Two Efficient Methods for Gas Distributive Network Calculation

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Two Efficient Methods for Gas Distributive Network Calculation

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Abstract: Today, two very efficient methods for calculation of flow distribution per branches of a looped gas pipeline are available. Most common is improved Hardy Cross method, while the second one is so-called unified node-loop method. For gas pipeline, gas flow rate through a pipe can be determined using Colebrook equation modified by AGA (American Gas Association) for calculation of friction factor accompanied with Darcy-Weisbach equation for pressure drop and second approach is using Renouard equation adopted for gas pipeline calculation. For the development of Renouard equation for gas pipelines some additional thermodynamic properties are involved in comparisons with Colebrook and Darcy-Weisbach model. These differences will be explained. Both equations, the Colebrook's (accompanied with Darcy-Weisbach scheme) and Renouard's will be used for calculation of flow through the pipes of one gas pipeline with eight closed loops which are formed by pipes. Consequently four different cases will be examined because the network is calculated using improved Hardy Cross method and unified node-loop method. Some remarks on optimization in this area of engineering also will be mentioned.

Keywords: Calculation methods, Flow rate equation, Hydraulic pipeline systems, Natural gas distribution systems, Pipeline networks.

1. Introduction

A pipeline network is a collection of elements such as pipes, compressors, pumps, valves, regulators, heaters, tanks, and reservoirs interconnected in a specific way. In this article focus is on pipes. The behavior of the network is governed by two factors: (i) the specific characteristics of the elements and (ii) how the elements are connected together. Our assumption is that pipes are connected in a smooth way, i.e. so called minor hydraulic loses are neglected. The difficulty to solve the turbulent flow problem in a single pipe lies in the fact that the friction factor is a complex function of relative surface roughness and the Reynolds number. Since the value of the hydraulic resistance depends on flow rate, problem of flow distribution per pipes in a gas distributive looped pipelines have to be solved using some kind of iterative procedures. Similar situation is with electrical resistances when diode is in circuit. With common resistors in electrical circuits where the electrical resistances are not depends on the value of electrical current in the conduit, problem is linear and no iterative procedure has to be used. So problem of flow through single tube is already complex. Despite of it, very efficient procedures are available for solution of flow problem in a complex pipeline such as looped pipeline like natural gas distribution network is.

Here has to be noted that in a municipal gas pipeline, natural gas can be treated as incompressible fluid (liquid) i.e. as water. Even under this circumstance, calculation of water pipelines cannot be literary copied and applied for calculation of gas pipelines. Assumption of gas incompressibility means that it is compressed and forced to convey through pipes, but inside the pipeline system pressure drop of already compressed gas is small and hence further changes in gas density can be neglected. This means that gas is compressible fluid in general, but in a distribution pipeline where the pressure drops can be neglected, natural gas can be treated as incompressible fluid. This is main difference between liquid and incompressible flow. According to this, water flow in pipelines is liquid incompressible flow, while the gas flow is gaseous incompressible flow. Fact is that gas is actually compressed and hence that volume of gas is decreased and then such compressed volume of gas is conveying with constant density through gas distribution pipeline. Hence, mass of gas is constant, but volume is decreased while gas density is according to this, increased.

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Operate pressure for distribution gas network is $4 \cdot 10^5$ Pa abs i.e. $3 \cdot 10^5$ Pa gauge and accordingly volume of gas is decreased four times compared to volume of gas at normal conditions. Hence, velocity of gaseous fluids depends on the pressure in pipe since they are compressible (1):

$$v = \frac{4 \cdot p_{st} \cdot Q_{st}}{p \cdot D_{in}^2 \cdot \pi} = \frac{Q}{D_{in}^2 \cdot \pi}, \quad (1)$$

2. Hydraulics frictions and gas flow rates in pipes

Each pipe is connected to two nodes at its ends. In a pipe network system, pipes are the channels used to convey fluid from one location to another. The physical characteristics of a pipe include the length, inside diameter, roughness coefficient, and minor loss coefficients. The pipe roughness coefficient is associated with the pipe material and age. When fluid is conveyed through the pipe, hydraulic energy is lost due to the friction between the moving fluid and the stationary pipe surface. This friction loss is a major energy loss in pipe flow. Losses of energy or head (pressure) losses depend on the shape, size and roughness of a channel, the velocity density and viscosity of a fluid.

Experiments show that in many cases pressure drop are approximately proportional to the square of the velocity (2):

$$p_1 - p_2 = \lambda \cdot \frac{L}{D_{in}} \cdot \frac{v^2}{2} \cdot \rho, \quad (2)$$

Equation (2) is called the Darcy-Weisbach equation, named after Henry Darcy, a French engineer of the nineteenth century, and Julius Weisbach, a German mining engineer and the scientist of the same era. In previous equation velocity and gas density must be correlated, since the gas is incompressible fluid, and hence for gas is more suitable equation in next form (3) because $Q \cdot \rho = Q_{st} \cdot \rho_{st}$:

$$p_1 - p_2 = \lambda \cdot \frac{L}{D_{in}^5} \cdot \frac{8 \cdot Q^2}{\pi^2} \cdot \rho = \lambda \cdot \frac{L}{D_{in}^5} \cdot \frac{8 \cdot Q_{st}^2}{\pi^2} \cdot \rho_{st}, \quad (3)$$

Density of gas can be noted as (4):

$$\rho = \frac{p \cdot M}{z \cdot R \cdot T}, \quad (4)$$

Considering the momentum equation applied to a portion of pipe length, inside which flows a compressible fluid with an average velocity, for example natural gas, and assuming steady state conditions, general equation for gas flow can be written as (5):

$$\int_1^2 dp + \int_1^2 \lambda \frac{dL}{D_{in}} \frac{v^2}{2} \rho = \int_1^2 \rho dp + \int_1^2 \lambda \frac{dL}{D_{in}} \frac{v^2}{2} \rho^2 = \\ = \frac{M}{z_{avr} \cdot R \cdot T_{avr}} \frac{p_1^2 - p_2^2}{2} + \lambda \frac{\Delta L}{D_{in}} \frac{v^2}{2} \rho^2 = 0, \quad (5)$$

General equation for steady-state gaseous flow includes kinetic energy term, pressure force work term, potential energy and energy dissipation by viscous friction. For a horizontal pipe the potential energy or elevation can be neglected. Also, the kinetic energy is negligible when compared with the other terms. In (6) flow can be used instead of velocity (5) and combined with (4) gives:

$$(v\rho)^2 = \frac{Q^2}{A^2} \rho^2 = \frac{Q_{st}^2}{A^2} \rho_{st}^2 = \frac{16 \cdot Q_{st}^2}{D_{in}^4 \cdot \pi^2} \frac{p_{st}^2 \cdot M^2}{z_{st}^2 \cdot R^2 \cdot T_{st}^2}, \quad (6)$$

Considering that gas density (4) at standard pressure conditions is equal as in average pressure in pipeline ($\rho_{st}=\rho_{avr}$), and finally assuming that for perfect gas $M=M_{air} \cdot \rho_r$, general equation for steady-state flow of gas can be written as (7) [1]:

$$C = p_1^2 - p_2^2 = \lambda \frac{16 \cdot \Delta L \cdot Q_{st}^2}{D_{in}^5 \cdot \pi^2} \frac{p_{st}^2 \cdot M_{air} \cdot \rho_{st}}{z_{st} \cdot R \cdot T_{st}}, \quad (7)$$

Equation (7) for gases is related to (2) for liquids. As it has been shown, for gaseous fluids some laws of thermodynamics also have to be included in general flow equation.

Main parameter related to the hydraulic regime is Darcy's friction factor (λ). Note that the Darcy friction factor is defined in theory as $\lambda=(8 \cdot \tau)/(\rho \cdot v2)$. To predict whether flow will be laminar, hydraulically 'smooth', partially turbulent or fully turbulent, it is necessary to explore the characteristics of flow. Hydraulically 'smooth' regime, characteristic for flow through plastic, i.e. polyethylene pipes is also sort of turbulent regime. For the steel pipes, partially turbulent is most common. In considerations related to the hydraulic frictions has to be very careful because some of the authors use Darcy's friction factor while the others use Fanning's factor. The Darcy's friction coefficient is four times larger than Fanning's while the physical meaning is equal. Graphically, friction factor for known Reynolds number and relative roughness can be determined using well known Moody diagram. The Darcy friction factor and the Moody friction factor are synonyms.

2.1. Gas flow through plastic pipes

When a gas is forced to flow through pipes it expands to a lower pressure and changes its density. Flow-rate, i.e. pressure drop equations for condition in gas distribution networks assumes a constant density of a fluid within the pipes. This means that gas is compressible fluid in general, but in a distribution pipeline where the pressure drops can be neglected, natural gas can be treated as incompressible fluid.

Inner surface of polyethylene pipes which are almost always used in gas distribution networks are practically smooth and hence flow regime in the typical network is hydraulically 'smooth'. For this regime is suitable Renouard's equation [2] adjusted for natural gas flow (8):

$$C = p_1^2 - p_2^2 = \frac{4810 \cdot Q_{st}^{1.82} \cdot L \cdot \rho_r}{D_{in}^{4.82}}, \quad (8)$$

In Renouard's formula flow rate is expressed for standard conditions of pressure and temperature. Renouard's formula is adjusted for gas flow calculations in plastic pipes with no determining of hydraulic resistances. This means that Darcy's friction factor is not necessary to be calculated. This is accomplished by simplification of general steady-state flow equation for gaseous fluids (7).

Using formulation for Darcy friction factor in hydraulically smooth region Renouard suggest his equation for liquid flow (9):

$$\lambda = \frac{0.172}{Re^{0.18}}, \quad (9)$$

In Renoard's equation adjusted for gas pipelines (8) friction factor is rearranged in the way to be expressed using other flow parameters and also using some thermodynamic properties of natural gas.

Using an absolute viscosity of $\mu=1.0757\cdot10^{-5}$ Pa·s, neglecting the potential energy term and assuming that temperature of natural gas is $T_{avr}=T_{st}=288.15$ K, pressure is $p_{st}=1.01325\cdot10^{5}$ Pa and compressibility factor is Z=1, general steady-state flow equation can be simplified as (10):

$$C = p_1^2 - p_2^2 = \frac{4088 \cdot Q_{st}^{1.82} \cdot L \cdot \rho_r^{0.82}}{D_{in}^{4.82}}, \quad (10)$$

It must be stressed that in the development leading to this more simplified equation (10), Renouard used a constant value for city gas kinematic viscosity $\nu=2.2\cdot10^{-5}$ m^2/s. Note that kinematic gas viscosity (ν) and dynamic gas viscosity (μ) is connected using gas density (ρ), this means that $\rho=0.49$ kg/m^3. Having regard that air density is $\rho=1.29$ kg/m^3, one can be concluded that relative density of natural gas is 0.37. Relative density of typical natural gas is 0.64. Assumed flow temperature in Renouard's equation is 15 ºC. This means that by fixing the value of gas cinematic viscosity, the density is also kept fixed, which is physically inaccurate when considering compressible gas flows at medium or high pressure, because the cinematic viscosity of gases is highly dependent upon pressure. Every time a gas with a cinematic viscosity different from the city gas is being used, a multiplying correction factor $(\nu/2.2\cdot10^{-5}\ \mathrm{m^2/s})^{0.18}$·must be applied into (8). This product multiplied with gas density is usually called the corrected density.

Comparing (8) and (10) and considering what has been previously exposed, the error obtained in the calculation of the pressure differential by using (8) instead of (10) is no more than 9%. The use of (8) without the viscosity correction, although quite common in the daily practice, leads to an overestimation on the calculation of the pressure drop, as the pressure difference of C is about 6% to 9% higher than the value obtained through (10).

Regarding to Renouard's formula has to be careful since it does not relate pressure drop but actually difference of the quadratic pressure at the input and the output of pipe. This means that $\sqrt{C}$ is not actually pressure drop in spite of the same unit of measurement is as used for the pressure (Pa). But, for gas pipeline calculation, fact that when $\sqrt{C}\rightarrow0$ that consecutive means that also $C\rightarrow0$ is very useful. Parameter $\sqrt{C}$ can be noted as pseudo-pressure drop.

2.2. Gas flow through steel pipes

For commercial steel pipes, Colebrook [3] showed the transition region of turbulence could be described by (11):

$$\frac{1}{\sqrt{\lambda}} = -2 \cdot \log\left(\frac{2.51}{Re \cdot \sqrt{\lambda}} + \frac{\varepsilon}{3.71 \cdot D}\right), \quad (11)$$

This empirical equation is developed using measurements conducted by Colebrook and White [4]. Colebrook equation also can be noted as (12):

$$\frac{1}{\sqrt{\lambda}} = 1.14 - 2 \cdot \log\left(\frac{\varepsilon}{D} + \frac{9.35}{Re\sqrt{\lambda}}\right), \quad (12)$$

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Colebrook's equation describes a monotonic change in the friction factor from smooth to fully rough (Figure 1).

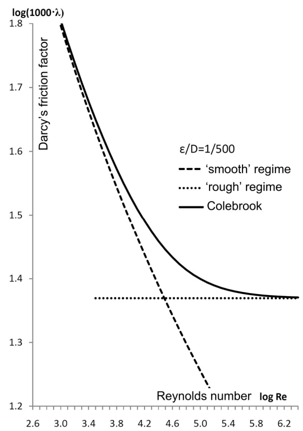

Fig. 1. Colebrook relation make transitional curve among hydraulically "smooth" regime and turbulent rough regime

It is also the basis for the widely used Moody diagram. Many researchers [1] adopt a modification of the Colebrook equation, using the 2.825 constant instead of 2.51 especially for gas flow calculation.

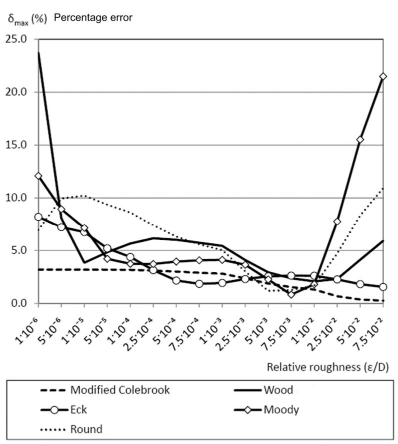

Fig. 2. Distribution of error for most inaccurate approximations of the Colebrook's equation

Colebrook's equation for determination of hydraulic resistances is implicit in fluid flow friction factor and hence it has to be approximately solved using iterative procedure or using some of the approximate explicit formulas developed by many authors. All presented approximations are very accurate (Table 1). Percentage error is less than 3% over the entire domain of Reynolds number [5]. Exceptions are Moody, Wood, Eck and Round approximations (Figure 2). These four formulas should not be used.

While iterative computations are trivial in the context of current computing power, iterative estimation of friction factor can significantly increase the computational burden in complex piping network like here presented where multiple calculations may be necessary. So, flow distribution problem in a complex looped gas pipeline has to be solved using an iterative procedure and further to be more complex, when Colebrook equation is used, additional iterative procedure for computing of friction pipe in every pipe has to be performed. Presented approximations are usually used in computer programs to avoid iterative scheme. But some computers codes continue to use Newton-Raphson iteration scheme for solving to the friction factor. For these methods, finding a good starting guess is often serious difficulty. This can be done very easily using common software tools like MS Excel 2007. Maximal number of iterations in MS Excel 2007 is 32767. To solve for unknown friction factor λ, one must start by somehow estimating the value of friction factor on the right side of the equation, solve for the new friction factor on the left, enter the new value back on the right side, and continue this process until there is a balance on both sides of the equation within an arbitrary difference. This difference must be small yet accommodate all values of relative roughness (ε/D) and values of Reynolds number values without causing endless computations. Note that Colebrook equation consists of two parts; first part is equal to zero in first iteration i.e. $2.51/(Re\cdot\lambda)=0$, but second part has value different than zero $\varepsilon/(D\cdot 3.71)\neq 0$, so estimation of the value in the first iteration is unnecessary. Initial value in the first iteration is $\varepsilon/(D\cdot 3.71)$. Sometimes, effective solutions are too simple on a first sight, and Excel is ideal tool to solve this kind of problem. Excel allows value of accuracy much more than 0.01 (maximal accuracy can be set to 0.0000001). To solve implicit Colebrook-White equation using Excel 'Office button' in the left corner at the top of the screen has to be pressed. Then 'Excel options', and 'Formulas', in sheet 'Formulas' tick box 'Enable iterative calculation' and finally maximum number of iterations (max. allowed is 32767) have to be chosen. Also maximal change allowed between two successive iterations has to be set.

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Table 1. Explicit approximations to Colebrook relation.

Relation	Name
$\lambda \approx 0.0055\cdot\left(1+\left(2\cdot 10^{4}\cdot \varepsilon/D+10^{6}/Re\right)^{1/3}\right)$	Moody
$\lambda \approx 0.094\cdot(\varepsilon/D)^{0.225}+0.53\cdot(\varepsilon/D)+88\cdot(\varepsilon/D)^{0.44}\cdot Re^{-\Psi}\quad \Psi=1.62\cdot(\varepsilon/D)^{0.134}$	Wood
$1/\sqrt{\lambda}\approx -2\cdot\log\left((1/3.715)\cdot(\varepsilon/D)+(15/Re)\right)$	Eck
$1/\sqrt{\lambda}\approx -2\cdot\log\left((1/3.7)\cdot(\varepsilon/D)+5.74/Re^{0.9}\right)\Leftrightarrow 1/\sqrt{\lambda}\approx 1.14-2\cdot\log\left[\varepsilon/D+21.25/Re^{0.9}\right]$	Swamee and Jain
$1/\sqrt{\lambda}=-2\cdot\log\left((1/3.71)\cdot(\varepsilon/D)+(7/Re)^{0.9}\right)$	Churchill
$1/\sqrt{\lambda}\approx -2\cdot\log\left((1/3.715)\cdot(\varepsilon/D)+(6.943/Re)^{0.9}\right)$	Jain
$\lambda\approx 8\cdot\left[(8/Re)^{12}+(\Theta_1+\Theta_2)^{-1.5}\right]^{\frac{1}{12}}$ $\Theta_1=\left[2.457\cdot\ln\left[(7/Re)^{0.9}+0.27\cdot(\varepsilon/D)\right]\right]^{16}$ $\Theta_2=(37530/Re)^{16}$	Churchill*
$1/\sqrt{\lambda}\approx -2.0\cdot\log\left[\begin{matrix}(1/3.7065)\cdot(\varepsilon/D)-\\-(5.0452/Re)\cdot\log\left((1/2.8257)\cdot(\varepsilon/D)^{1.1098}+\left(5.8506/Re^{0.8981}\right)\right)\end{matrix}\right]$	Chen
$1/\sqrt{\lambda}\approx 1.8\cdot\log\left[Re/(0.135\cdot Re\cdot(\varepsilon/D)+6.5)\right]$	Round
$1/\sqrt{\lambda}\approx -2\cdot\log\left(\begin{matrix}(1/3.7)\cdot(\varepsilon/D)+\\+(4.518\cdot\log(Re/7))/\left(Re\left(1+\left(Re^{0.52}/29\right)\cdot(\varepsilon/D)^{0.7}\right)\right)\end{matrix}\right)$	Barr
$1/\sqrt{\lambda}\approx -2\cdot\log\left[(1/3.7)\cdot(\varepsilon/D)-5.02/Re\cdot\log\left(\begin{matrix}(1/3.7)\cdot(\varepsilon/D)-\\-5.02/Re\cdot\log((1/3.7)\cdot(\varepsilon/D)+13/Re)\end{matrix}\right)\right]$ $1/\sqrt{\lambda}\approx -2\cdot\log\left[(1/3.7)\cdot(\varepsilon/D)-5.02/Re\cdot\log((1/3.7)\cdot(\varepsilon/D)+13/Re)\right]$	Zigrang and Sylvester
$1/\sqrt{\lambda}\approx -1.8\cdot\log\left[((1/3.7)\cdot(\varepsilon/D))^{1.11}+6.9/Re\right]$	Haaland
$\lambda\approx\left[\Psi_1-(\Psi_2-\Psi_1)^2/(\Psi_3-2\Psi_2+\Psi_1)\right]^{-2}\quad \lambda\approx\left[4.781-(\Psi_1-4.781)^2/(\Psi_2-2\Psi_1+4.781)\right]^{-2}$ $\Psi_1=-2\cdot\log((1/3.7)\cdot(\varepsilon/D)+12/Re)\quad \Psi_2=-2\cdot\log((1/3.7)\cdot(\varepsilon/D)+(2.51\cdot\Psi_1)/Re)$ $\Psi_3=-2\cdot\log((1/3.7)\cdot(\varepsilon/D)+(2.51\cdot\Psi_2)/Re)$	Serghides
$1/\sqrt{\lambda}\approx -2\cdot\log\left((1/3.7)\cdot(\varepsilon/D)+95/Re^{0.983}-96.82/Re\right)$	Manadilli
$1/\sqrt{\lambda}\approx -2\cdot\log\left\{\begin{matrix}(1/3.7065)\cdot(\varepsilon/D)-\\-(5.0272/Re)\cdot\log\left[\begin{matrix}(1/3.827)\cdot(\varepsilon/D)-\\-(4.567/Re)\cdot\log\left(\begin{matrix}((1/7.7918)\cdot(\varepsilon/D))^{0.9924}+\\+(5.3326/(208.815+Re))^{0.9345}\end{matrix}\right)\end{matrix}\right]\end{matrix}\right\}$	Romeo, Royo and Monzón
$1/\sqrt{\lambda}\approx 0.8686\cdot\ln\left[(0.4587\cdot Re)/S^{S/(S+1)}\right]$ $S=0.124\cdot Re\cdot(\varepsilon/D)+\ln(0.4587\cdot Re)$	Sonnad and Goudar
$1/\sqrt{\lambda}=\alpha-\left[(\alpha+2\cdot\log(\beta/Re))/(1+2.18/\beta)\right]\quad \alpha=((0.774\cdot\ln(Re))-1.41)/\left(1+1.32\cdot\sqrt{\varepsilon/D}\right)$ $\beta=(1/3.7)\cdot(\varepsilon/D)\cdot Re+2.51\cdot\alpha$	Buzzelli [6]
$1/\sqrt{\lambda}\approx -2\cdot\log\left(10^{-0.4343\cdot S}+(1/3.71)\cdot(\varepsilon/D)\right)\; 1/\sqrt{\lambda}\approx -2\cdot\log((2.18\cdot S)/Re+(1/3.7)\cdot(\varepsilon/D))$ $S=\ln(Re/(1.816\cdot\ln(1.1\cdot Re/\ln(1+1.1\cdot Re))))$	Brkić [7]

*also cover laminar regime

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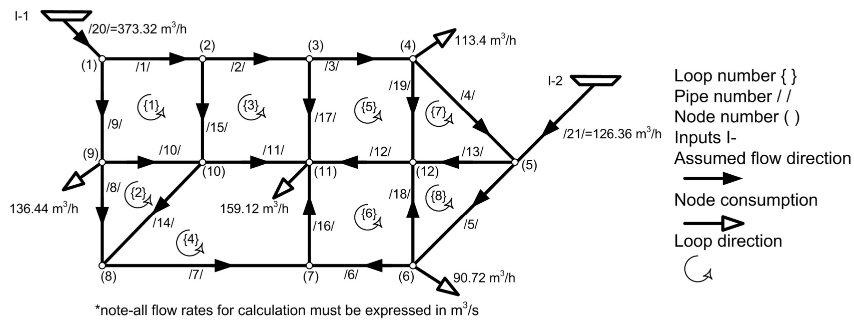

Fig. 3. Example of looped gas distributive pipeline network

When the Darcy's friction factor is finally calculated using Colebrook's equation, it has to be put in general steady-state gas flow equation (7).

3. Looped gas pipeline calculations

All methods for looped gas pipeline calculations assume equilibrium among pressure and friction forces in steady and incompressible flow. As a result, they cannot be successfully used in unsteady and compressible flow calculations with large pressure drop where inertia force is important. Minor drop of pressure in the networks for gaseous fluid distribution enables to treat this fluid as incompressible, i.e. as water. Of course, some different approach must exist, but problem is not much different. Since, the resistances in hydraulic networks depend on flow, problem is not linear like in electric circuits, and iterative procedure must be used.

$$\left.\begin{array}{ll}
-Q_{/1/}-Q_{/9/}+Q_{(1)-\text{input}}=0 & \text{node}_{(1)}\\
Q_{/1/}-Q_{/2/}-Q_{/15/}=0 & \text{node}_{(2)}\\
Q_{/2/}-Q_{/3/}-Q_{/17/}=0 & \text{node}_{(3)}\\
Q_{/3/}-Q_{/4/}-Q_{/19/}-Q_{(4)-\text{output}}=0 & \text{node}_{(4)}\\
Q_{/4/}-Q_{/5/}-Q_{/13/}+Q_{(5)-\text{input}}=0 & \text{node}_{(5)}\\
Q_{/5/}-Q_{/6/}-Q_{/18/}-Q_{(6)-\text{output}}=0 & \text{node}_{(6)}\\
Q_{/6/}+Q_{/7/}-Q_{/16/}=0 & \text{node}_{(7)}\\
-Q_{/7/}+Q_{/8/}+Q_{/14/}=0 & \text{node}_{(8)}\\
-Q_{/8/}+Q_{/9/}-Q_{/10/}-Q_{(9)-\text{output}}=0 & \text{node}_{(9)}\\
Q_{/10/}-Q_{/11/}-Q_{/14/}+Q_{/15/}=0 & \text{node}_{(10)}\\
Q_{/11/}+Q_{/12/}+Q_{/16/}+Q_{/17/}-Q_{(11)-\text{output}}=0 & \text{node}_{(11)}\\
-Q_{/12/}+Q_{/13/}+Q_{/18/}+Q_{/19/}=0 & \text{node}_{(12)}-\text{ref.}
\end{array}\right\}, \quad (13)$$

To solve flow distribution problem in the looped pipeline shown in figure 3, maximal consumption for each node including one of more inlet nodes has to be determined. In figure 3 inlet nodes are 1 (through pipe 20) and 5 (through pipe 21) with inlet rates shown in Figure 1. Four outlet nodes also exist in the example network from Figure 1 and these nodes are 4, 6, 9 and 11. Outlet flow rates for these nodes are also shown in Figure 3. All other nodes are neither inlet nor outlet nodes. First assumed flows are chosen to satisfy first Kirchhoff's law (13). Pipe diameters and node input and output cannot be changed during the iterative procedure. Goal is to find final flow distribution for pipeline system from Figure 3. Second Kirchhoff's law has to be satisfied with demanded accuracy at the end of calculation (14), i.e. $L_{\{x\}}\approx 0$.

$$\left.\begin{array}{ll}
-C_{/1/}+C_{/9/}+C_{/10/}-C_{/15/}=L_{\{1\}} & \text{Loop}_{\{1\}}\\
C_{/8/}-C_{/14/}-C_{/10/}=L_{\{2\}} & \text{Loop}_{\{2\}}\\
C_{/2/}+C_{/15/}+C_{/11/}-C_{/17/}=L_{\{3\}} & \text{Loop}_{\{3\}}\\
-C_{/11/}+C_{/14/}+C_{/7/}+C_{/16/}=L_{\{4\}} & \text{Loop}_{\{4\}}\\
-C_{/3/}+C_{/17/}-C_{/12/}-C_{/19/}=L_{\{5\}} & \text{Loop}_{\{5\}}\\
-C_{/6/}+C_{/12/}-C_{/16/}+C_{/18/}=L_{\{6\}} & \text{Loop}_{\{6\}}\\
-C_{/4/}+C_{/19/}-C_{/13/}=L_{\{7\}} & \text{Loop}_{\{7\}}\\
C_{/13/}-C_{/18/}-C_{/5/}=L_{\{8\}} & \text{Loop}_{\{8\}}
\end{array}\right\}, \quad (14)$$

3.1. The improved Hardy Cross method

Hardy Cross iterative method with its modification by Epp and Fowler [8] today is widely used for calculation of fluid flow through pipes or related pipes' diameters in loops-like distribution networks of conduits with known node fluid consumptions. Original Hardy Cross method is some sort of single adjustment methods threats equations by equations, while the improved version treats whole system of equations simultaneously. In both version of the Hardy Cross method, results of calculation per iterations is correction of flow ΔQ rather than flow Q (in optimization problem, results of calculation per iterations is correction of pipe diameter rather than diameter). Unfortunately, these corrections should be added to or subtracted from flow (or diameter in inverse problem) calculated in previous iteration using some kind of complex algebraic rules [9].

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$$\begin{bmatrix}
L'_{\{1\}} & -\frac{\partial C(Q)_{/10/}}{\partial Q_{\{1\}}} & -\frac{\partial C(Q)_{/15/}}{\partial Q_{\{1\}}} & 0 & 0 & 0 & 0 & 0 \\
-\frac{\partial C(Q)_{/10/}}{\partial Q_{\{2\}}} & L'_{\{2\}} & 0 & -\frac{\partial C(Q)_{/14/}}{\partial Q_{\{2\}}} & 0 & 0 & 0 & 0 \\
-\frac{\partial C(Q)_{/10/}}{\partial Q_{\{3\}}} & 0 & L'_{\{3\}} & -\frac{\partial C(Q)_{/11/}}{\partial Q_{\{3\}}} & -\frac{\partial C(Q)_{/17/}}{\partial Q_{\{3\}}} & 0 & 0 & 0 \\
0 & -\frac{\partial C(Q)_{/14/}}{\partial Q_{\{4\}}} & -\frac{\partial C(Q)_{/11/}}{\partial Q_{\{4\}}} & L'_{\{4\}} & 0 & -\frac{\partial C(Q)_{/16/}}{\partial Q_{\{4\}}} & 0 & 0 \\
0 & 0 & -\frac{\partial C(Q)_{/17/}}{\partial Q_{\{3\}}} & 0 & L'_{\{5\}} & -\frac{\partial C(Q)_{/12/}}{\partial Q_{\{5\}}} & -\frac{\partial C(Q)_{/19/}}{\partial Q_{\{5\}}} & 0 \\
0 & 0 & 0 & -\frac{\partial C(Q)_{/16/}}{\partial Q_{\{6\}}} & -\frac{\partial C(Q)_{/12/}}{\partial Q_{\{6\}}} & L'_{\{6\}} & 0 & -\frac{\partial C(Q)_{/18/}}{\partial Q_{\{6\}}} \\
0 & 0 & 0 & 0 & -\frac{\partial C(Q)_{/19/}}{\partial Q_{\{7\}}} & 0 & L'_{\{7\}} & -\frac{\partial C(Q)_{/13/}}{\partial Q_{\{7\}}} \\
0 & 0 & 0 & 0 & 0 & -\frac{\partial C(Q)_{/18/}}{\partial Q_{\{8\}}} & -\frac{\partial C(Q)_{/13/}}{\partial Q_{\{8\}}} & L'_{\{8\}}
\end{bmatrix}
x
\begin{bmatrix} \Delta Q_{\{1\}} \\ \Delta Q_{\{2\}} \\ \Delta Q_{\{3\}} \\ \Delta Q_{\{4\}} \\ \Delta Q_{\{5\}} \\ \Delta Q_{\{6\}} \\ \Delta Q_{\{7\}} \\ \Delta Q_{\{8\}} \end{bmatrix}
=
\begin{bmatrix} L_{\{1\}} \\ L_{\{2\}} \\ L_{\{3\}} \\ L_{\{4\}} \\ L_{\{5\}} \\ L_{\{6\}} \\ L_{\{7\}} \\ L_{\{8\}} \end{bmatrix}, \quad (15)$$

These rules can be implemented in a MS Excel spreadsheet. Lack of space prevents here detail discussion on these rules.

3.2. The node-loop method

The node matrix with all node included are not linearly independent [10]. To obtain linear independence any row of the node matrix [N] has to be omitted (16). No information on the topology in that way will be lost. Node 12 will be noted as referential and hence will be virtually omitted from the calculation. For the node-loop method calculation, using matrix [V] (16) and the node-loop matrix [NL] formed to unite both, the node matrix [N] (17) and the loop matrix [L] (18). First eleven rows in [NL] matrix are from the first Kirchhoff's law (matrix [N]), and next eight rows are from the second Kirchhoff's law (matrix [L]) where each term is multiplied with first derivative (for each pipe) of C where Q is treated as variable.

$$[V]=\begin{bmatrix}
-\left|Q_{(1)-input}\right| \\
0 \\
0 \\
\left|Q_{(4)-output}\right| \\
-\left|Q_{(5)-input}\right| \\
\left|Q_{(6)-output}\right| \\
0 \\
0 \\
\left|Q_{(9)-output}\right| \\
0 \\
\left|Q_{(11)-output}\right| \\
-L_1+\left(\left|C'_{/1/}\right|\cdot Q_{/1/}+\left|C'_{/9/}\right|\cdot Q_{/9/}+\left|C'_{/10/}\right|\cdot Q_{/10/}+\left|C'_{/15/}\right|\cdot Q_{/15/}\right) \\
-C_2+\left(\left|C'_{/8/}\right|\cdot Q_{/8/}+\left|C'_{/10/}\right|\cdot Q_{/10/}+\left|C'_{/14/}\right|\cdot Q_{/14/}\right) \\
-L_3+\left(\left|C'_{/2/}\right|\cdot Q_{/2/}+\left|C'_{/11/}\right|\cdot Q_{/11/}+\left|C'_{/15/}\right|\cdot Q_{/15/}+\left|C'_{/17/}\right|\cdot Q_{/17/}\right) \\
-L_4+\left(\left|C'_{/7/}\right|\cdot Q_{/7/}+\left|C'_{/11/}\right|\cdot Q_{/11/}+\left|C'_{/14/}\right|\cdot Q_{/14/}+\left|C'_{/16/}\right|\cdot Q_{/16/}\right) \\
-L_5+\left(\left|C'_{/3/}\right|\cdot Q_{/3/}+\left|C'_{/12/}\right|\cdot Q_{/12/}+\left|C'_{/17/}\right|\cdot Q_{/17/}+\left|C'_{/19/}\right|\cdot Q_{/19/}\right) \\
-L_6+\left(\left|C'_{/6/}\right|\cdot Q_{/6/}+\left|C'_{/12/}\right|\cdot Q_{/12/}+\left|C'_{/16/}\right|\cdot Q_{/16/}+\left|C'_{/18/}\right|\cdot Q_{/18/}\right) \\
-L_7+\left(\left|C'_{/4/}\right|\cdot Q_{/4/}+\left|C'_{/13/}\right|\cdot Q_{/13/}+\left|C'_{/19/}\right|\cdot Q_{/19/}\right) \\
-L_8+\left(\left|C'_{/5/}\right|\cdot Q_{/5/}+\left|C'_{/13/}\right|\cdot Q_{/13/}+\left|C'_{/18/}\right|\cdot Q_{/18/}\right)
\end{bmatrix} \quad (16)$$

$$[N]=\begin{bmatrix}
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0
\end{bmatrix}, \quad (17)$$

$$[L]=\begin{bmatrix}
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0
\end{bmatrix}, \quad (18)$$

Now, unknown flows can be calculated directly using (19):

$$[Q]=inv[NL]x[V], \quad (19)$$

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4. Optimization problem

In previous text, flow distribution problem is solved using (20):

$$\frac{\partial\left(p_1^2-p_2^2\right)}{\partial Q}=\frac{\partial C(Q)}{\partial Q}=\frac{\partial\left(\frac{4810\cdot Q_{st}^{1.82}\cdot L\cdot \rho_r}{D_{in}^{4.82}}\right)}{\partial Q}=,\qquad (20)$$
$$=\frac{1.82\cdot 4810\cdot Q_{st}^{0.82}\cdot L\cdot \rho_r}{D_{in}^{4.82}}$$

In the problem of optimization of pipe diameters, flow is not any more treated as variable (21):

$$\frac{\partial\left(p_1^2-p_2^2\right)}{\partial D_{in}}=\frac{\partial C(D_{in})}{\partial D_{in}}=\frac{\partial\left(\frac{4810\cdot Q_{st}^{1.82}\cdot L\cdot \rho_r}{D_{in}^{4.82}}\right)}{\partial D_{in}}=,\qquad (21)$$
$$=\frac{-4.82\cdot 4810\cdot Q_{st}^{1.82}\cdot L\cdot \rho_r}{D_{in}^{5.82}}$$

Of course, some other adaptations of previously show method should be done for optimization problem. As the diameters have to be chosen among a finite set of available nominal values, optimization problem is highly combinatorial.

5. Conclusions

Compared modified Hardy Cross method and the node loop method, taking as a criterion the number of iteration to achieve final results, both presented methods are equally good. For more complex networks, using the node-loop method, number of required iteration is smaller even compared with the modified Hardy Cross method. But main strength of the node-loop method lays in the fact that it does not required complex numerical scheme for algebraic addition of corrections in each of iterations. In the node-loop method, final results of each of the iterations are flows directly and not correction of flows.

Both methods can be used for calculation of gas pipelines made with steel or plastic pipes using the appropriate equation according to discussion in this paper.

Nomenclature

p pressure, (Pa)

L length of pipe, (m)

D diameter of pipe, (m)

v velocity, (m/s)

Q flow (m^3/s)

T temperature (K)

z gas compressibility factor (-)

M relative molecular mass (-)

R universal gas constant = 8314.41 J/(kmol·K)

Greek symbols

μ gas dynamic viscosity (Pa·s)

ε inside pipe wall roughness (m)

λ Darcy friction factor or coefficient (-)

ρ density (kg/m^3)

Subscripts and superscripts

r relative

st standard (T_{st}=288.15 K, p_{st}=101325 Pa)

avr average

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