INSTRUCTIONS TO PREPARE A FULL PAPER FOR THE IX INTERNATIONAL CONFERENCE ON COMPUTATIONAL METHODS FOR COUPLED PROBLEMS IN SCIENCE AND ENGINEERING – COUPLED PROBLEMS 2021

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Abstract. In this work, a study of micrometeoroid protection (MMP) is carried out. It is a thin-walled shell designed to protect the equipment on the nose of the spacecraft from micrometeorites. However, an equally dangerous equipment is posed by an astronaut who can accidentally affect the shell during repair work and damage the equipment. To counteract this, the shell is specially supported by supports from the inside. To develop a method for calculating the number of supports and their position, a simplified version of the problem is considered. The interaction of a plate hinged along the edges and having additional supports over the area with a special load in the form of a non-stationary Delta function is studied. The Kirchhoff-Lowe plate was chosen as the model of the plate. The origin is placed at the top left corner of the plate. It is required to determine the optimal location of additional supports, based on the fact that the maximum deflection should not allow the maximum permissible value. The deflection function is defined as the sum of the convolutions of the influence functions with the corresponding external load and reactions in the additional supports. To determine the value of the influence function, all functions included in the expressions for the motion of the plate are expanded into Fourier series in such a way that the boundary conditions at the edges of the plate are satisfied and the Laplace transform in time is applied. Further, in the equation of motion of the plate, both the external load and the convolution of the Delta function with reactions in additional supports and in time are taken into account. The values of the reactions in the supports are determined from the boundary conditions, based on the fact that the displacements of the fixed points are equal to zero. After that, from the obtained equation of normal displacements, the coordinates of the location of the supports around the applied external load are determined, so that the condition of not exceeding the specified displacement value is satisfied. The normal displacements are determined, the inverse Laplace transform is performed

in time, and the sum of the series is found.

1 INTRODUCTION

The problem under study has a practical meaning, since at first it was supposed to study the shell, which is a truncated cone. This shell was a micrometeoroid protection, it was a thin-walled shell designed to protect the equipment on the nose of the spacecraft from micrometeorites and other influences. To counteract this, the shell is specially supported by supports from the inside. However, the complexity of the calculations and the lack of a methodology forced the authors to simplify the initial task. To develop a method for calculating the number of supports and their position, a simplified version of the problem was considered.

In this paper, we study the interaction of a plate hinged along the edges and having additional supports along the area with a special load in the form of a non-stationary delta- function. The Kirchhoff-Lowe plate was chosen as the model of the plate [1]. The main task is to determine the optimal location of additional supports, based on the fact, that the maximum deflection should not allow the maximum allowable value. The deflection function is defined as the sum of the convolutions of the influence functions with the corresponding external load and reactions in the additional supports.

Non-stationary problems have recently become widespread; various types of interaction of external loads and plates and shells, both homogeneous and anisotropic, are being investigated. One of the methods for solving such problems is the method for determining the influence functions, presented in [2-8]. This work also uses this approach to determine the deflection influence function.

In addition, at the moment, inverse problems for non-stationary loads are widely studied, such as problems for a Timoshenko-type beam of finite length under the action of a non-stationary load, issues related to the identification of defects in an elastic rod [4-6]. In fact, the problem being solved can be defined as the inverse, since the stiffness condition is a known displacement, on the basis of which the position of the additional supports and the values of the reactions in them are determined.

The problem being solved is an intermediate stage in solving the problem of designing micrometeroid protection and its results allow determining the optimal position of the supports based on their strength conditions.

2 FORMULATION OF THE PROBLEM

In this paper, we consider a rectangular thin hinged-supported plate with dimensions a by b of constant thickness h, which has additional supports inside. Additional supports are set with the same step along the coordinate axes, forming identical segments. At the initial moment of time, an unsteady concentrated force is applied to a random place of the plate. It is necessary to determine the size of the segment and the position of the additional supports, at which the stiffness condition would be met: the maximum deflection does not exceed the specified meaning.

To determine the size of the segment, we will solve the following problem. We have a thin rectangular plate of constant thickness pivotally supported on all sides. An unsteady concentrated load is applied to the middle of the plate. Four additional supports are installed around the point of application of the load at a certain radius y_{max} to be determined, forming a square segment

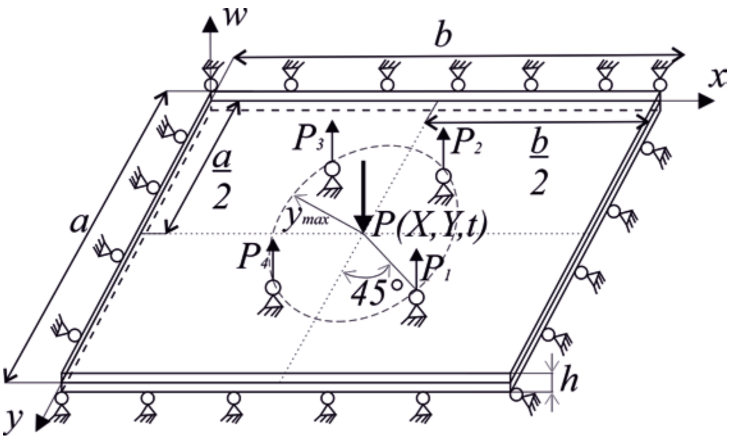

Figure 1: Statement of the problem for determining the size of the segment

. The Kirchhoff-Lowe plate was chosen as the model of the plate [1]. The origin is placed at the top left corner of the plate. It is required to determine the optimal location of additional supports, based on the fact that the maximum deflection should not exceed the maximum permissible value.

3 FORMULATION OF THE PROBLEM OF AN INFINITE HINGED OPERATED STRIP

Due to the complexity of the calculations, the initial problem of determining the dimensions of a segment with supports for a plate is simplified to the problem of an infinite strip in a planar setting. This approach will make it possible to determine the position of the supports, after which the original problem is simplified by one parameter y_{max}.

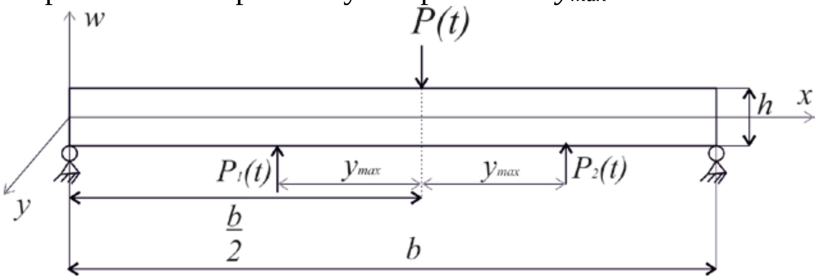

Figure 2: Endless strip with additional supports under the influence of a non-stationary load

Consider the problem of an infinite hinged strip along the short side of a plate with two additional supports. An unsteady concentrated load is applied to the center of the strip. The initial conditions are equal to zero. The boundary conditions correspond to a hinge, which allows all functions to be decomposed into trigonometric series. It is necessary to determine the coordinates of the position of the additional supports. In the figure (Figure 2), the additional supports have already been replaced with the corresponding reactions.

The equation of motion of an endless strip in displacements is written as follows:

$$\rho h\ddot{w}(x,t) = -D\Delta\Delta w(x,t) + P(t),$$
$$\Delta = \frac{\partial}{\partial x}, \tag{1}$$

Initial conditions:

$$w\big|_{t=0} = \left.\frac{\partial w}{\partial t}\right|_{t=0} = 0, \tag{2}$$

Border conditions:

$$w\big|_{x=0,b} = \left.\frac{\partial^2 w}{\partial x^2}\right|_{x=0,b} = 0. \tag{3}$$

The boundary conditions for additional supports are as follows:

$$w(q_i, z_i, t) = 0, \qquad i = 1, 2, \tag{4}$$

where the points of the position of the supports have coordinates

$$q_1 = \frac{b}{2} - y_{\max}, q_2 = \frac{b}{2} + y_{\max}. \tag{5}$$

The equations of motion in displacements, initial conditions and boundary conditions form the initial-boundary value problem of unsteady forced oscillations. The solution to the initial-boundary value problem will be sought using the influence function. Let's imagine the deflection as the convolution of this function with the actual load:

$$w(x,t) = G(x,t;X) * P(t) + \sum_{i=1}^{2} G(x,t;q_i) * P_i(q_i,t). \tag{6}$$

Then the equation of motion (1) and the corresponding conditions (2), (3) will be rewritten as follows:

$$\rho h\ddot{G}(x,t) = -D\Delta\Delta G(x,t) + \delta(x-X)\delta(t),$$
$$G(x,0) = \left.\frac{\partial G(x,t)}{\partial t}\right|_{t=0} = 0, \tag{7}$$
$$G(x,t)\big|_{x=0,b} = \left.\frac{\partial^2 G(x,t)}{\partial x^2}\right|_{x=0,b} = 0.$$

Let's perform the Laplace transform for the equation of motion. Then, taking into account the initial conditions, we get:

$$\rho h s^2 G^L(x,s) = -D\Delta\Delta G^L(x,s) - \delta(x-X)1(s). \tag{8}$$

Let us expand into trigonometric Fourier series in the coordinates of all functions (8) included in the equations of motion of the plate in such a way that the boundary conditions at the edges of the plate are satisfied [9, 10].

$$G^L(x,s)=\sum_{n=1}^{\infty}G_n^L(s)\sin(\lambda_n x),$$
$$\delta(x-\xi)=\sum_{n=1}^{\infty}\delta_n(\xi)\sin(\lambda_n x), \tag{9}$$

where

$$\lambda_n=\frac{\pi n}{b},$$
$$\delta_{nm}(\xi)=\frac{2}{b}\sin(\lambda_n\xi). \tag{10}$$

Then we get:

$$\sum_{n=1}^{\infty}s^2G_n^L(s)\sin(\lambda_n x)=-\frac{D}{\rho h}\sum_{n=1}^{\infty}\lambda_n^4G_n^L(s)\sin(\lambda_n x)+\sum_{n=1}^{\infty}\frac{\delta_n(\xi)}{\rho h}\sin(\lambda_n x). \tag{11}$$

Omitting the summation signs and reducing the trigonometric factors, we obtain the equation of motion in the coefficients of the series of the influence function in the images:

$$s^2G_n^L(s)=-\frac{D}{\rho h}\lambda_n^4G_n^L(s)+\frac{\delta_n(\xi)}{\rho h}. \tag{12}$$

Having solved the equation for the coefficients of the influence function in the images, we get:

$$G_n^L(s)=\frac{\delta_n(\xi)}{\rho F\left(s^2+\frac{D\lambda_n^4}{\rho h}\right)}. \tag{13}$$

Let's return to the originals of the coefficients of the influence function by performing the inverse Laplace transform:

$$G_n(t)=\frac{\delta_{nm}(\xi)}{\rho F\sqrt{\frac{D\lambda_n^4}{\rho h}}}\sin\left(\sqrt{\frac{D\lambda_n^4}{\rho h}}t\right). \tag{14}$$

Then the sought influence function has the form:

$$G(x,t)=\sum_{n=1}^{\infty}\frac{\delta_n(\xi)}{\sqrt{D\rho h\lambda_n^4}}\sin\left(\sqrt{\frac{D\lambda_n^4}{\rho h}}t\right)\sin(\lambda_n x). \tag{15}$$

The next step is to determine the reactions in the supports. They are determined from the boundary conditions in the supports (4). To do this, let us open the convolution of the influence function with the acting load and reactions in the supports:

$$w(x,t)=\int_0^t G(x,t-\tau;X)P(\tau)d\tau+\sum_{i=1}^{2}\int_0^t G(x,t-\tau;q_i)P_i(q_i,\tau)d\tau. \tag{16}$$

For integrals with unknown reaction functions in supports, we apply discretization with partial integration according to [11]:

$$w\left(x,t_j\right)=G\left(x,t_j;X\right)P\left(t_j\right)+ \\ +\sum_{i=1}^{2}\sum_{k=2}^{j+1}P_i\left(q_i,t_k\right)\int_{t_{k-1}}^{t_k}G\left(x,t_j-\tau;q_i\right)d\tau,\quad j=2..T+1. \tag{17}$$

In this case, the integrals of the influence function are taken, which in what follows will also be borne in mind. Let us apply the boundary conditions in the supports to this record of the deflection function. The reactions in the supports at each moment of time are sought iteratively. Time sampling is performed unevenly. This is due to the large type of records. We use 11 points of division in time, where we concentrate most of them in the area of maximum application of efforts. Then we get:

$$t_1=0,\ t_2=0.001,\ t_3=0.002,\ t_4=0.003,\ t_5=0.004,\ t_6=0.005,\ t_7=0.006,\ t_8=0.007,$$
$$t_9=0.008, t_{10}=0.05,\ \ t_{11}=0.1.$$

At time t1 = 0, there is no deflection from the initial conditions. Consequently, the reactions in the supports are equal to zero. At the moment t2, the reactions in the supports are calculated by the Cramer method with the use of the GU in additional supports from the SLAE, written in matrix form

$$\begin{pmatrix} G\left(q_1,t_2,t_1,t_2;q_1\right) & G\left(q_1,t_2,t_1,t_2;q_2\right) \\ G\left(q_2,t_2,t_1,t_2;q_1\right) & G\left(q_2,t_2,t_1,t_2;q_2\right)\end{pmatrix}\cdot\begin{pmatrix}P_1\left(q_1,t_2\right)\\ P_2\left(q_2,t_2\right)\end{pmatrix}=\begin{pmatrix}\text{-}P\left(t_2\right)G\left(q_1,t_2;X\right)\\ \text{-}P\left(t_2\right)G\left(q_2,t_2;X\right)\end{pmatrix}, \tag{18}$$

$$P_1\left(t_2\right)=\frac{-P\left(t_2\right)G\left(q_1,t_2;X\right)G\left(q_2,t_2,t_1,t_2;q_2\right)+P\left(t_2\right)G\left(q_2,t_2;X\right)G\left(q_1,t_2,t_1,t_2;q_2\right)}{G\left(q_1,t_2,t_1,t_2;q_1\right)G\left(q_2,t_2,t_1,t_2;q_2\right)-G\left(q_1,t_2,t_1,t_2;q_2\right)G\left(q_2,t_2,t_1,t_2;q_1\right)}, \\ P_2\left(t_2\right)=\frac{-P\left(t_2\right)G\left(q_2,t_2;X\right)G\left(q_1,t_2,t_1,t_2;q_1\right)+P\left(t_2\right)G\left(q_1,t_2;X\right)G\left(q_2,t_2,t_1,t_2;q_1\right)}{G\left(q_1,t_2,t_1,t_2;q_1\right)G\left(q_2,t_2,t_1,t_2;q_2\right)-G\left(q_1,t_2,t_1,t_2;q_2\right)G\left(q_2,t_2,t_1,t_2;q_1\right)}. \tag{19}$$

In subsequent iterations, the previous values of the reactions in the supports are taken into account. And the system of equations takes the form:

$$\begin{pmatrix} G\left(q_1,t_j,t_{i-1},t_i;q_1\right) & G\left(q_1,t_j,t_{i-1},t_i;q_2\right) \\ G\left(q_2,t_j,t_{i-1},t_i;q_1\right) & G\left(q_2,t_j,t_{i-1},t_i;q_2\right)\end{pmatrix}\cdot\begin{pmatrix}P_1\left(q_1,t_j\right)\\ P_2\left(q_2,t_j\right)\end{pmatrix}=\begin{pmatrix}Sum_1\\ Sum_2\end{pmatrix}, \tag{20}$$

where

$$Sum_1=\text{-}P\left(t_j\right)G\left(q_1,t_j;X\right)-\sum_{i=2}^{j-1}\left(P_1\left(t_i\right)G\left(q_1,t_j,t_{i-1},t_i;q_1\right)+P_2\left(t_i\right)G\left(q_1,t_j,t_{i-1},t_i;q_2\right)\right), \\ Sum_2=\text{-}P\left(t_j\right)G\left(q_2,t_j;X\right)-\sum_{i=2}^{j-1}\left(P_1\left(t_i\right)G\left(q_2,t_j,t_{i-1},t_i;q_1\right)+P_2\left(t_i\right)G\left(q_2,t_j,t_{i-1},t_i;q_2\right)\right). \tag{21}$$

Then the reactions in the supports at the moment of time $t_j,\quad j=3..T+1$ will be determined as:

$$P_1(t_j)=\frac{Sum_1\cdot G(q_2,t_j,t_{j-1},t_j;q_2)-Sum_2\cdot G(q_1,t_j,t_{j-1},t_j;q_2)}{G(q_1,t_j,t_{j-1},t_j;q_1)G(q_2,t_j,t_{j-1},t_j;q_2)-G(q_1,t_j,t_{j-1},t_j;q_2)G(q_2,t_j,t_{j-1},t_j;q_1)},$$
$$P_2(t_j)=\frac{Sum_2\cdot G(q_1,t_j,t_{j-1},t_j;q_1)-Sum_1\cdot G(q_2,t_j,t_{j-1},t_j;q_1)}{G(q_1,t_j,t_{j-1},t_j;q_1)G(q_2,t_j,t_{j-1},t_j;q_2)-G(q_1,t_j,t_{j-1},t_j;q_2)G(q_2,t_j,t_{j-1},t_j;q_1)}. \quad (22)$$

The reactions in the supports at each moment of time are found.

Let the acting load have the following form: $P(t)=-750e^{-50t}$.

The geometrical and physical characteristics of the strip under study are presented in the table.

Table ~~1~~ Geometric and physical characteristics of the plate

Characteristics	Value
Elastic modulus of the first kind (Young), *E*	$71\cdot10^9$ Pa
Poisson's ratio , *v*	0,31
Plate thickness , *h*	0.002 m
Vertical plate size , *a*	1 m
Horizontal plate size , *b*	2 m
Density of plate material , *ρ*	2640 kg/m^3
Number of members of the series ,*N*	80
Amount of time split points , *T*	11

Substituting all the numbers into the equations found earlier, we arrive at the equation of the stiffness condition with respect to the radius of the circle y_{max}, on which additional supports are located .

$$w(X,t_{ist})+0.006=0. \quad (23)$$

Analyzing the resulting graphs at different times, we come to the conclusion that the time at which it is necessary to search for the value of y_{max} is $t_7=0.006$.

Then the stiffness condition equation has the following graphic solution:

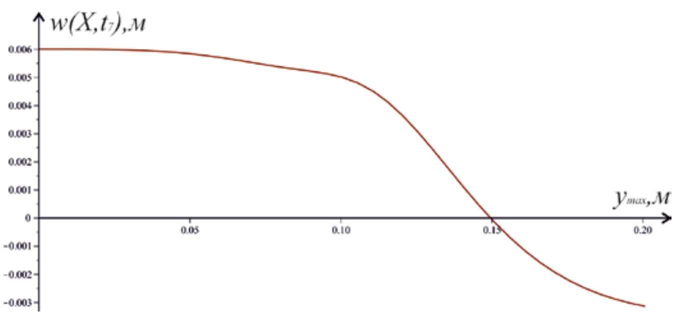

Figure 3: Graphical solution of the stiffness condition equation

The numerical answer, rounded down to integers, is as follows:

$$y_{\max} = 0.145 \text{ м}. \tag{24}$$

4 SOLVING THE PROBLEM OF DETERMINING THE DEFLECTION FUNCTION FOR A PLATE SEGMENT

Let us return to the problem statement for a plate with four additional supports. Since the formulation of the problem for a plate with four supports is tougher than the problem of an infinite strip, the previously found result should also be suitable for a bounded plate. Knowing the radius on which the supports are located, we determine their position from the following formulas:

$$q_1 = \frac{b}{2} + y_{\max}\cos\left(\frac{\pi}{4}\right), z_1 = \frac{a}{2} + y_{\max}\sin\left(\frac{\pi}{4}\right),$$
$$q_i = q_{i-1} + \cos\left(\frac{\pi}{4}\right) y_{\max}\sin\left(\frac{i\alpha\pi}{180}\right), z_i = z_{i-1} + \sin\left(\frac{\pi}{4}\right) y_{\max}\cos\left(\frac{i\alpha\pi}{180}\right). \quad w\left(X, t_{ist}\right) + 0.006 = 0. \tag{25}$$

Next, the problem is formulated and the reactions in the supports are determined in the same way as for an infinite strip. The equation of motion of the Kirchhoff-Lowe plate in displacements is written as follows:

$$\rho h\ddot{w} = -D\Delta\Delta w + P(t), \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}. \tag{26}$$

Initial conditions:

$$w\big|_{t=0} = \frac{\partial w}{\partial t}\bigg|_{t=0} = 0, \tag{27}$$

And boundary conditions:

$$w(x,y,t)\big|_{x=0,b} = \frac{\partial^2 w(x,y,t)}{\partial x^2}\bigg|_{x=0,b} = 0, w(x,y,t)\big|_{y=0,a} = \frac{\partial^2 w(x,y,t)}{\partial y^2}\bigg|_{y=0,a} = 0. \tag{28}$$

The boundary conditions for additional supports are as follows:

$$w(q_i, z_i, t) = 0, \qquad i = 1,4. \tag{29}$$

The solution to the initial-boundary value problem will be sought using the influence function. Let's imagine the deflection as the convolution of this function with the actual load:

$$w(x,y,t) = G(x,y,t;X,Y) * P(t) + \sum_{i=1}^{4} G(x,y,t;q_i,z_i) * P_i(t). \tag{30}$$

Then the problem statement through the influence function will be rewritten as follows:

$$\rho h\frac{\partial^2 G}{\partial t^2}=-D\Delta\Delta G+\delta\left(x-\xi,y-\zeta\right)\delta\left(t\right),$$
$$G\big|_{t=0}=\frac{\partial G}{\partial t}\bigg|_{t=0}=0,G\big|_{x=0,a}=G\big|_{y=0,b}=\frac{\partial^2 G}{\partial x^2}\bigg|_{x=0,a}=\frac{\partial^2 G}{\partial y^2}\bigg|_{y=0,b}=0. \tag{31}$$

Let's perform the Laplace transform for the equation of motion. Then, taking into account the IC, we get:

$$\rho hs^2 G^L=-D\Delta\Delta G^L+\delta\left(x-\xi,y-\zeta\right). \tag{32}$$

Let us divide both parts by ρh and expand the influence function in the images into trigonometric Fourier series.

$$G^L\left(x,y,s\right)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}G_{nm}^L\left(s\right)\sin\left(\lambda_n x\right)\sin\left(\lambda_m y\right),$$
$$\delta\left(x-\xi,y-\zeta\right)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\delta_{nm}\left(\xi,\zeta\right)\sin\left(\lambda_n x\right)\sin\left(\lambda_m y\right), \tag{33}$$

where

$$\lambda_n=\frac{\pi n}{a},\lambda_m=\frac{\pi m}{b},\delta_{nm}\left(\xi,\zeta\right)=\frac{4}{ab}\sin\left(\lambda_n\xi\right)\sin\left(\lambda_m\zeta\right). \tag{34}$$

Then:

$$\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}s^2G_{nm}^L\left(s\right)\sin\left(\lambda_n x\right)\sin\left(\lambda_m y\right)=-\frac{D}{\rho h}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left(\lambda_n^2+\lambda_m^2\right)^2G_{nm}^L\left(s\right)\sin\left(\lambda_n x\right)\sin\left(\lambda_m y\right)+$$
$$+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{\delta_{nm}\left(\xi,\zeta\right)}{\rho h}\sin\left(\lambda_n x\right)\sin\left(\lambda_m y\right). \tag{35}$$

Omitting the summation signs and reducing the trigonometric factors, we obtain the equation of motion in the coefficients of the series of the influence function in the images:

$$s^2G_{nm}^L\left(s\right)=-\frac{D}{\rho h}\left(\lambda_n^2+\lambda_m^2\right)^2G_{nm}^L\left(s\right)+\frac{\delta_{nm}\left(\xi,\zeta\right)}{\rho h}. \tag{36}$$

Having solved the equation for the coefficients of the influence function in the images, we get:

$$G_{nm}^L\left(s\right)=\frac{\delta_{nm}\left(\xi,\zeta\right)}{\rho h\left(s^2+\frac{D}{\rho h}\left(\lambda_n^2+\lambda_m^2\right)^2\right)}. \tag{37}$$

Let's return to the originals of the coefficients of the influence function by performing the inverse Laplace transform:

$$G_{nm}\left(t\right)=\frac{\delta_{nm}\left(\xi,\zeta\right)}{\sqrt{D\rho h\left(\lambda_n^2+\lambda_m^2\right)^2}}\sin\left(\sqrt{\frac{D}{\rho h}\left(\lambda_n^2+\lambda_m^2\right)^2}t\right). \tag{38}$$

Then the sought influence function has the form:

$$G(x,y,t)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{\delta_{nm}(\xi,\zeta)}{\sqrt{D\rho h\left(\lambda_n^2+\lambda_m^2\right)^2}}\sin\left(\sqrt{\frac{D}{\rho h}\left(\lambda_n^2+\lambda_m^2\right)^2}\,t\right)\sin(\lambda_n x)\sin(\lambda_m y). \quad (39)$$

The next step is to determine the reactions in the supports. They are determined from the boundary conditions in the supports. To do this, let us open the convolution of the influence function with the acting load and reactions in the supports:

$$w(x,y,t)=\int_0^t G(x,y,t-\tau;X,Y)P(\tau)\,d\tau+\sum_{i=1}^{4}\int_0^t G(x,y,t-\tau;q_i,z_i)P_i(\tau)\,d\tau. \quad (40)$$

For integrals with unknown reaction functions in supports, we apply discretization with partial integration according to [11]:

$$\begin{gathered}w(x,y,t_j)=G(x,y,t_j;X,Y)P(t_j)+\\+\sum_{i=1}^{2}\sum_{k=2}^{j+1}P_i(t_k)\int_{t_{k-1}}^{t_k}G(x,y,t_j-\tau;q_i,z_i)\,d\tau,\quad j=2..T+1.\end{gathered} \quad (41)$$

In this case, the integrals of the influence function are taken, which in what follows will also be borne in mind. Let us apply the boundary conditions in the supports to this record of the deflection function. The reactions in the supports at each moment of time are sought iteratively. Time discretization is performed evenly according to the following formula:

$$t_1=0,t_j=\frac{0.1}{T}(j-1),\quad j=2..T+1. \quad (42)$$

where 50 points were used as T.At the moment t2, the reactions in the supports are calculated by the Cramer method with the use of the BC in additional supports from the SLAE, written in matrix form

$$M_{4\times4}\cdot\begin{pmatrix}P_1(t_2)\\P_2(t_2)\\P_3(t_2)\\P_4(t_2)\end{pmatrix}=\begin{pmatrix}-P(t_2)G(q_1,z_1,t_2;X,Y)\\-P(t_2)G(q_2,z_2,t_2;X,Y)\\-P(t_2)G(q_3,z_3,t_2;X,Y)\\-P(t_2)G(q_4,z_4,t_2;X,Y)\end{pmatrix}, \quad (43)$$

$$M_{4\times4}=\begin{pmatrix}G(q_1,z_1,t_2;q_1,z_1)&G(q_1,z_1,t_2;q_2,z_2)&G(q_1,z_1,t_2;q_3,z_3)&G(q_1,z_1,t_2;q_4,z_4)\\G(q_2,z_2,t_2;q_1,z_1)&G(q_2,z_2,t_2;q_2,z_2)&G(q_2,z_2,t_2;q_3,z_3)&G(q_2,z_2,t_2;q_4,z_4)\\G(q_3,z_3,t_2;q_1,z_1)&G(q_3,z_3,t_2;q_2,z_2)&G(q_3,z_3,t_2;q_3,z_3)&G(q_3,z_3,t_2;q_4,z_4)\\G(q_4,z_4,t_2;q_1,z_1)&G(q_4,z_4,t_2;q_2,z_2)&G(q_4,z_4,t_2;q_3,z_3)&G(q_4,z_4,t_2;q_4,z_4)\end{pmatrix}, \quad (44)$$

Then the reactions in the supports are determined: $\Delta=|M_{4\times4}|$.

$$P_i(t_2)=\frac{\Delta_i}{\Delta},\Delta=|M_{4\times4}|, \quad (45)$$

$$\Delta_i = \begin{vmatrix} G(q_1,z_1,t_2;q_1,z_1)\ldots\text{-}P(t_2)G(q_1,z_1,t_2;X,Y)\ldots G(q_1,z_1,t_2;q_4,z_4) \\ G(q_2,z_2,t_2;q_1,z_1)\ldots\text{-}P(t_2)G(q_2,z_2,t_2;X,Y)\ldots G(q_2,z_2,t_2;q_4,z_4) \\ G(q_3,z_3,t_2;q_1,z_1)\ldots\text{-}P(t_2)G(q_3,z_3,t_2;X,Y)\ldots G(q_3,z_3,t_2;q_4,z_4) \\ G(q_4,z_4,t_2;q_1,z_1)\ldots\text{-}P(t_2)G(q_4,z_4,t_2;X,Y)\ldots G(q_4,z_4,t_2;q_4,z_4) \end{vmatrix}, \quad (46)$$

In subsequent iterations, the previous values of the reactions in the supports are taken into account. And the system of equations takes the form:

$$M_{4\times4}\cdot\begin{pmatrix} P_1(t_j) \\ P_2(t_j) \\ P_3(t_j) \\ P_4(t_j) \end{pmatrix} = \begin{pmatrix} Sum_1(t_j) \\ Sum_2(t_j) \\ Sum_3(t_j) \\ Sum_4(t_j) \end{pmatrix}, \quad (47)$$

$$\begin{aligned} Sum_1(t_j) &= \text{-}P(t_j)G(q_1,t_j;X) - \sum_{i=1}^{4}\sum_{k=2}^{j-1} P_i(t_k)G(q_1,t_j,t_{k-1},t_k;q_i), \\ Sum_2(t_j) &= \text{-}P(t_j)G(q_2,t_j;X) - \sum_{i=1}^{4}\sum_{k=2}^{j-1} P_i(t_k)G(q_2,t_j,t_{k-1},t_k;q_i), \\ Sum_3(t_j) &= \text{-}P(t_j)G(q_3,t_j;X) - \sum_{i=1}^{4}\sum_{k=2}^{j-1} P_i(t_k)G(q_3,t_j,t_{k-1},t_k;q_i), \\ Sum_4(t_j) &= \text{-}P(t_j)G(q_4,t_j;X) - \sum_{i=1}^{4}\sum_{k=2}^{j-1} P_i(t_k)G(q_4,t_j,t_{k-1},t_k;q_i). \end{aligned} \quad (48)$$

Then support reactions at the moment $t_j, \quad j = 3..T+1$ are defines as:

$$P_i(t_j) = \frac{\Delta_i}{\Delta},$$

$$\Delta_i = \begin{vmatrix} G(q_1,z_1,t_j;q_1,z_1)\ldots Sum_1(t_j)\ldots G(q_1,z_1,t_j;q_4,z_4) \\ G(q_2,z_2,t_j;q_1,z_1)\ldots Sum_2(t_j)\ldots G(q_2,z_2,t_j;q_4,z_4) \\ G(q_3,z_3,t_j;q_1,z_1)\ldots Sum_3(t_j)\ldots G(q_3,z_3,t_j;q_4,z_4) \\ G(q_4,z_4,t_j;q_1,z_1)\ldots Sum_4(t_j)\ldots G(q_4,z_4,t_j;q_4,z_4) \end{vmatrix}, \quad (49)$$

$$\Delta = |M_{4\times4}|.$$

Substituting the found values and influence functions into the deflection function, we will build several graphs of the deflection of the segment under study (**Figure 5**).

5 CONCLUSIONS

This technique makes it possible to determine the optimal location of additional supports inside a hingedly supported Kirchhoff-Love plate of known thickness and material characteristics that satisfies the condition of structural rigidity: not exceeding the normal deflection to a given value, through the formulation of the problem of an infinite hingedly supported strip with two additional supports.

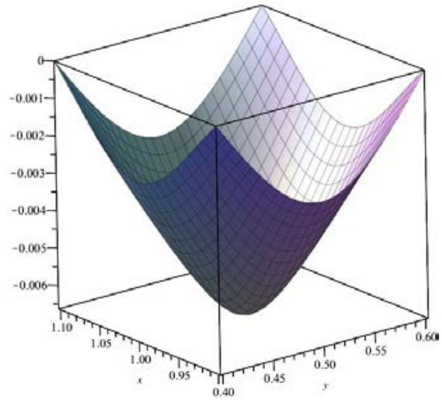

Figure 5: Graph of the deflection of a plate segment at a point in time *t=0.012 c*

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