1st Latin-American Workshop on Structural Health Monitoring
LATAM-SHM 2023
December, 5-8 2023

MODAL FILTRATION WITHOUT CALCULATION OF THE MODAL MODEL

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Key words: Modal filtration, Modal analysis, Evolutionary computation, Optimization, Frequency-based quality metric

Abstract. Modal filtration is a prominent tool in mechanical systems research. However, the application of modal filtration requires a prior calculation of the modal model consisting of a set of natural frequencies, modal damping coefficients, and mode shapes. Such a procedure can be costly and difficult to perform in practice.
In this work, an alternative approach is presented: modal filtration can be performed with perfect accuracy without knowledge of the modal model of the structure, provided only that natural frequencies and frequency response functions for all the sensors are known. In our approach, we take the frequency spectrum of the object in question and optimize from scratch spatial filters built with the same goal in mind as in standard modal filtration. An evolutionary algorithm is used to this end. Such a choice is dictated by the necessity to maintain the reliability of optimization in the presence of local minima. The fitness of individuals is assessed using a quality metric that weighs output frequencies according to their distance to the filtered natural frequency. The solution is presented using simulated data and enables filtration quality comparable with standard modal filtration based on a pre-calculated modal model.

1 INTRODUCTION

Modal filtering is a type of spatial filtration that, using a modal model, allows the removal of components related to natural vibration modes other than the one to which the filter is set from the vibration response of the object [1]. The original purpose of this tool was modal control [2], but over the years, other applications have been developed and implemented. Here you can mention:
- comparison of modal models, and, more precisely, the mode shapes obtained by numerical simulation and experimental research [3],
- identification of forces exciting objects [4],
- damage detection and localization [5]–[9],
In each of the above-mentioned applications, the method's effectiveness is influenced by the achieved filtration quality. This is important in modal control, where imperfect filter operation may lead to the spill-over phenomenon, i.e. the escape of the control signal energy to higher

modes, which may further result in loss of stability [10]. The problem arises when the controlled object begins to change parameters during operation. The causes may be, for example, material degradation or changes in external conditions. To avoid deterioration of modal filtration due to these changes, an adaptive modal filter was developed [11], which adjusts the coefficients to ensure the best performance throughout the operation time.

Perfect filtration quality is also required in the damage detection method proposed by Deraemaeker and Preumont [7], which was later extended to include the possibility of localizing damage [9]. In both cases, the idea behind the method is that the quality of modal filtration deteriorates as a result of structural changes occurring in the object. This deterioration should be identified by the formation of peaks in the modal filter responses at the filtered natural frequencies. Therefore, if the filtration quality is not ideal at the beginning for an undamaged object, false alarms may be generated. Therefore, it is essential that the filtration quality is as good as possible.

To improve the quality of filtration, a number of methods for synthesizing modal filter parameters have been developed. The first historical formulation of a modal filter [2] used a system model with a continuous distribution of parameters:

$$Lu(x,t) + M(x)\frac{\partial^2 u(x,t)}{\partial t^2} = f(x,t) \tag{1}$$

where: x – given point of area D ,

L – linear, Hermitian (self-adjoint) differential operator,

$u(x, t)$ – displacement of point x,

$M(x)$ – mass distribution function,

$f(x,t)$ – load distribution function.

Knowing displacement $u(x,t)$ of every point x of area D, one can define modal filter with the following equation:

$$\eta_r(t) = \int_D M(x)\phi_r(x)u(x,t)dD \tag{2}$$

Since eigenfunctions ϕ_r are orthogonal, the contribution of other modes in vibration signals recorded by sensors is removed from the modal coordinate η_r .

The continuous modal filter has a basic drawback: it operates on continuous functions of the displacement distribution (velocities or accelerations) of the research object. Such a distribution is not available through measurement, so to solve this practical problem, measurements taken at a finite number of points located on the object were interpolated. However, the accuracy of this approach depends on the type of interpolation function and the number of analysed measurement points. In order to achieve satisfactory accuracy, it is necessary to use expensive multi-channel systems. Therefore, in the late 1980s, a discrete modal filter was developed [1], which could be easily implemented on the basis of measurement data taken at a limited number of measurement points. For this case, the synthesis of filter parameters is as described below.

The synthesis of *r*-th modal filter begins with the definition of one degree of freedom transfer function $H_{pp}(\omega)$:

$$H_{pp}(\omega) = \frac{1j}{j\omega - \lambda_r} + \frac{1j^*}{j\omega - {\lambda_r}^*} \tag{3}$$

where: λ_r - r - th pole of the system,
$*$ - complex conjugate number.

Next, to the calculation of reciprocal modal vectors matrix (coefficients of filter modal) Ψ_p the following formula is applied:

$$[\Psi_p] = [H_{kN}]^+ \cdot [H_{pp}] \tag{4}$$

where: $^+$ - pseudo-inverse of the matrix
$H_{kN}(\omega)$ - matrix transfer functions of the object.
r-th reciprocal modal vector ψ_r, is orthogonal to all modal vectors except *r*-th, to which the filter is set. Thanks to this, the system's response is transformed from coordinates related to the object's geometry to a modal coordinate η_r.

$$\eta_r(\omega) = \{\psi_r\}^T \cdot \{y(\omega)\} = \left(\frac{\{\phi_r\}^T}{j\omega - \lambda_r} + \{\psi_r\}^T\{{\phi_r}^*\}\frac{\{{\phi_r}^*\}^T}{j\omega - {\lambda_r}^*}\right) \cdot \{f(\omega)\} \tag{5}$$

where: ϕ_r – *r*-th modal vector
$y(\omega)$ – object responses vector.

Another procedure can be used to determine reciprocal modal vectors for experimental data from objects with small nonlinearities [7]. It uses the same data as the method shown above, i.e. the transfer functios matrix $H_{kN}(\omega)$, and the set of *r* system's poles (giving information about the natural frequencies ω_r and the modal damping coefficients ξ_r). In the first step, the theoretical transfer function at the output of the modal filter is determined:

$$G_{rt}(\omega) = \frac{2\xi_r\omega_r^2}{\omega_r^2 - \omega^2 - 2j\xi_r\omega_r\omega} \tag{6}$$

Next one can calculate reciprocal modal vectors solving the equation:

$$[H_{kN}]\{\psi_r\} = G_{rt}(\omega) \tag{7}$$

There is also a method for determining the parameters of a spatial filter based on operating deflection shapes instead of mode shapes [12]. In order to improve the quality of filtration, methods are also used to optimize the arrangement of sensors measuring response [13] or the already mentioned adaptation of the filter to changes in object parameters. However, each of the methods shown for experimental data does not provide perfect filtration and there is room for improvement of the obtained results. Therefore, in this work it was decided to use optimization techniques based on artificial intelligence to obtain a spatial filter with the best possible filtration quality. What is more, the proposed approach does not require a prior modal analysis: the objective function measuring the quality of filtration is enough to built an operational modal filter equivalent to the one obtained through traditional means and then further improvement to allow for an optimal filtration.

The rest of this paper is as follows: Chapter 2 introduces natural optimization algorithm based on a 1+1 optimization scheme, Chapter 3 describes simulation-based experiments that show the concept in-action, finally, Chapter 4 concludes and summarizes the paper.

2 NATURAL OPTIMIZATION APPROACH TO MODAL FILTERING

From a computational perspective, the objective of modal analysis and modal filter calculation is to determine a set of vectors that meet a specific requirement: Each vector, when employed to filter the recorded structural response, should isolate a single frequency component corresponding to a chosen natural frequency of the object. Traditionally, these filters are computed through analytical methods based on modal analysis. However, it's possible to obtain or optimize these filters using natural optimization techniques, with the aim of maximizing the response at a specific natural frequency while damping all other frequencies.

The optimization process begins by obtaining the frequency response for the object in question. Subsequently, an initial starting point for the optimization is generated. This starting point can be chosen randomly or based on a precomputed modal filter. The optimization process then employs a 1+1 evolutionary scheme [14], which gradually refines the filter.

The fundamental principle behind the 1+1 optimization scheme is to perform a systematic yet partially randomized search within the parameter space. Successive candidate solutions for the minimum are generated in the vicinity of the best result found so far. The rate of improvement is controlled by a variable denoted as "R," which determines the size of the search neighborhood. Larger values of R facilitate rapid improvement at the outset of the optimization, while smaller values allow for more meticulous exploration of the minimum. As the algorithm progresses, R is reduced if the results become stagnant, thus ensuring a balance between exploring different options and exploiting the best solutions. Figure 1 illustrates a flowchart of the algorithm.

The motivation behind adopting this straightforward 1+1 search approach is rooted in the finding that, in preliminary testing, all the local minima of the problem were situated at depths comparable to the global minimum. Therefore, there was no necessity to employ a full-fledged genetic algorithm, making the 1+1 search method a suitable choice.

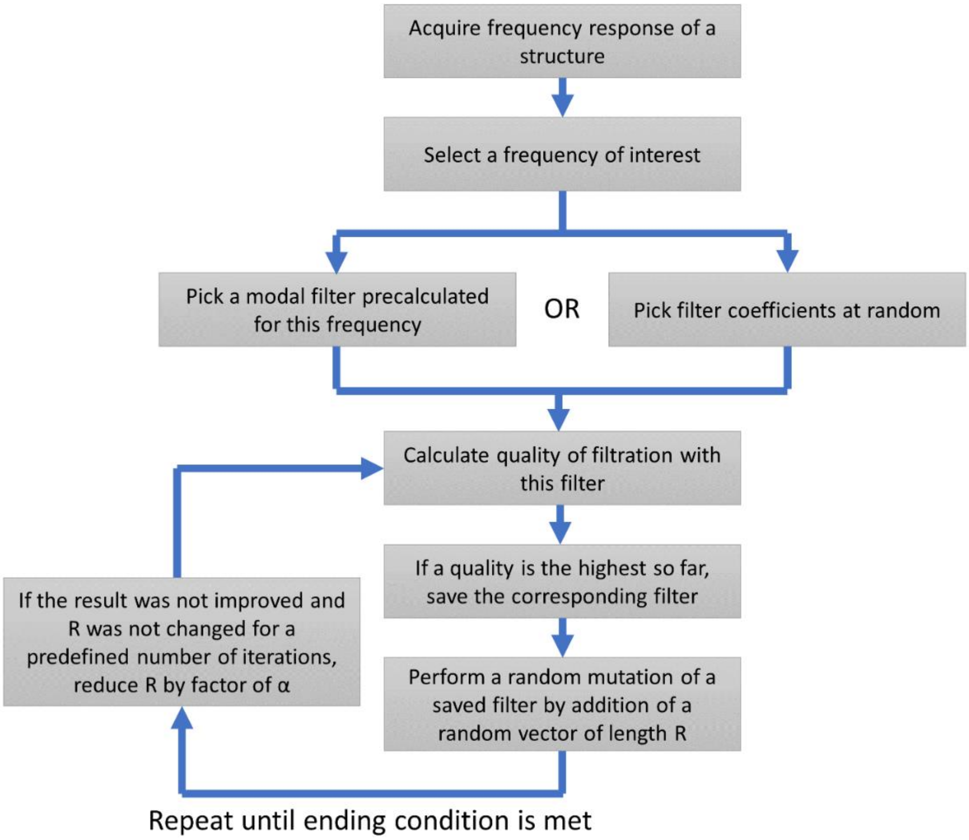

Figure 1: A flowchart of the optimization routine for calculation and improvement of the modal filter for a given natural frequency.

3 EXPERIMENTAL EVALUATION

4.1 Model design

To demonstrate the method's functionality in a simulated setting, we have developed a 5-degree-of-freedom model. This model, depicted in Figure 2, comprises five masses interconnected in a series-parallel configuration. The values for damping, stiffness, and mass coefficients are calculated from the state matrices provided below:

$$M = \begin{bmatrix} 5 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1.5 \end{bmatrix} \quad K = \begin{bmatrix} 105 & -15 & -20 & 0 & 0 \\ -15 & 15 & 0 & 0 & 0 \\ -20 & 0 & 49 & -12 & -17 \\ 0 & 0 & -12 & 12 & 0 \\ 0 & 0 & -17 & 0 & 17 \end{bmatrix} * 10^3 \; C = \begin{bmatrix} 35 & -5 & -5 & 0 & 0 \\ -5 & 5 & 0 & 0 & 0 \\ -5 & 0 & 15 & -5 & -5 \\ 0 & 0 & -5 & 5 & 0 \\ 0 & 0 & -5 & 0 & 5 \end{bmatrix}$$

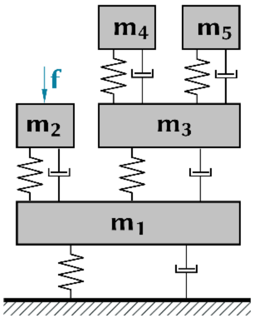

Figure 2: A scheme of the model used in simulation.

4.2 Quality function

The quality function is established with the premise that each modal filter is centered around a specific predefined frequency, through which the vibration signal should be allowed to pass. The equation that defines the mask function is given by equation (1):

$$M(f) = \frac{\tau}{|f - f_n| + \tau} \tag{1}$$

Here, f represents the frequency under consideration, f_n is the selected natural frequency to which the filter is tuned, and τ is a coefficient that governs how abruptly the mask attenuates undesired frequencies. In this study, τ is set to 0.4. Figure 3 illustrates the shape of the mask for frequency filtration, calculated for the 4^{th} natural frequency.

The quality metric that the algorithm aims to minimize is defined as follows (2):

$$Q = Q_0 + w_b \cdot \sum (1 - M) \cdot S - \sum M \cdot S \tag{2}$$

Here:

- S represents the normalized frequency response.
- M stands for the mask.
- w_b is a weight that determines the relative importance of minimizing frequencies across a broad spectrum. In simpler terms, this parameter controls whether the optimization algorithm places more emphasis on reducing unwanted frequencies or preserving frequencies centered around the target frequency.
- Q_0 serves as a quality-of-life constant, useful for displaying all the convergence curves in a one logarithmic-scale plot and is predetermined for each frequency under consideration. It does not influence the actual computation.

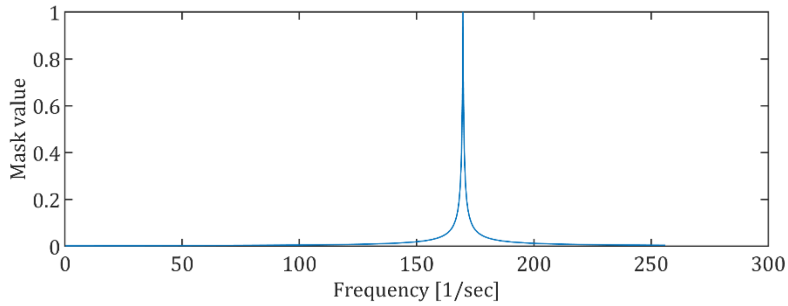

Figure 3: Mask shape for frequency filtration.

4.3 Optimization results

Figure 4 displays the convergence curves for the algorithm employed to optimize modal filters. It's important to note that the level of convergence is influenced by the constant Q_0 integrated into the quality function. As a result, the relative levels in these curves should not be compared with one another. What is meaningful to compare, however, is the convergence rate. Filter optimization for some natural frequencies proves to be more straightforward than for others. Nevertheless, in all cases, the optimized filters, initiated from random vectors without relying on modal analysis, outperform analytically derived filters.

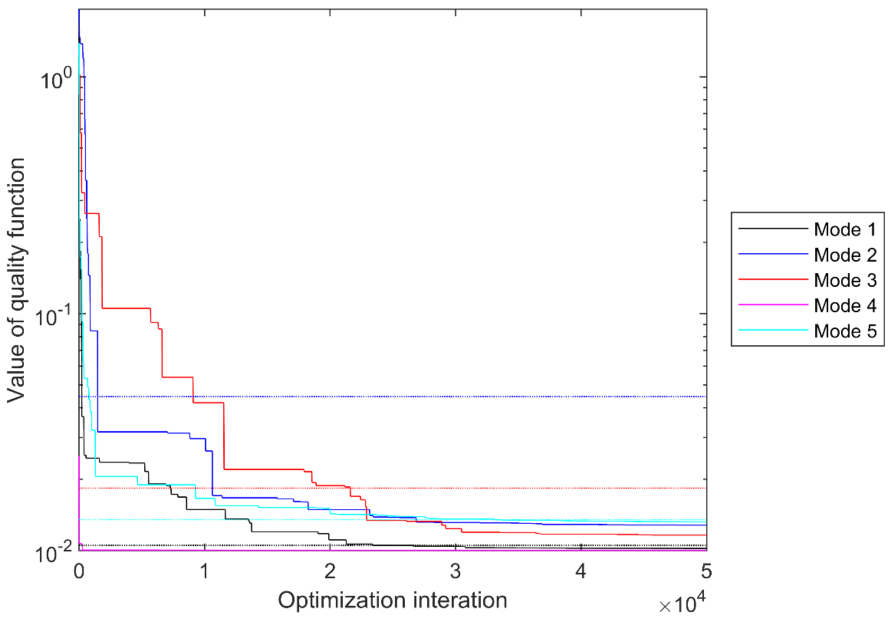

Figure 4: Convergence curves for optimization of filters for five natural modes of vibration. Level of filtration quality provided by analytically computed modal filters is provided in dotted lines.

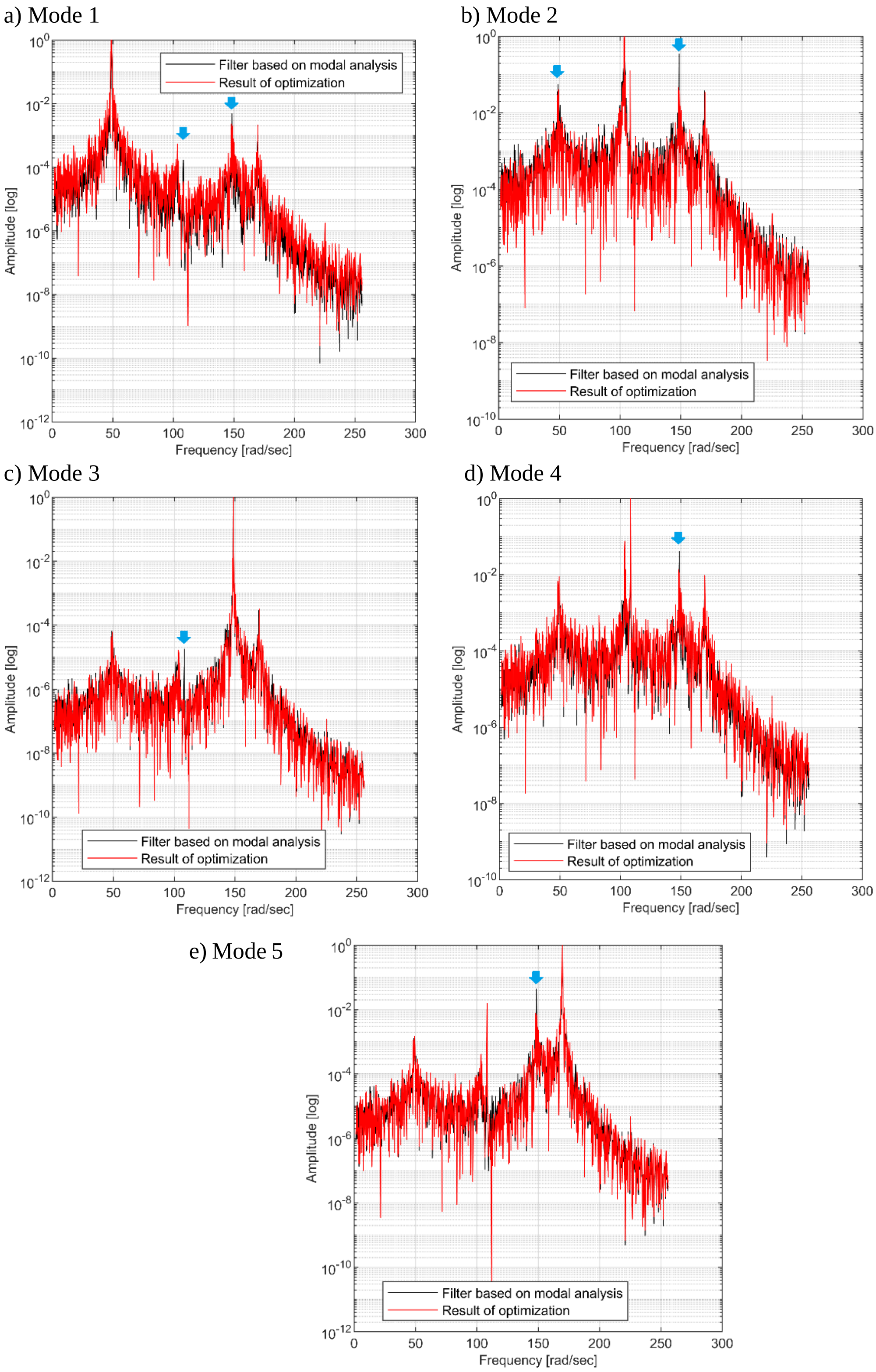

Figure 5: Results of filtration for modal filters tuned to five modes of vibration. Blue arrows point to frequencies for which optimized filter enabled better filtration

Figure 5 presents filtration results in the frequency domain. It compares a response filtered using a modal filter tuned through a modal analysis-based process (in black) and our modal-analysis-free method (in red). While the results in the broad frequency spectrum are similar, the optimized filters result in lower response values for natural frequencies to which the respective filters are not tuned. This indicates an overall superior filtration quality.

A direct comparison between the calculated and optimized filters is not possible due to their differing lengths. However, after normalization based on the vectors' norms, the discrepancies between the optimized and reference vectors (as shown in Table 1) are minimal. This demonstrates that not only does the solution yield superior results compared to the analytical approach, but it also identifies and optimizes the same set of modal filters.

Table 1: Residua between normalized results of optimization and normalized reference vectors

Mode	1	2	3	4	5
Residuum	$r_1 = \begin{bmatrix} 0.0007 \\ -0.0004 \\ 0.0001 \\ 0.0033 \\ -0.0028 \end{bmatrix}$	$r_2 = \begin{bmatrix} 0.0054 \\ -0.0060 \\ 0.0040 \\ 0.0118 \\ -0.0004 \end{bmatrix}$	$r_3 = \begin{bmatrix} 0.0043 \\ 0.0019 \\ 0.0002 \\ 0.0003 \\ 0.0003 \end{bmatrix}$	$r_4 = \begin{bmatrix} 0.0096 \\ 0.0008 \\ -0.0081 \\ 0.0010 \\ 0.0063 \end{bmatrix}$	$r_5 = \begin{bmatrix} -0.0002 \\ 0.0004 \\ 0.0000 \\ -0.0002 \\ 0.0015 \end{bmatrix}$

5 SUMMARY AND CONCLUSIONS

In this study, a method for optimizing modal filters was explored in a simulated environment using a 5-degree-of-freedom model. The model incorporated five masses connected in a series-parallel arrangement. The research primarily focused on the 1+1 optimization scheme that forms the basis of filter optimization: the filters are iteratively optimized starting from a random solution and ending in a fine-tuned filters that surpass results obtained through modal analysis consistently for all five natural frequencies under evaluation.

In conclusion, this study introduced a novel approach for optimizing modal filters without relying on modal analysis. The results highlighted the method's ability to outperform analytically derived filters, particularly in reducing responses at undesired frequencies. These findings suggest the potential for broader application in vibration control and mechanical systems research, especially in detection of structural damages.

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