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EFFECT OF VEHICLE-BRIDGE-INTERACTION ON THE VIBRATION OF THE BRIDGE

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Abstract. This paper studies the effect of vehicle-bridge interaction (VBI) on the vibration of the supporting bridge, and subsequently proposes a decoupled analysis scheme for the VBI problem with reference to high-speed railway systems. The study examines the VBI problem analytically and reveals the main coupling parameters between vehicles and bridges. It proves that, except for the stiffness ratio, the impedance ratio, defined as the ratio of the vehicle's damping and bridge's mechanical impedance, is also a dominant coupling parameter between vehicles and bridges. Following, the study shows that VBI alters the mechanical system of the bridge via an additional damping, an additional stiffness and a modified loading term. The coupling terms (i.e., the vehicle response) appear solely in the modified loading term. Assuming small stiffness ratio, which is realistic for practical train-bridge systems, the proposed decoupling scheme eliminates the vehicle response from the bridge's equation of motion in a systematic manner. With respect to the fully coupled system, the proposed method returns more accurate results compared to well-known decoupling methodologies, such as the moving load approximation.

1 INTRODUCTION

Vehicle-bridge interaction (VBI) constitutes an area of increasing interest during the last decades [1, 2]. This is mainly due to the vast expansion of high-speed railway lines worldwide and the growing number of bridges they are made up of [3]. Despite the extensive research on VBI ([1, 2] and references therein), only a limited number of studies focused on the examination of the physical mechanisms behind the interaction between vehicles and bridges [4–12].

Focusing on resonance and cancellation phenomena due to VBI, Yang et al. [4, 5] examined analytically a series of single-degree of freedom (SDOF) vehicles traversing a simple beam. These studies [4, 5] proposed a dimensionless speed parameter S_v, intrinsically related to such phenomena, and defined critical values of S_v for resonance and cancellation. Later studies showed that traversing vehicles change the frequency and mode shape of the supporting bridge [6, 7, 13]. Specifically, they showed that for high vehicle-to-bridge mass and eigenfrequency ratios the bridge's fundamental frequency increases [6, 7]. Moreover, VBI introduces a favorable damping effect that decreases the response of the underlying bridge [8–12]. Eurodoce suggested the Additional Damping Method (ADM) to consider this additional damping on the mechanical system of simply supported bridges, based on the bridge's length [14]. However, later studies indicated that considering solely the bridge's length is not adequate, as the additional damping effect of VBI is affected by the mechanical characteristics of both vehicle and bridge [8–11]. These studies [10, 11] also proposed formulas to consistently estimate the additional damping of bridges due to VBI.

This study aims to clearly illustrate the coupling mechanisms of VBI and subsequently characterize the VBI effects on the bridge's mechanical system. Therefore, it examines analytically a simple SDOF vehicle - SDOF bridge configuration that can adequately reveal the major coupling parameters between the two systems. In this context, it proposes a decoupling methodology oriented to simply supported railway bridges; the Modified Bridge System (MBS) method. Although more complicated than the commonly adopted moving load approximation, MBS method returns more accurate results with respect to the solution of the coupled VBI system.

2 VEHICLE-BRIDGE SYSTEM MODELLING

Assume an SDOF vehicle traversing a simply supported bridge considered with its first mode. The vehicle has N wheels in contact with the bridge with stiffness k_i^V and damping c_i^V each (Fig. 1). The distance between adjacent wheels is $\tilde{d}_i$ (Fig. 1). The equation of motion (EOM) of the vehicle about its statically deformed configuration is:

$$m^V \ddot{\tilde{z}}^V(t) = \mathbf{W}^V \tilde{\boldsymbol{\lambda}}(t) \tag{1}$$

with $\ddot{\tilde{z}}^V(t)$ being the acceleration and m^V the mass of the vehicle. t is the time and $\tilde{\boldsymbol{\lambda}}(t)$ is the contact force vector defined as:

$$\tilde{\boldsymbol{\lambda}}(t) = \begin{bmatrix} \tilde{\lambda}_1(t) & \cdots & \tilde{\lambda}_i(t) & \cdots & \tilde{\lambda}_N(t) \end{bmatrix}^{\mathrm{T}} \in \Re^{N\times 1}. \tag{2}$$

$\tilde{\lambda}_i(t)$ is the contact force at each contact point i. Lastly, $\mathbf{W}^V$ is the contact direction matrix of the vehicle:

$$\mathbf{W}^V = \begin{bmatrix} 1 & \cdots & 1 & \cdots & 1 \end{bmatrix} \in \Re^{1\times N}. \tag{3}$$

The angular frequency ω^V of the vehicle in the vertical direction is:

$$\omega^V = \sqrt{\sum_{i=1}^{N} k_i^V / m^V}\,. \tag{4}$$

Tilde above a symbol denotes a dimensional quantity to distinguish it from the corresponding dimensionless quantity (without tilde) (later, in Section 3).

Similar to the vehicle subsystem, the EOM of the SDOF bridge is:

$$m^B \ddot{\tilde{z}}^B(t) + c^B \dot{\tilde{z}}^B(t) + k^B \tilde{z}^B(t) = -\mathbf{W}^B(x)\left(\tilde{\boldsymbol{\lambda}}(t) + \mathbf{f}^B\right),\ 0 \le x \le L. \tag{5}$$

L is the length of the bridge and $\tilde{z}^B(t)$ is the generalized displacement at the midpoint of the bridge. m^B, k^B and c^B are, respectively, the generalized mass, stiffness and damping of the bridge [15], while its fundamental frequency ω^B is:

$$\omega^B = \sqrt{\frac{k^B}{m^B}}. \tag{6}$$

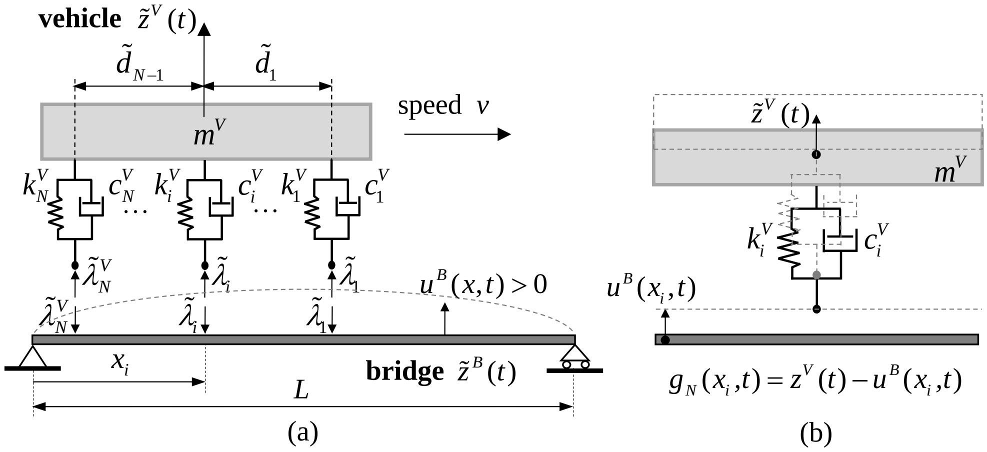

Figure 1: (a) SDOF vehicle - SDOF bridge system and (b) gap function $g_N(x_i,t)$ between the vehicle and the bridge at point i. Solid lines correspond to the initial state of the vehicle and the bridge, and dashed lines to the deformed configuration as the vehicle traverses the bridge.

$\mathbf{f}^B = \left[f_i^B\right]$ is the vector containing the external forces f_i^B acting on the bridge due to the vehicle's weight. For equidistant contact points, $f_i^B = m^V g/N$, where g is the gravitational acceleration. $\mathbf{W}^B(x)$ is the contact direction matrix pertaining to the bridge:

$$\mathbf{W}^B(x) = \left[\psi^B(x_1) \quad \cdots \quad \psi^B(x_i) \quad \cdots \quad \psi^B(x_N)\right] \in \Re^{1\times N}. \tag{7}$$

$\psi^B(x)$ is the mode shape corresponding to the fundamental mode of the bridge. For a simply supported bridge, $\psi^B(x)$ is :

$$\psi^B(x) = \sin\left(\frac{\pi x}{L}\right),\ 0 \leq x \leq L \tag{8}$$

where $x = vt$ (v is the moving speed). x_i is the location of each contact point on the bridge:

$$x_i = x - \sum_{i=1}^{N} \tilde{d}_{i-1} = vt - \sum_{i=1}^{N} \tilde{d}_{i-1},\ 0 \leqslant x_i \leqslant L \text{ and } \tilde{d}_0 = 0. \tag{9}$$

The response of the beam at any point x is:

$$\tilde{u}^B(x,t) = \psi^B(x)\,\tilde{z}^B(t). \tag{10}$$

Adopting the compliance method [1], the contact force vector between the vehicle and the bridge is:

$$\tilde{\boldsymbol{\lambda}}(t) = -\mathbf{c}^V \dot{\mathbf{g}}_N(x,t) - \mathbf{k}^V \mathbf{g}_N(x,t). \tag{11}$$

$\mathbf{c}^V = diag\left\{c_i^V\right\}$ and $\mathbf{k}^V = diag\left\{k_i^V\right\}$ are the matrices containing the contact damping c_i^V and stiffness k_i^V at each contact point. $\mathbf{g}_N(x,t) = \left[g_{Ni}(x,t)\right]$ is the gap function vector, containing the gap function $g_N(x_i,t)$ at each point i (Fig. 1(b)):

$$\mathbf{g}_N(x,t) = \left(\mathbf{W}^V\right)^{\mathrm{T}} \tilde{z}^V(t) - \left(\mathbf{W}^B(x)\right)^{\mathrm{T}} \tilde{z}^B(t). \tag{12}$$

Differentiating the gap function (Eq. (12)) once with respect to time t yields:

$$\dot{\mathbf{g}}_N(x,t) = \left(\mathbf{W}^V\right)^{\mathrm{T}} \dot{\tilde{z}}^V(t) - \frac{\pi v}{L}\left(\mathbf{W}^B(x)\right)'^{\mathrm{T}} \tilde{z}^B(t) - \left(\mathbf{W}^B(x)\right)^{\mathrm{T}} \dot{\tilde{z}}^B(t). \tag{13}$$

Hence, $\tilde{\boldsymbol{\lambda}}(t)$ (Eq. (11)) becomes:

$$\begin{aligned}\tilde{\boldsymbol{\lambda}}(t) = -\mathbf{c}^V &\left(\left(\mathbf{W}^V\right)^{\mathrm{T}} \dot{\tilde{z}}^V(t) - \frac{\pi v}{L}\left(\mathbf{W}^B(x)\right)'^{\mathrm{T}} \tilde{z}^B(t) - \left(\mathbf{W}^B(x)\right)^{\mathrm{T}} \dot{\tilde{z}}^B(t)\right) \\ -\mathbf{k}^V &\left(\left(\mathbf{W}^V\right)^{\mathrm{T}} \tilde{z}^V(t) - \left(\mathbf{W}^B(x)\right)^{\mathrm{T}} \tilde{z}^B(t)\right).\end{aligned} \tag{14}$$

Subsequently, the bridge's EOM (Eq. (5)) is:

$$\begin{aligned} & m^B \ddot{\tilde{z}}^B(t) + \left(c^B + \mathbf{W}^B(x)\,\mathbf{c}^V\left(\mathbf{W}^B(x)\right)^{\mathrm{T}}\right)\dot{\tilde{z}}^B(t) \\ & + \left(k^B + \frac{\pi v}{L}\mathbf{W}^B(x)\,\mathbf{c}^V\left(\mathbf{W}^B(x)\right)'^{\mathrm{T}} + \mathbf{W}^B(x)\,\mathbf{k}^V\left(\mathbf{W}^B(x)\right)^{\mathrm{T}}\right)\tilde{z}^B(t) \\ & = -\mathbf{W}^B(x)\left(\mathbf{f}^B - \mathbf{c}^V\left(\mathbf{W}^V\right)^{\mathrm{T}}\dot{\tilde{z}}^V(t) - \mathbf{k}^V\left(\mathbf{W}^V\right)^{\mathrm{T}}\tilde{z}^V(t)\right)\end{aligned} \tag{15}$$

and the vehicle's EOM (Eq. (1)) becomes:

$$\begin{aligned} & m^V \ddot{\tilde{z}}^V(t) + \sum_{i=1}^{N} c_i^V \dot{\tilde{z}}^V(t) + \sum_{i=1}^{N} k_i^V \tilde{z}^V(t) \\ & = \mathbf{W}^V\left[\mathbf{c}^V\left(\frac{\pi v}{L}\left(\mathbf{W}^B(x)\right)'^{\mathrm{T}}\tilde{z}^B(t) + \left(\mathbf{W}^B(x)\right)^{\mathrm{T}}\dot{\tilde{z}}^B(t)\right) + \mathbf{k}^V\left(\mathbf{W}^B(x)\right)^{\mathrm{T}}\tilde{z}^B(t)\right].\end{aligned} \tag{16}$$

3 DIMENSIONLESS DESCRIPTION OF VBI

This section demonstrates a set of dimensionless groups that sufficiently describe the coupled problem [16, 17], and subsequently demonstrates the dimensionless EOMs of the system. According to Buckingham's Π-theorem [18, 19] the number of dimensionless groups is equal to the number of dimensional parameters minus the number of involved dimensions: mass [M], length [L] and time [T]. The involved parameters are: the displacement of the vehicle $\tilde{z}^V(t)$, the displacement of the bridge $\tilde{z}^B(t)$, the mass m^V, stiffness k_i^V and damping c_i^V of the vehicle, the mass m^B, stiffness k^B and damping c^B of the bridge, the bridge's length L, the distance $\tilde{d}_i$ between adjacent contact points, the speed v, the time t, the force f_i^B acting on the bridge at each point i and the number of contact points N. Consequently, the number of dimensionless groups is $14-3=11$. Formulating the dimensionless groups with reference to the bridge's mass m^B, frequency ω^B and length L, the following groups result:

$$\begin{gathered} z^V = \frac{\tilde{z}^V}{L},\ z^B = \frac{\tilde{z}^B}{L},\ \zeta^B = \frac{c^B}{2m^B\omega^B},\ d_i = \frac{d_i}{L},\ \tau = \omega^B t,\ N, \\ S_v = \frac{\pi v}{\omega^B L},\ F_i^B = \frac{\tilde{f}_i^B}{m^B(\omega^B)^2 L},\ M = \frac{m^V}{m^B},\ K_i = \frac{k_i^V}{k^B},\ C_i = \frac{c_i^V}{m^B\omega^B}.\end{gathered} \tag{17}$$

z^V and z^B are the dimensionless displacements of the vehicle and bridge, respectively. ζ^B is the damping ratio of the bridge, d_i is the scaled distance between adjacent contact points and τ is the dimensionless time. S_v is the speed parameter [1] and F_i^B is the normalized vehicle weight at point i. K_i and C_i are the stiffness and *impedance* ratios at each contact point and M is the mass ratio of the vehicle with respect to the bridge. The impedance ratio is the ratio of the vehicle's damping c_i^V and bridge's mechanical impedance $m^B\omega^B$. $m^B\omega^B$ is the inertial resistance of the bridge to free vibrations.

According to the dimensionless groups of Eq. (17) the EOM of the bridge becomes:

$$\begin{aligned}\ddot{z}^B + \left(2\zeta^B + \mathbf{W}^B(x)\,\mathbf{C}\left(\mathbf{W}^B(x)\right)^{\mathrm{T}}\right)\dot{z}^B + \left(1 + \mathbf{W}^B(x)\,S_v\mathbf{C}\left(\mathbf{W}^B(x)\right)'^{\mathrm{T}} + \mathbf{W}^B(x)\,\mathbf{K}\left(\mathbf{W}^B(x)\right)^{\mathrm{T}}\right)z^B \\ = -\mathbf{W}^B(x)\left(\mathbf{F}^B - \mathbf{C}\left(\mathbf{W}^V\right)^{\mathrm{T}}\dot{z}^V - \mathbf{K}\left(\mathbf{W}^V\right)^{\mathrm{T}}z^V\right).\end{aligned} \tag{18}$$

$\mathbf{K} = diag\{K_i\}$ and $\mathbf{C} = diag\{C_i\}$ are diagonal matrices consisting of K_i and C_i, and $\mathbf{F}^B = \left[F_i^B\right]$ is a dimensionless force vector consisting of F_i^B. Accordingly, the dimensionless EOM of the vehicle (Eq. (16)) is:

$$M\ddot{z}^V + \sum_{i=1}^{N} C_i\dot{z}^V + \sum_{i=1}^{N} K_i z^V = \mathbf{W}^V\left[\mathbf{C}\left(S_v\left(\mathbf{W}^B(x)\right)'^{\mathrm{T}}z^B + \left(\mathbf{W}^B(x)\right)^{\mathrm{T}}\dot{z}^B(t)\right) + \mathbf{K}\left(\mathbf{W}^B(x)\right)^{\mathrm{T}}z^B\right]. \tag{19}$$

3.1 Effect of VBI on the mechanical system of the bridge

The bridge's EOM (Eq. (18)) can be expressed as:

$$\ddot{z}^B + \left(2\zeta^B + C_I(x)\right)\dot{z}^B + \left(1 + K_I(x,S_v)\right)z^B = -F_I\left(x,S_v,z^V,\dot{z}^V\right) \tag{20}$$

where $C_I(x)$ represents an *additional damping* term, $K_I(x,S_v)$ an *additional stiffness* term and $F_I\left(x,S_v,z^V,\dot{z}^V\right)$ a modified loading term. Those terms denote the effect of VBI on the supporting bridge. The additional damping term $C_I(x)$ is:

$$C_I(x) = \mathbf{W}^B(x)\,\mathbf{C}\left(\mathbf{W}^B(x)\right)^{\mathrm{T}} = \sum_{i=1}^{N} C_i\left(\psi^B(x_i)\right)^2 \tag{21}$$

and depends on the impedance ratio C_i and the position of the vehicle on the bridge x. For this case, the additional damping can only attain positive values, which confirms the *favorable damping effect* of VBI demonstrated in previous studies [8, 9, 20, 21]. A recent study by the authors [11] further elaborates on this additional damping effect of VBI. The additional stiffness term $K_I(x,S_v)$ is:

$$\begin{aligned}K_I(x,S_v) = K_{I,0}(x) + K_{I,v}(x,S_v) = \\ = \mathbf{W}^B(x)\,\mathbf{K}\left(\mathbf{W}^B(x)\right)^{\mathrm{T}} + \mathbf{W}^B(x)\,S_v\mathbf{C}\left(\mathbf{W}^B(x)\right)'^{\mathrm{T}} = \sum_{i=1}^{N} K_i\left(\psi^B(x_i)\right)^2 + S_v\sum_{i=1}^{N} C_i\psi^B(x_i)\left(\psi^B(x_i)\right)'\end{aligned} \tag{22}$$

and can obtain both positive and negative values. The additional stiffness constitutes an alternative explanation to the change of bridge's frequencies due to VBI [6, 7]. $K_{I,0}(x)$ is the "static" part of the additional stiffness term, dependent solely on the position of the vehicle $x = vt$ and stiffness ratio K_i. $K_{I,v}(x,S_v)$ is the "dynamic" part, which depends on x, the impedance ratio C_i and speed parameter S_v. Similarly to previous studies [6, 7], Eq. (22) shows that the stiffness of the bridge, and subsequently, the natural frequency, changes with the vehicle-to-bridge mass M and eigenfrequency $\Omega = \omega^V/\omega^B$ ratios. Those two groups (M and Ω) appear in Eq. (22) through the stiffness ratio K_i, as $\sum_{i=1}^{N} K_i = M\Omega^2$.

However, Eq. (22) proves that the impedance ratio C_i and speed parameter S_v also affect the natural frequency of the bridge. Finally, the modified loading term is:

$$F_I\left(x, S_v, z^V, \dot{z}^V\right) = \mathbf{W}^B\left(\mathbf{F}^B - \mathbf{C}\left(\mathbf{W}^V\right)^{\mathrm{T}}\dot{z}^V - \mathbf{K}\left(\mathbf{W}^V\right)^{\mathrm{T}}z^V\right) = \sum_{i=1}^{N}\psi^B\left(x_i\right)\left(F_i^B - C_i\dot{z}^V - K_i z^V\right) \tag{23}$$

and contains the external forces acting on the bridge due to the vehicle's self weight, as well as the coupling terms, i.e., the additional load on the bridge due to the response of the traversing vehicle $-\sum_{i=1}^{N}\psi^B\left(x_i\right)\left(C_i\dot{z}^V + K_i z^V\right)$.

4 DECOUPLING OF THE VBI PROBLEM

This section decouples the VBI problem by eliminating the coupling terms (vehicle response) from the EOM of the bridge (Eq. (20)). This decoupling relies on the assumption of small total stiffness ratio between the vehicle and the bridge, i.e., $O\left(\sum_{i=1}^{N}K_i\right) \ll 1$ [12]. First, consider the modified loading term (Eq. (23)) of the bridge's EOM (Eq. (20)), as this is the only term including the vehicle response. The first term of Eq. (23) corresponds to the sum of the external forces acting on the bridge due to the vehicle's self-weight, i.e., $\sum_{i=1}^{N}\psi^B\left(x_i\right)F_i^B$. $\psi^B\left(x_i\right)$ is the bridge's shape function at point i with maximum value $\psi^B\left(x_i\right) = 1$ according to Eq. (8). Thus, the limiting value of $\sum_{i=1}^{N}\psi^B\left(x_i\right)F_i^B$ is $\sum_{i=1}^{N}F_i^B$. The order of magnitude of this term depends on the weight of the vehicle and it is:

$$O\left(\sum_{i=1}^{N}F_i^B\right) = O\left(\frac{m^V g}{k^B L}\right) \approx 10^{-4}. \tag{24}$$

The calculation of the order of magnitude of $O\left(\sum_{i=1}^{N}F_i^B\right)$ involves values for m^B, ω^B and L, as well as for the vehicle's weight derived from the European Rail Research Institute (ERRI) [22].

Moving to the coupling terms, the dimensionless displacement of the vehicle z^V, that equals its dimensional displacement scaled with reference to the bridge's length L, is typically of order of magnitude $O\left(z^V\right) = 10^{-4}$ [1]. The order of magnitude of the dimensionless velocity $\dot{z}^V$ is:

$$O\left(\dot{z}^V\right) = O\left(\frac{\dot{\tilde{z}}^V}{\omega^B L}\right) = O\left(\frac{1}{\omega^B L}\frac{\tilde{z}^V}{T^V}\right) = O\left(\frac{\omega^V}{2\pi\omega^B}\frac{\tilde{z}^V}{L}\right) = O\left(\frac{\Omega}{2\pi}z^V\right) \tag{25}$$

where $\Omega = \omega^V/\omega^B$ is the vehicle-to-bridge eigenfrequency ratio. z^V appears into Eq. (23) in the term $\sum_{i=1}^{N}\psi^B\left(x_i\right)K_i z^V$. The limiting value of $\sum_{i=1}^{N}\psi^B\left(x_i\right)K_i z^V$ is $\sum_{i=1}^{N}K_i z^V$. According to the assumption $O\left(\sum_{i=1}^{N}K_i\right) \ll 1$, the order of magnitude of this term is:

$$O\left(\sum_{i=1}^{N}K_i z^V\right) \approx 10^{-6} \ll O\left(\sum_{i=1}^{N}F_i^B\right). \tag{26}$$

Accordingly, the velocity of the vehicle appears in the term $\sum_{i=1}^{N} \psi^B(x_i) C_i \dot{z}^V$, with limiting value $\sum_{i=1}^{N} (x_i) C_i \dot{z}^V$. The order of magnitude of the later term is:

$$O\left(\sum_{i=1}^{N} C_i \dot{z}^V\right) = O\left(2M\Omega\zeta^V \frac{\Omega}{2\pi} z^V\right) = O\left(\frac{\zeta^V}{\pi}\sum_{i=1}^{N} K_i z^V\right) \approx 10^{-6} \ll O\left(\sum_{i=1}^{N} F_i^B\right). \tag{27}$$

Note that the impedance ratio can be written as $\sum_{i=1}^{N} C_i = 2M\Omega\zeta^V$, where $\zeta^V = \sum_{i=1}^{N} c_i^V / \left(2m^V \omega^V\right)$ is the vehicle's damping ratio. According to Eqs. (26) and (27) the dominant quantity of the modified loading term (Eq. (23)) is $\sum_{i=1}^{N} \psi^B(x_i) F_i^B$. Thus, for small total stiffness ratio $O\left(\sum_{i=1}^{N} K_i\right) \ll 1$ the vehicle response can be eliminated from the bridge's EOM.

$K_{I,0}(x)$ term of the additional stiffness term (Eq. (22)) depends on the stiffness ratio K_i, and is negligible compared to the structural stiffness of the bridge (Eq. (20)). The limiting value of $K_{I,0}(x) = \sum_{i=1}^{N} K_i \left(\psi^B(x_i)\right)^2$ is $\sum_{i=1}^{N} K_i$, thus for $O\left(\sum_{i=1}^{N} K_i\right) \ll 1$, $K_{I,0}(x) \ll 1$. The "dynamic" stiffness part $K_{I,v}(x, S_v)$ is not always negligible as it depends on the impedance ratio C_i, which can obtain large values, especially for locomotives [12, 22]. Finally, the additional damping term (Eq. (21)), dependent solely on the impedance ratio is comparable or even higher than the structural damping of the bridge [11], thus cannot be omitted form the bridge's EOM.

According to the preceded analysis, for total stiffness ratio $\sum_{i=1}^{N} K_i = M\Omega^2 \ll 1$, the EOM of the bridge (Eq. (20)) becomes:

$$\ddot{z}^B + \left(2\zeta^B + C_{\mathrm{MBS}}\right)\dot{z}^B + \left(1 + K_{\mathrm{MBS}}\right) z^B = -F_{\mathrm{MBS}} \tag{28}$$

where the additional damping term is:

$$C_{\mathrm{MBS}} = \mathbf{W}^B(x)\,\mathbf{C}\left(\mathbf{W}^B(x)\right)^{\mathrm{T}} = \sum_{i=1}^{N} C_i \left(\psi^B(x_i)\right)^2, \tag{29}$$

the additional stiffness term is:

$$K_{\mathrm{MBS}} = S_v \mathbf{W}^B(x)\,\mathbf{C}\left(\mathbf{W}^B(x)\right)'^{\mathrm{T}} = S_v \sum_{i=1}^{N} C_i \psi^B(x_i) \left(\psi^B(x_i)\right)', \tag{30}$$

and the loading term is:

$$F_{\mathrm{MBS}} = \mathbf{W}^B \mathbf{F}^B = \sum_{i=1}^{N} \psi^B(x_i) F_i^B. \tag{31}$$

Equation (28) constitutes the proposed *Modified Bridge System* (MBS) method [12]. This method accounts for the effect of vehicles on the supporting bridge by changing the mechanical system of the bridge via an additional damping term C_{MBS}, an additional stiffness term K_{MBS} and a loading term F_{MBS}. Crucially, Eq. (28) is uncoupled as it does not depend on the vehicle response. The MBS method can also estimate the vehicle response by substituting the bridge response from Eq. (28) into the vehicle's EOM (Eq. (19)).

When except for the total stiffness ratio $\sum_{i=1}^{N} K_i$, the total impedance ratio $\sum_{i=1}^{N} C_i$ also obtains very small values, Eq. (28) simplifies to:

$$\ddot{z}^B + 2\zeta^B \dot{z}^B + z^B = -\sum_{i=1}^{N} \psi^B(x_i) F_i^B. \tag{32}$$

This is the commonly used *moving load approximation*. The moving load approximation usually relies on the assumption of low mass and eigenfrequency ratios or small stiffness ratio of the vehicle with respect to the supporting bridge [6, 8]. This study reveals that the impedance ratio C_i also dominates vehicle-bridge coupling, giving an insight into the coupling mechanisms of VBI.

5 NUMERICAL EXAMPLES

5.1 Parametric analysis on a simply supported bridge traversed by a 10-vehicle passenger train

To examine the performance of the proposed MBS method, this section conducts a parametric analysis for a simply supported bridge traversed by a 10-vehicle train, for different speed parameters S_v. The considered bridge is the steel composite, simply supported Skidträsk bridge in Sweden [12]. The Skidträsk bridge is $L = 36$ m long, with fundamental frequency $f^B = 3.86$ Hz and damping ratio $\zeta^B = 0.5\%$ [9]. The bridge is modelled with its first mode. The adopted vehicle is a Pioneer passenger train [23]. The MBS method considers the realistic vehicle on the bridge as a series of SDOF oscillators. The number of oscillators depends on the number of contact points of the vehicle with the bridge, and the distance between adjacent oscillators is the same as the distance between adjacent wheels. Thus, for a typical two-dimensional vehicle with four wheels, the MBS method assumes four SDOF oscillators with mass equal to one fourth of the vehicle's total mass (Fig. 2(a)), and stiffness and damping same as that of the vehicle's primary suspension system. Eventually, the MBS approach solves a modified system of the bridge that considers the oscillators' weight f_i^B and damping c_i^V (Fig. 2(a)) [12]. f_i^B acts as an external force at each contact point (Eq. (31)), while c_i^V creates an additional damping and an additional stiffness to the bridge (Eqs. (29) and (30)).

Figure 2 illustrates the displacement (Fig. 2(b)) and acceleration (Fig. 2(c)) spectra of the midpoint of the bridge for various speed parameters S_v according to the solution of the coupled system (Eq. (18)), the MBS method (Eq. (28)) and the moving load approximation (Eq. (32)). Eurocode's ADM [14] does not consider any additional damping for the bridge due to VBI, as the bridge is longer than 30 m, and thus coincides with the moving load approximation. The MBS method is in very good agreement with the coupled solution, while some small differences appear at resonant velocities (Fig. 2(b) and (c)). On the other hand, the moving load approximation, which completely neglects the dynamic interaction between trains and bridges, overestimates significantly the bridge response (Fig. 2(b) and (c)). Moreover, the proposed MBS method is computationally more efficient than the coupled solution (for this example, the MBS method is 1.6 times faster than the coupled solution) and as efficient as the moving load approximation.

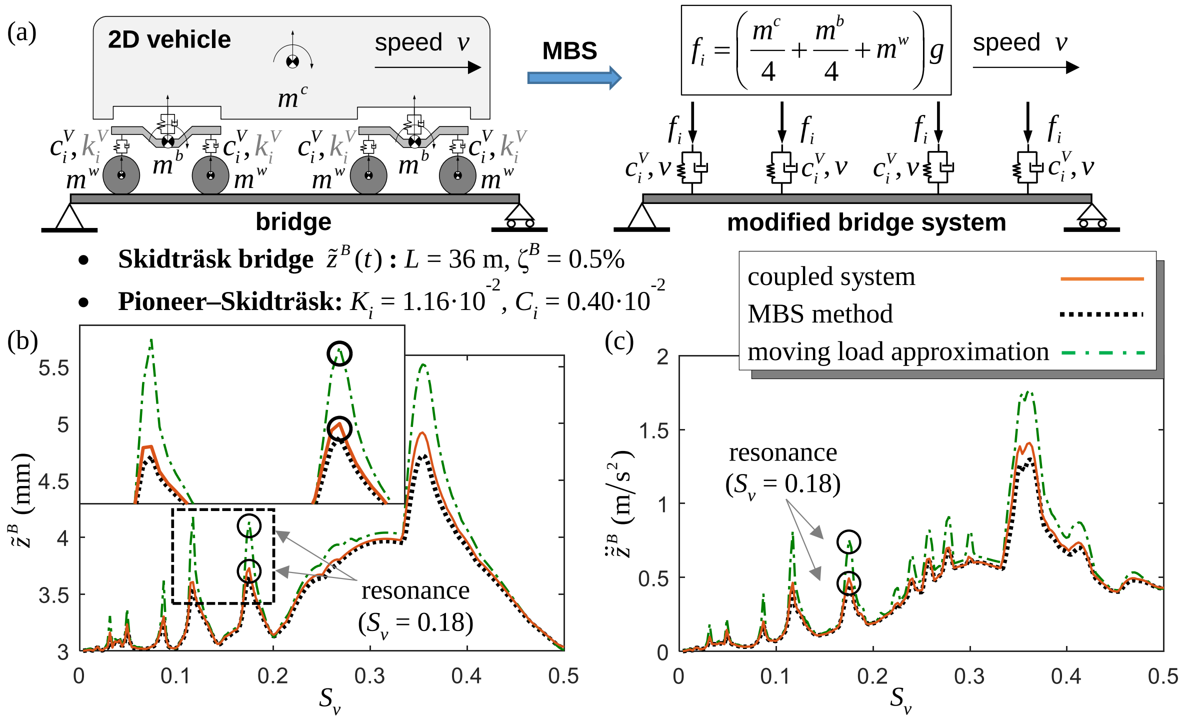

Figure 2: (a) Modified bridge system according to the MBS method, and (b) displacement and (c) acceleration spectra of the midpoint of the simply supported Skidträsk bridge [9] under the passage of a 10-vehicle Pioneer passenger train [23].

5.2 Response-history analysis of a simply supported bridge traversed by a 10-vehicle locomotive

Typically, the impedance ratio C_i, which drastically affects the coupled interaction between vehicles and bridges, is higher when locomotives instead of passenger trains traverse a bridge [12]. Therefore, this section examines the response-history of a 10-vehicle ICE2 locomotive [22] traversing the Skidträsk bridge of Section 5.1 at speed $v = 145$ km/h ($S_v = 0.15$). The study compares the response of the vehicle and bridge according to the solution of the coupled system (Eqs. (18) and (19)), the MBS method (Eqs. (28) and (19)) and the moving load/virtual path method [24] (Eqs. (32) and (19)) (Fig. 3). The latter approach solves the bridge system with the moving load approximation (Eq. (32)) and then substitutes the bridge response into the vehicle's EOM (Eq. (19)) [24]. All three methods account for the first mode of the bridge only. Note that, again, ADM coincides with the moving load approximation as the bridge's length exceeds 30 m. According to Fig. 3, the response of the bridge from the MBS method is in very good agreement with the coupled solution, while the moving load approximation considerably overestimates the bridge response (Fig. 3(a) and (b)). The results are the same for the acceleration of the car-body of the tenth vehicle of the train (Fig. 3(c)).

For comparison, this section also examines the solution of the coupled vehicle-bridge system considering the finite element model of the bridge and "rigid contact" between the vehicle's wheels and the rails [23] (complete coupled system). The response of the coupled system considering only the first mode of the bridge, as well as the response of the system according to the MBS method, is very close to that of

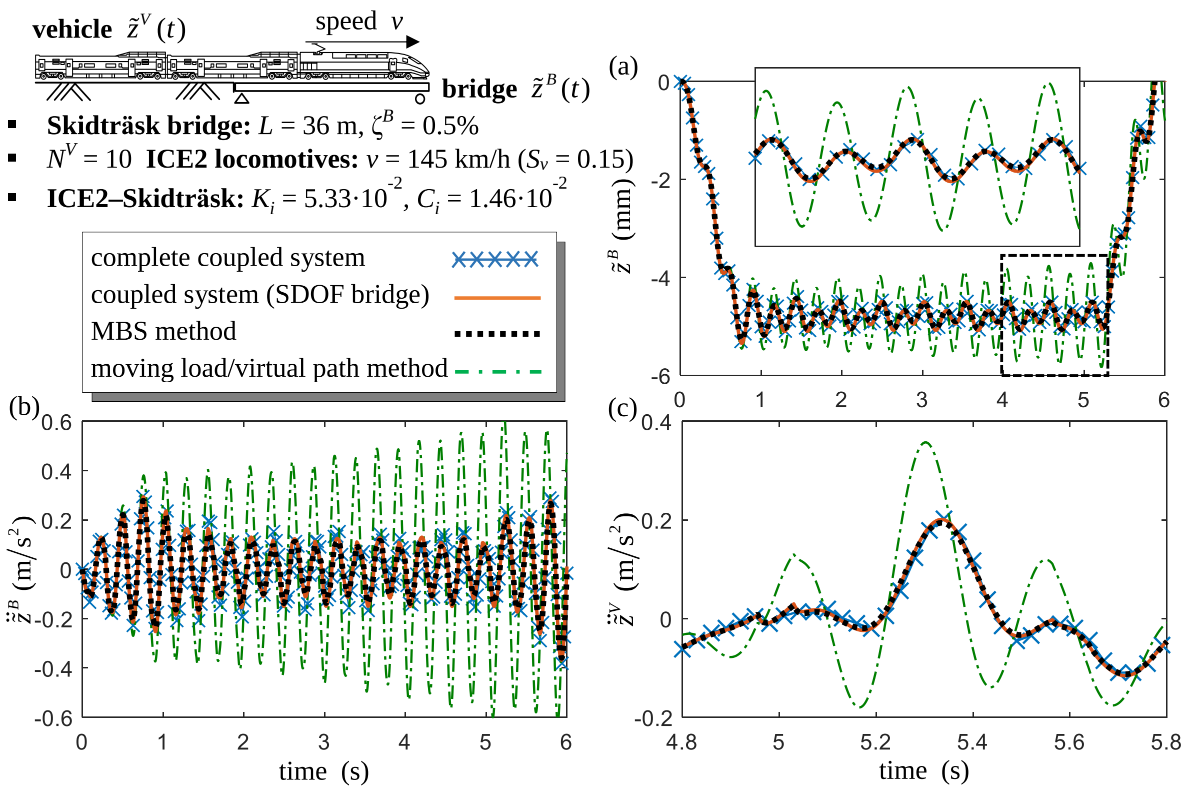

Figure 3: A ten-vehicle ICE-2 locomotive [22] running on the simply supported Skidträsk bridge [9]: (a) displacement and (b) acceleration histories at the midpoint of the bridge, and (c) acceleration history of the car-body of the tenth vehicle.

the complete coupled system for the considered vehicle-bridge configuration. This, again, validates the accuracy of the proposed methodology for simply supported railway bridges.

6 CONCLUSIONS

The present study examines the effect of traversing vehicles on the vibration of the supporting bridge via an SDOF vehicle - SDOF bridge configuration. It reveals that VBI translates into an additional damping, an additional stiffness and a modified loading term on the bridge's mechanical system. Moreover, it shows that a main coupling parameter, in addition to the vehicle-to-bridge mass and stiffness ratios, is the impedance ratio, defined as the ratio of the vehicle's damping to bridge's mechanical impedance.

Based on the assumption of small total stiffness ratio of the vehicle with respect to the bridge, the study proposes a decoupling approach to estimate the bridge response independently of the vehicle; the Modified Bridge System (MBS) method. This method takes into account the effect of VBI indirectly, by altering the mechanical system of the bridge. Numerical examples on simply supported bridges indicate that the MBS approach is equally accurate with the solution of the coupled system, while outperforms existing decoupling methods, such as the moving load approximation and the Additional Damping Method (ADM) of Eurocode. At the same time, the MBS method is computationally more efficient than the coupled solution, as it eliminates the vehicle's DOFs from the solution of the bridge system. The proposed scheme is adequate for simply supported railway bridges. For more complicated bridge configurations, e.g., continuous bridges, more elaborate (multi-degree of freedom) vehicle and bridge models are required [25].

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