Model Predictive Control applied to thermostatic controlled systems

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Abstract

A Model Predictive Control (MPC) was developed to optimize the performance of a residential heat pump. To test the MPC, the real system was replaced by a Modelica emulator of the entire house. The building's thermal behavior model was developed as an Auto-Regressive linear model with eXogenous variables (ARX) and the optimization algorithm was based in a genetic algorithm. The proposed predictive model to characterize the building's thermal behavior was developed successfully. And the MPC is already implemented and validated, but the total integration with the emulator of the system and the feedback loop has not been yet assembled. In this way, we are looking towards to end this task and complete the further analysis.

I. Introduction

A Model Predictive Control (MPC) is a mechanism which, by controlling several variables comprising a system, allows to keep some parameters between the margins of some prefixed constrains. Figure 1 shows a scheme of a typical environment in which a system is controlled by a MPC. Control inputs are set by the MPC into the system and measurements of other parameters of the system are used as feedback to the MPC. The measurements are used as the starting values to run the predictive model of the MPC and an optimization algorithm is ran over the model results in order to achieve the best new control actions under the constrain requirements.

In this study, a MPC was developed to optimize the performance of a heat pump installed in the Twin house O5. The real system was nor controlled nor measured in reality, but a Modelica emulator of the house with its heat pump is going to be used to test the MPC. The building's thermal behavior model was developed as an Auto-Regressive linear model with eXogenous variables (ARX) and the optimization algorithm was based in a genetic algorithm.

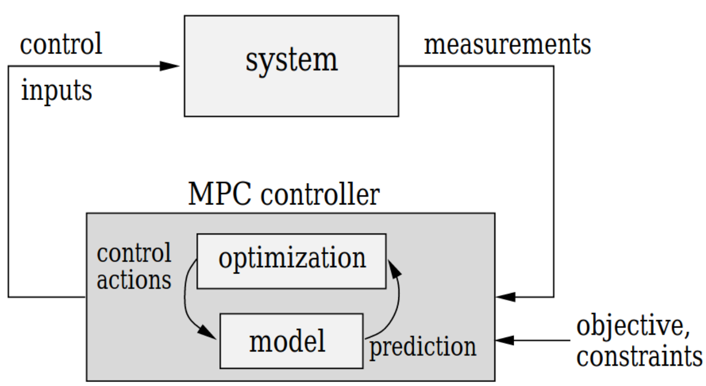

Fig. 1: MPC environment.

II. Model for building's thermal behaviour

The energy performance of a household depends on several features, these can be grouped in: indoor climate and outdoor weather parameters, construction characteristics of the building, type of heating ventilation and air conditioning (HVAC) system, and thermostatic control (which in most cases is managed by the users). The governing equations proposed to model the electric consumption of the thermostatic system intends to replicate the relations between the mentioned variables in the most complete and at the same time computationally economical way. With this focus, in this study a methodology is developed to accurately predict the energy consumption of the thermostatic-controlled heating system of the emulated Twin house O5 when set point temperature is modified.

The implemented model to predict the building's thermal behaviour is composed of three equations proposed in a highly intelligible formulation. Prior transformations of the input variables are made to improve the parameters fitting and the models accuracy. Transformations such as low pass filter, Fourier or base spline transformations where applied. The first equation describes the electric consumption (ϕ) behaviour and has the following form:

$$\begin{aligned}\phi(t=0) = \alpha_0 + &\sum_{t=-N}^{0} \alpha_1(t)\, T_s(t) + \\ &+ \alpha_2 \left(T_s(t=-1) \times T_i(t=-1)\right) + \\ &+ \sum_{t=-N}^{0} \alpha_3(t)\, T_e(t)\end{aligned} \tag{1}$$

where t is the time, T_s is the heat pump's supply temperature, T_i is the house's interior temperature, T_e is the external temperature, α_i are the auto-regressive coefficients to be fit and N is the order (number of time lags) of the auto-regressive model.

The next equation characterizes the supply temperature (T_s):

$$\begin{aligned}T_s(t=0) = &\sum_{t=-N}^{0} \beta_1 \ \left(T_s(t-1) \times \text{hp}(t)\right) \\ &+ \sum_{t=-N}^{0} \beta_2(t) \left(\phi(t) \times T_e(t)\right)\end{aligned} \tag{2}$$

where T_e is the exterior temperature, hp if the heat pump's status which take binary values (ON/OFF), ϕ is the heat pump's electric consumption and β_i are the auto-regressive coefficients.

And finally, the third equation represents the dynamics of the interior temperature (T_i):

$$\begin{aligned}T_i(t=0) = &\sum_{t=-N}^{-1} \gamma_1(t) T_i(t) + \sum_{t=-N}^{0} \gamma_2(t) \left(T_s(t) \times \text{hp}(t)\right) \\ &+ \sum_{t=-N}^{0} \gamma_3(t) T_e(t) + \sum_{t=-N}^{0} \gamma_4(t) \text{hg}(t)\end{aligned} \tag{3}$$

where hg are the heat gains and γ_i are the auto-regressive coefficients.

Moreover, the thermostatic control is done by the set point temperature of the heat pump (T_{set}) which is defined as

$$T_{set}(t) = \begin{cases} T_s(t) & if \ \text{hp} = \text{ON} \\ \text{NA} & if \ \text{hp} = \text{OFF} \end{cases}$$

which takes a value equal to the supply temperature is the heat pump is on (hp $= ON$) and a non numeric value if the heat pump is off (hp $= OFF$). Values for historical set point temperature were rearranged as shown in T_{set} so as to use it as unique parameter of control in the MPC.

Once the three models were trained with the historical data given by the emulator (α_i, β_i, γ_i were fitted), the models are coupled in the following way to do the predictions for the next hour.

```
FOR each of the next 24 hours (t):
    phi[t]= 0
    T_s_FF = T_s[t] = predict using model for T_s
    T_i_FF = T_i[t] = predict using model for T_i
    IF T_set[t] is not NA:
        T_s[t] = T_set[t]
        phi[t] = predict using model for phi
        IF phi[t]>0:
            T_i[t] = predict using model for T_i
        IFELSE phi[t]<=0:
            phi[t] = 0
            T_set[t] = NA
            T_s[t] = T_s_FF
            T_i[t] = T_i_FF
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The presented loop shows a starting point in which the electric consumption is assumed to be zero. The temperature of supply (T_s) and the interior temperature (T_i) are calculated with their respective models using $\phi = 0$. This condition in called Free Floating (FF). Once this is obtained, a conditional is presented over T_{set} so that if the heat pump is ON ($T_{set} \neq$ NA), T_s is recalculated. With the value for T_s obtained, electric consumption (ϕ) is obtained. And finally, the T_i is adjusted. The loop is ran over 24 hours in order to obtain the prediction for a whole day.

Next hour predictions were done to compare the predicted results with the historical measurements. The validation graphs showing the results of the prediction model (in red) and the historical data (in black) for the three variables are shown in the figure 2.

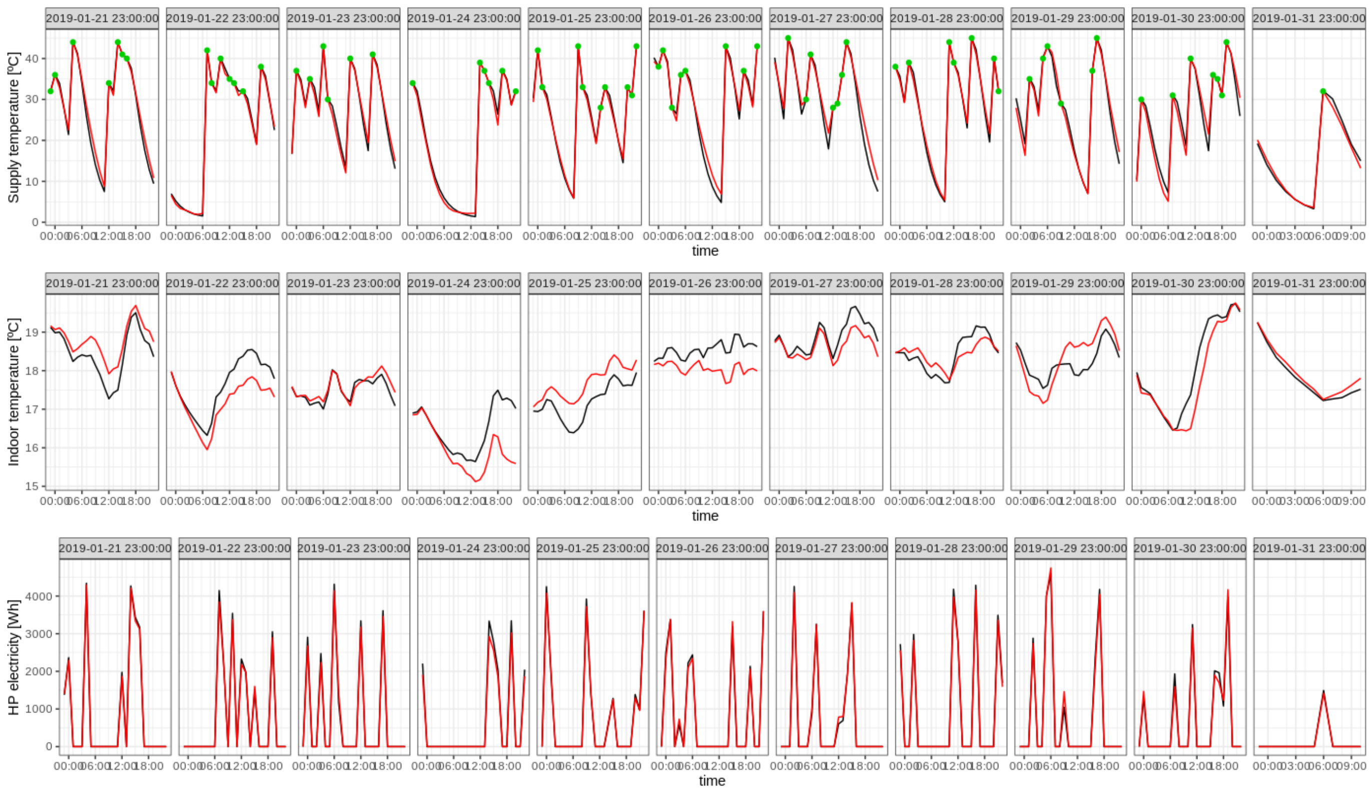

Fig. 2: Validation graphs showing the results of the prediction model (in red) and the historical data (in black) for T_s, T_i and ϕ.

III. Implementation of MPC

In a genetic algorithm a population of possible solutions to an optimization problem are evolved towards better solutions. The evolution starts from a population of a randomly generated possible solution (called individual), which it is randomly initialized. In each iteration the fitness of every individual is obtained using an objective function.

To optimize the heat pump's performance the vector constituting the individual to be optimized is

$$\boldsymbol{T_{set}} = (T_{set}(t=0), ..., T_{set}(t=24))$$

$\boldsymbol{T_{set}}$ (vector of set point temperatures) is randomly initialized inside the temperature range defined by the historical limits of supply temperature. The constrains which are to be satisfied by the MPC are determined by the thermal comfort band, which is assigned to be 20-24 C° during the day (7:00 to 23:00) and 18-22 C° during the night (23:00 to 7:00).

And the objective function (f) calculated over a day (d) to be minimized by the optimization problem is

$$f(d) = \sum_{t=0}^{24} \phi(t) \times \text{price}(t) + \text{daily penalty}(d) \tag{4}$$

which considers the total daily cost of the electric consumption and a *daily penalty* that is defined in order to quantify how good the new solution is in terms of thermal comfort.

A symmetrical penalty function is proposed (same penalty if the temperature bound in violated for overestimation or underestimation). Taking $\Delta(t)$ as the distance between internal temperature predicted and temperature limit for each hour and Δ_{limit} as the maximum delta allowed for each hour, the hourly penalty function suggested has the following exponential form (with λ the exponential parameter)

$$\text{hourly penalty}(t) = exp\left(\frac{-\lambda\Delta(t)}{(\Delta(t) - \Delta_{limit})}\right)$$

which is added to make the

$$\text{daily penalty}(d) = \sum_{t=0}^{24} \text{hourly penalty(t)}$$

The MPC was implemented and tested using the cost function defined in equation 4 and proper results were obtained. The figure 3 shows the results for a validation day in which savings of 37% were obtained. The red curves are the proposed new solution and the black curves represent the historical data.

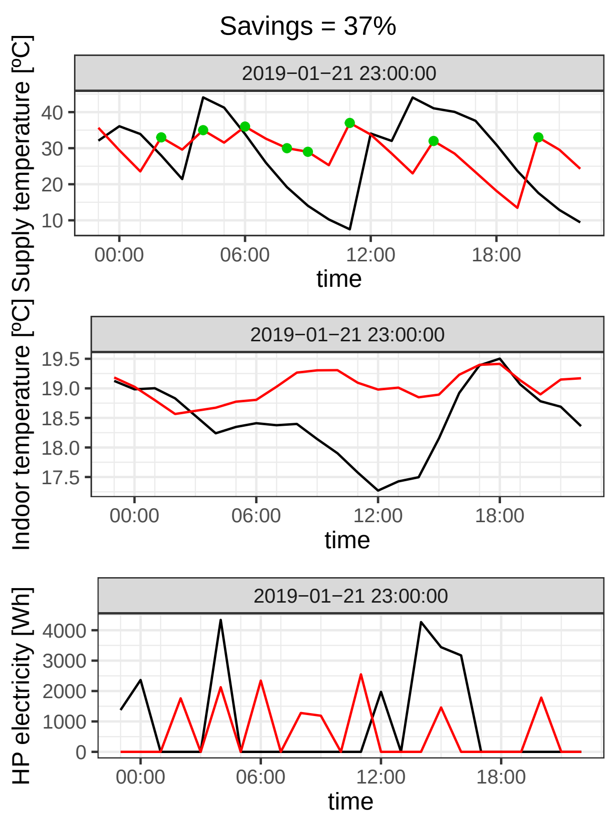

Fig. 3: Validation day with savings of 37%. The red curves shows the proposed new solution and the black curves represent the historical data.

IV. Conclusions and future work

The aim of this study is twofold. The first goal is to develop a predictive model to characterize building's thermal behavior, which was obtained. And secondly, the proposal was done to compare the performance of MPC with respect to conventional controllers such as PID and Rule Based Controller. Regarding the first aspect, it is evident that a trustworthy model to predict building behaviour was obtained. And when focusing on the second goal, while the MPC is already implemented and validated, the total integration with the emulator of the system and the feedback loop has not been yet integrated. In this way, we are looking towards to end this task and complete the further analysis.