A posteriori error estimates for the Crank-Nicolson method: application to parabolic partial differential equations with small random input data – ADMOS 2023

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ABSTRACT

In this article, we present residual-based *a posteriori* error estimates for the parabolic partial differential equation (PDE) with small random input data in the $L^2_P(\Omega; L^2(0,T;H^1(D)))$-norm, where $(\Omega, \mathcal{F}, P)$ is a complete probability space, D is the physical domain, $T > 0$ is the final time. Such a class of PDEs arises due to a lack of complete understanding of the physical model. To this end, the perturbation technique [2019, Arch. Comput. Methods Eng., 26, pp. 1313-1377] is exploited to express the exact random solution in terms of the power series with respect to the uncertainty parameter, whence we obtain decoupled deterministic problems. Each problem is then discretized in space by the finite element method and advanced in time by the Crank-Nicolson scheme. Quadratic reconstructions are introduced to obtain optimal bounds in the temporal direction. The work generalizes the isotropic results obtained in [2009, SIAM J. Sci. Comput., 31, pp. 2757-2783] for the deterministic parabolic PDEs to the parabolic PDE with small random input data. Numerical results demonstrate the effectiveness of the bounds.

REFERENCES

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