More Equilibrium, More Complementarity!

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ABSTRACT

This lecture addresses the relevance of using strong forms of equilibrium, combined with strong forms of compatibility, to assess the quality of finite element solutions.

Advocating a more generalised use of equilibrated approaches may be seen, from an historical perspective, as "just" a continuation of the work developed by Fraeijs de Veubeke [1] in the sixties and seventies, which is probably unknown for most members of our community.

This raises a question: Are equilibrium approaches nothing more than a footnote in the (distant) history of Computational Mechanics?

My answer is a clear no. What is involved is not a race to know which type of formulation, compatibility or equilibrium, leads to better solutions, to smaller systems, is faster to solve, or easier to program. The relevant point is complementarity, a race that requires both contenders crossing the finish line.

The orthogonality of the error of complementary solutions [2], was [3] and should continue to be, the main argument to nurture the development of equilibrium formulations. In recent years this has been reinforced with new ways to obtain local outputs, with smaller errors [4, 5], that can be directly applied to reduced order models [6, 7].

It is true that equilibrium finite element models tend to be temperamental, prone to spurious modes, and do not follow the utterly simple assembly procedures of their displacement based counterparts. However, when properly characterised such inconveniences are minor, and can be easily overcome. All in all it is not substantially more difficult to program and use equilibrium formulations, you just need to know what to do. Some of the most relevant aspects required to achieve this goal will also be addressed in the lecture.

After more than thirty years working on the topic [8, 9] I do believe that much more can be done with strong forms of equilibrium. Then, a more extensive use of complementarity will allow for a better comprehension of the problems that we solve in Computational Mechanics.

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