the pseudoconcentration method,[4,5] i.e. the concentration is redefined at each time step to avoid large gradients. This smoothing eliminates the wiggles in the contact surface as may be seen in Figure 5(b) but, on the other hand, erroneous deformations are obtained in the corners. Figure 5(d) shows that both spurious behaviours are eliminated with the present formulation. No porous boundary condition has been used in this case. For this reason, the lens is not able to contact the mould and a spurious layer of pseudomaterial remains between the mould and the lens as shown in Figure 5(d). Finally, a new mesh shown in Figure 6(a) has been used in order to improve the precision of the method at the corners. The final position of the lens obtained with the Lagrangian free surface update method is presented in Figures 6(b) and 6(c). In the first of them, the porous boundary condition has not been used. In the last Figure, the porous boundary condition has been imposed. Note that now the lens is fully in contact with the mould and the problem of the boundary layer of incompressible pseudomaterial disappears.

CONCLUSIONS

Moving surfaces with sharp corners, usually found in forming processes, may be accurately treated using a pseudomaterial approach when the interface between two materials is followed with an arbitrary Lagrangian mesh. This technique seems to be more accurate than the standard pseudoconcentration method which sometimes leads to unacceptable oscillations of the free surface or roundness of the corners.

Furthermore, the porous boundary condition proposed is a very easy way to enhance the robustness of the pseudomaterial method to deal accurately with contact problems.

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