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NEOMUDEJAR ARCHITECTURE AND ANALYSIS OF LOCAL STRESSES OF MASONRY STRUCTURES: THE ESCUELAS AGUIRRE CASE STUDY

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Abstract. *At the end of the 19th century, an architectural style called Neomudéjar became quite popular in some areas of Spain. Very much like other historicist European styles in the same years, the Neomudéjar sought to recreate the local medieval architecture. The use of faced brick façades with complicated bonds -formed by stretchers and headers in and out the main wall plane- lead in term to a wide variety of results that resembled Arabic architecture. The brick façades of the Neomudéjar buildings are ideal case studies for the analysis of the local behaviour of masonry structures, especially regarding problems of stress concentration. There are several methods for studying the global behaviour of masonry structures – from the classic thrust line to the limit analysis tools – but, as the average stresses taken over by structural masonry elements are usually well below the compression strength of the constituent material, the classical methods of analysis are designed to verify only the global stability. Local behaviour, on the other hand, is quite elusive, especially when the properties of the material are uncertain. In such cases stress concentrations might appear, resulting on stress currents and low stress islets. A particular case of these phenomena occurs in the bonding of Neomudéjar façades. Local concentration of stresses is especially likely in these bonds, given the peculiar relative position of some bricks with respect to others. The paper proposed will use one of these buildings, the Aguirre Schools (Rodríguez Ayuso, Madrid, 1886), as a case study to evaluate local behaviour. Starting from a geometrical hypothesis of the internal distribution of the material based on recent photogrammetric surveys, and using conventional software of parametric design, the paper will describe a numerical model based on a non-deterministic random algorithm, although limited in its number of solutions, to discuss later the validity and scope of them. The limitations of the standard hardware in which these design tools are usually handled will also be considered in the discussion.*

1 INTRODUCTION

The geometric complexity of the bonds used in the masonry walls of *neomudejar* buildings makes them interesting case studies for the analysis of one of the least studied particularities of the mechanical behaviour of masonry structures: local phenomena. Several methods allow the analysis of the global behaviour of this type of structures (thrust line, limit analysis, etc.) – being one or the other more or less adequate depending on the characteristics of the material, the available data and the phenomenon studied. However, the analysis of local behaviour is elusive, especially when the properties and even the existence of the joint material is in doubt and the strength of the bond depends on friction.

The fundamental premise of most methods of global analysis is that the average working stresses are always well below the characteristic strengths of the material. This premise is true for most masonry structures; however, local problems arise, especially on irregular elements such as those in the façades of the *neomudejar* buildings (whose specificity lies in the complexity and the variety of the indented brick bonds). These local problems are not related to the average compression values, but to the possible concentrations of stresses in the contacts between blocks. The patterns that characterize the distributions of the blocks of this architecture normally seek to achieve aesthetic effects (rhythms, regularity or irregularity of the surface, etc) or to improve a certain property of the constructive element (ventilation or shading, for example, when it comes to facades), but they are not intended to respond to specific structural problems.

A mathematical model for the analysis of these stress concentrations, designed specifically for geometries of high complexity built in dry-stack masonry, is proposed in this paper. For this purpose, a constructive and geometric description of a case study (the facade of the Las Aguirre School building, in Madrid) is first performed; then the theoretical framework of the proposal (within the studies on percolation in discontinuous media, understood as branching and regrouping of stress currents) is presented; and, finally, a general description of the model (which, from a strictly mathematical point of view, can be considered a stochastic Markovian process in discrete time) and its implementation (carried out by means of graphical-analytical computer tools) are introduced.

2. BACKGROUND AND CASE STUDY

2.1. Case study: the Escuelas Aguirre building

In order to carry out the following analysis, the facade of a building located in Madrid, the Escuelas Aguirre building (Fig. 1a) will be taken as a case study. The building was built between 1881 and 1887 by the architect Emilio Rodríguez Ayuso at the confluence of the streets of Alcalá and O'Donnell. Rodríguez Ayuso, a historicist architect who had already rehearsed the *neomudejar* style in different buildings (such as the disappeared Plaza de Toros de Madrid, 1874) designed for Lucas Aguirre, promoter of the schools, a two-storey building in a rectangular and symmetrical plan, organized around two large covered interior courtyards (initially an open U-floor at the back) and a central core of stairs, which is topped with a tower located in the central part. The masonry façade was built with brick load-bearing walls adorned with abundant recesses and protrusions.

The numerous extensions that the building underwent in the following years, and the reforms carried out throughout the 20th century, were generally respectful of the *neomudejar* style, to the point of being currently difficult to distinguish from the original construction. A recent study [1] has documented, in addition to the construction in 1896 of the attached pavilions of warehouse and counselling (which were erected following designs by Rodríguez Ayuso), actions of various kinds in 1908-28, 1933, 1999 and 2008. The following analysis is carried out exclusively on the 19th century part of the construction.

Figure 1a. The Escuelas Aguirre building in the 1920s. (Lacoste, circa 1925). **Figures 1b and 1c.** Some views of the brick bonds in different areas of the façade. [1]

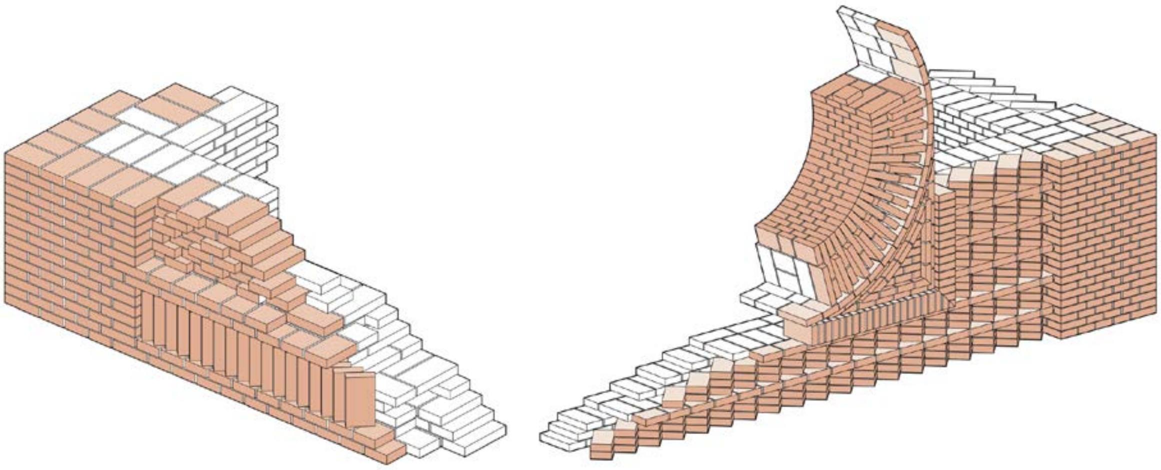
The brick of the main building, brown in colour and 27x13x4.5 cm in measures, is a product of the Jarama riverbank (fig. 1b). It is a piece of regular faces, double frogged, with a special depression on both beds for the placement of the mortar (although it may seem a dry-stack system, this type of masonry rarely is so) that allows the visible joint to be reduced. References to the manufacturer company, SABO [1], appear in the recesses. The bricks of the pavilions, of intense red colour, are mould manufactured and their measures are slightly different, 25.5x12.5x4.8 cm. but the geometry is very similar. The manufacturer's record can also be tracked: in this case it was Eloy Silió's, a factory located in Valladolid [2].

The constructive configuration of the facade is the most common in *neomudejar* buildings. Over the typical headers bond (also called *spanish* in the area) different recesses are made to create diverse figures, mostly rectangular or rhomboid, or to form imposts and cornices (fig. 1c). Jambs and lintels are formed by revoked lintels imitating stone, a mortar line that can also be seen in several impost lines, as well as in the formation of a small decorative bib. The thickness of the wall is variable, decreasing from two and a half bricks (69 cm) on the ground floor, to two (55 cm) on the first floor and lower body of the tower, and to one a half (41 cm) on the Second body of the tower.

The geometric complexity of the bonds used can be noted in figures 2a and 2b. The recesses of the pieces are always proportional to the fourth part of the main dimension of the same. Thus, most common recesses or protrusions are ¼, ½ or ¾ of the main stretcher dimension –that is, the 27 cm. aforementioned. Only occasionally recesses of 1/3 or 1/8, or multiples of these numbers, appear.

2.1. Theoretical framework

This paper attempts to extend the use of methods that have already been used, first in the study of granular media [3] and later in dry-stack masonry, for the analysis of the local behaviour of structures with other types of discontinuities and geometric irregularities caused by the elimination of parts or parts of them and/or by deterioration of the joint material. It is convenient to briefly review some examples of these methods.



Figures 2a and 2b. Brick disposition of the façades of the Escuelas Aguirre. [1]

A large majority of these studies consist of an experimental part, very often carried out by photoelastic tests [4], and some kind of theoretical approach: analytical, or more often numerical, consisting of a simulation of the behaviour of the studied media [5]. Photoelastic tests allow visualizing a locally random behaviour with the appearance of force chains or, more properly, force networks.

The application of these methods to ordered (orthotropic) granular media was carried out, according to Bigoni, first by Rajchenbach [6]. Bigoni's studies on dry masonry walls [7] included not only a photoelastic analysis but also, in a second article, a numerical simulation model compatible with the photoelastic results obtained. In this model, and in order to comply with the equilibrium conditions in each node, a branching occurs, which can be regrouped into subsequent nodes: "The result of the procedure is a form of random cascade admitting random coalescence, in addition to random branching". The photoelastic tests have been partially repeated later by Baig [8] and Magdalena [9], with different objectives and scope, obtaining coincident results.

Although the branching and regrouping model proposed by Bigoni comes to qualitatively similar solutions to the results of photoelastic tests, a later article [10] using a Monte Carlo Simulation through Linear Programming with random contact points, shows, based on the experimental results from a previous load test published by Magdalena [11] that the branching scheme can be improved. However, attempts to transfer this last method to truly three-dimensional cases have so far resulted in high rates of rejection that are excessively expensive from a computational point of view.

This article proposes assuming a model based on percolation, such as that cited by Bigoni or Shahriar's [12]. The concept of percolation corresponds in geology to phenomena such as the slow passage of fluids through a porous medium. The use of the concept of percolation has been extended to a large number of physics topics, as well as other fields: complex networks, epidemiology, etc.

A combinatorial approach is often used to study percolation. This has sometimes materialized as a Random Walk on Graph [13], which implies starting at some vertex and traversing the graph according to a random algorithm. The random walk, a concept introduced by Pearson in a very brief annotation in Nature [14], is a mathematical formalization of the result obtained from taking a series of random steps and has been used in many fields [15]. The concept refers in essence to a unique walk that advances through a series of steps in time.

The formulation proposed below retains the concept of randomness and is based on the resolution of a sequence of equilibrium equations in which the actions are obtained from the results of the previous stage and the reactions are established in a series of points of random application. To meet the equilibrium conditions in each node, a branch can be produced, so the path is not unique but branched in most of them. The procedure refers to an oriented sequence in which the randomness in each stage is conditioned by the results of the previous stage, thus giving a much lower rejection rate than that resulting from the random fixation of the application points independently. In the case of the proposed study, and returning to the percolation image, it is a matter of random diffusion but subjected to the action of the gravity of a system of charges through an orthotropic medium. The name of Directed Stress Percolation [again 13] is applicable, as it has been used, in the field of granular media, to study the transmission of stresses between –grains.

3. MODEL DESCRIPTION

3.1. The constituent element

The constituent element of the model (fig. 3a) is a solid of parallelepiped geometry with sides A, B and C –the relationship between sides being $A=2B=4C$. In the model described below, each element will be superimposed on another of identical geometry, resting on the larger faces AB, at as many heights as necessary, although the basic tool is designed to work in a number of up to 6 to limit the computational cost. The distribution of the element’s overlays may vary, of course, depending on the bond of the masonry.

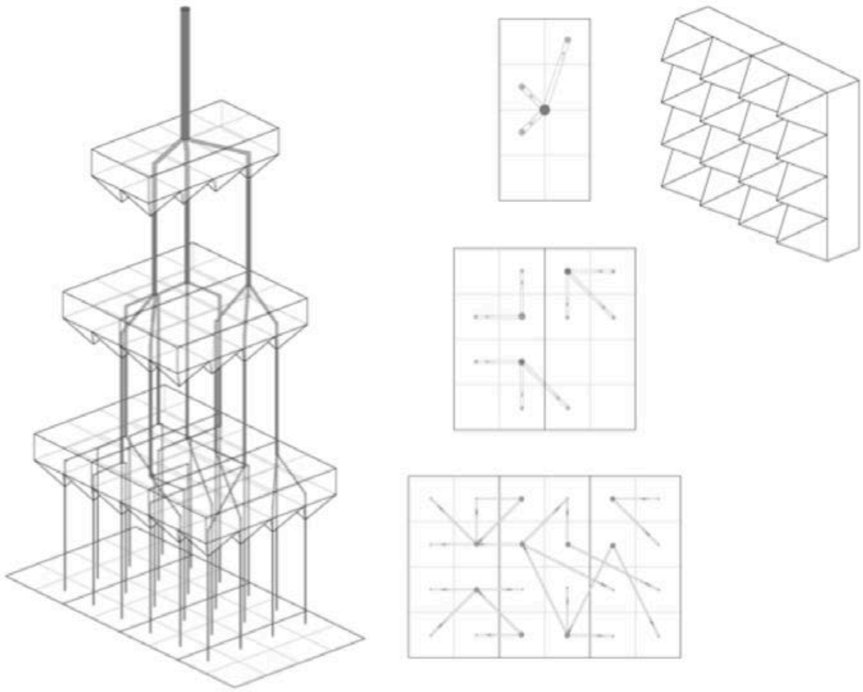
The contact between solids can occur through a limited number of points n, located exclusively at the base AB of each element. One of the premises of the model is that there is no contact on the AC or BC side faces. These n points are distributed homogeneously on the contact surfaces. To determine n, the generator algorithm establishes a grid on the surface AB and allows the values of n_a and n_b to be adjusted –both being positive integers. Obviously, the value of n is n_ax n_b. Typically, n_a will be of the order of double n_b, (being, as stated, A=2B) but it should not necessarily be so. The distance d between the outer plane of the masonry and the recessed plane cannot be any: it will be determined by the relationship between A and n_a, in the form

$$d = p\frac{A}{n_a}$$

p being a number to represent the indentations of the recessed plane, an integer that meets:

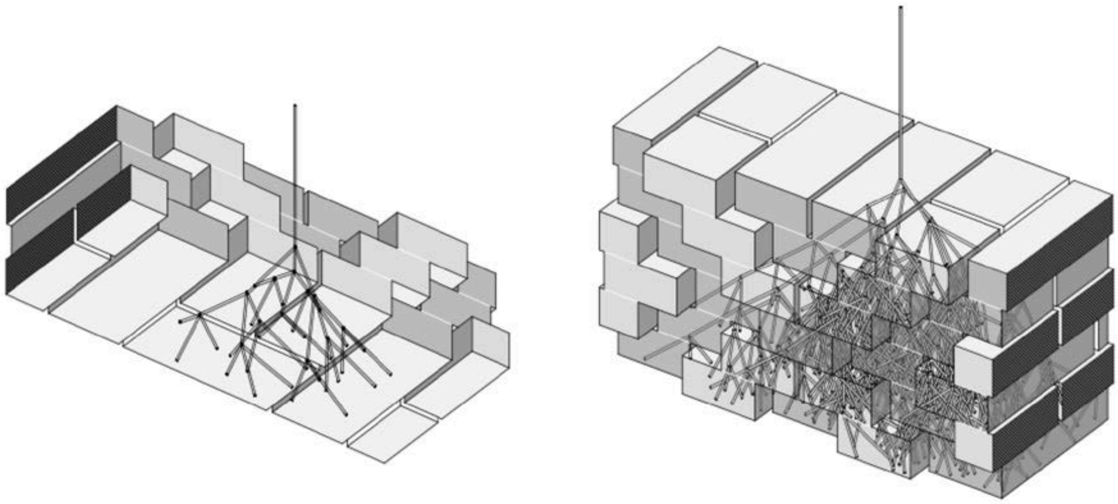
$$0 < p \leq n_a - 2$$

The figures attached (figs. 3a and 3b) are intended to facilitate the physical understanding of the constituent element by representing the possible points of contact as diamond tips. The numerical model does not need to replicate this type of geometry to offer verifiable results. In a physical model, the contact surface at those points would depend on the resistance of the material; in the numerical model, as no limitations for compression stresses are established, this consideration is unnecessary.



Figures 3a y 3b: Description of the constituent element. Contacts points are represented as diamond tips in the lower surfaces of the blocks.

On a finite set of elements such as those described above, stacked vertically and connected through a specific and identical number of points under each block, the proposed model will draw, through a computer algorithm, a tree of paths, a particular case of stochastic process in discrete time in which the time factor is replaced by vertical advance. From a generic point of view, this is a non-deterministic model, although limited in its number of solutions, and subject to various boundary conditions –the fundamental being the one referring to the action and reaction points in the constituent element, since the algorithm forces a certain degree of discretization to always be in one or more of the *n* positions described in the previous section, both on the upper face and in the bottom of the block. This will affect not only the initial punctual action but the entire chain of reactions described below.



Figures 4a y4b: Application to one of the walls in the Escuelas Aguirre building.

The algorithm will start from a value of the initial action Q_y and its position A (which can be entered by the user or self-generated) to randomly search for a triad of points B, C and D (among the n possible contact points described above under the first block) such that the reactions in B, C and D make possible the static equilibrium of the piece without introducing additional compatibility equations. As is evident, only a limited number of BCD triads can offer static equilibrium possibilities against the action exerted on A. Once the triad of contacts has been determined, the algorithm automatically assigns the R_B, R_C, and R_D, reactions, that must compensate Q_A so that

$$Q_A = R_B + R_C + R_D$$

A conventional isostatic system would require not only the obvious equilibrium of actions and reactions but also that of the moments resulting from them, $\sum \mathrm{M}_A = 0$. However, this premise cannot be fulfilled for any random point: the position of A with respect to B, C and D should be such that A is contained, in plan, in the triangle BCD (in the understanding that the reactions R_B, R_C, and R_D must always be of opposite sign to the action; in dry-stack masonry constructions the tensile strength should be considered zero). It is obvious that if A is found in one of the corners of the element, the only possible reaction would be at a single point B, being $Q_A = R_B$ and C and D redundant, given that necessarily $R_C=R_D=0$. In the event that A was found on one of the faces of the element, a similar situation would occur, although in this case there could be two reactions, so that

$$Q_A = R_B + R_C$$

Only one position, D, would be redundant in this case, logically being $R_D = 0$. Thus, the probability of an event occurring is conditioned by the probability of the immediately preceding event. This possibility must be taken into account in bonds such as the one discussed here, given the special arrangement of the blocks (figs. 4a and 4b).

With regard to mechanical analysis, the final model can therefore be considered a set of rigid solids in unilateral, dry and direct contact, with finite friction and following a nonassociative friction-slip law. This model tries to represent a material with compressive strength far superior to that needed to assume the stresses to which it is subjected and reduced tensile strength, although not zero. This last consideration significantly increases the set of statically permissible solutions within the possible combinations.

3.2 Geometric model

The model corresponds to the outer elements of one of facades of the Aguirre Schools, at the height of the second floor (Fig. 5). Six layers have been modelled using computer aided design software Rhinoceros [16]. Each of the bricks is made up of a Brep Box type entity, which is arranged by separating the space corresponding to the joint. The base piece has dimensions of 20 x 10 x 5 cm, with vertical joints 0.5 cm thick and horizontal joints 1 cm thick.

The studied surface has a total dimension of 66 x 23 x 35 cm, composed of both complete pieces and half pieces at the ends (Fig. 5). In the upper part there is a centred piece that will be the one that receives the load in the first place before it is distributed to the rest of the wall.

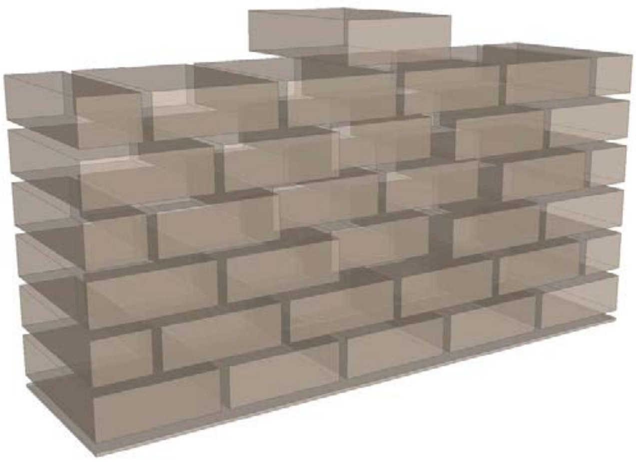


Figure 5: Rhinoceros model of studied piece of wall.

4. IMPLEMENTATION OF THE ALGORITHM

Since the geometric model was made using Rhinoceros, the algorithm implementation to determine the stochastic load path has been programmed using Python language within the Grasshopper parametric design plugin [17] (Fig. 6)

It starts from a piece placed on top of the wall, which receives a vertical load of unit value. This load is distributed among the possible supports that correspond to each of the indentations described in the previous section. The grid of the contact points is parameterized in a number $m \times n$, being $2 \leq n \leq 8$ and $2 \leq m \leq 8$ as well. The number of possible combinations increases very significantly to a greater number of supports –and so does the computing time necessary for processing.

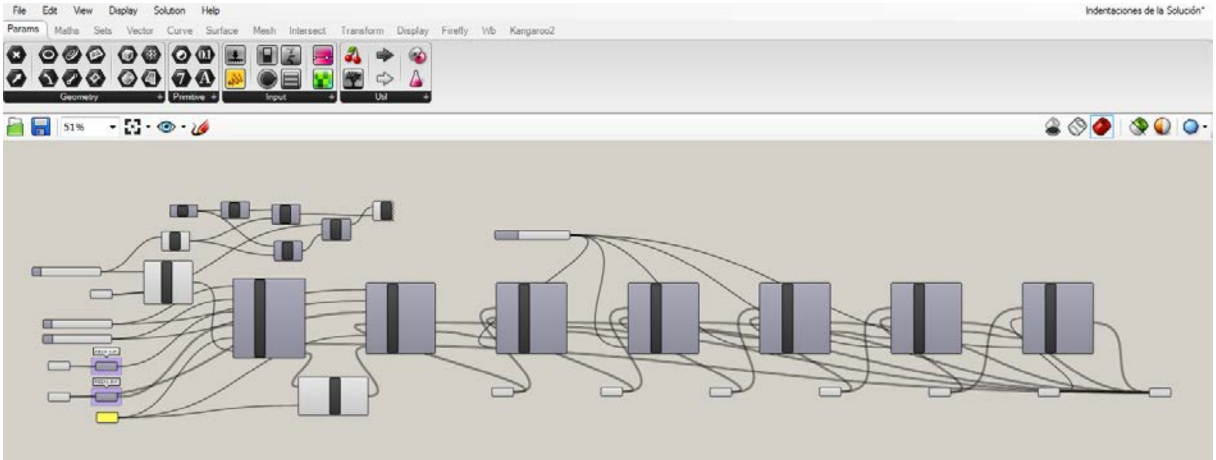


Figure 6: Grasshopper programming scheme.

After establishing these contact points the algorithm evaluates whether it is possible to transmit the load, since some of the blocks can be found in a cantilever and have no support (fig. 7). Once the valid supports under the piece have been determined, one of the combinations that meets the conditions of isostatic equilibrium is randomly assigned (figs. 8a and 8b) These conditions are established based on the equilibrium equations in space, with the additional hypothesis that it is possible to transmit tensile stresses with a maximum value of 10% of the elastic compression limit – a condition that is usual in the regulations and standards for the design of masonry walls.

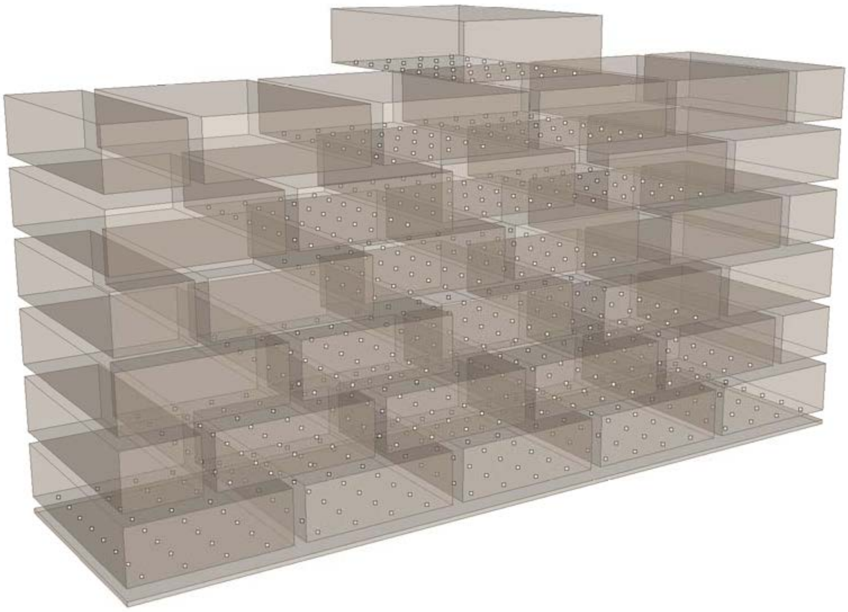
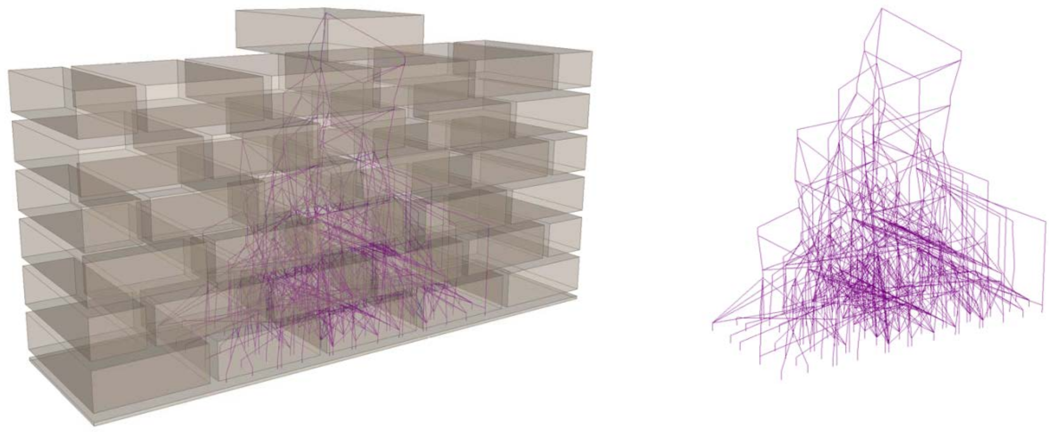


Figure 7: Support possibilities in the Rhinoceros model.

This process is iterated successively in each of the pieces that make up the set studied. In all cases, the load with which the iteration begins is the result of those transmitted on each block in each course, so it is necessary to determine, as the process progresses, the waypoint and the value of the resultant action in each of the pieces.

4. DISCUSSION

From a strictly mathematical point of view, the described model can be considered a stochastic Markov process in discrete time. There are two fundamental reasons for this ascription. On the one hand, the fact that it is not possible to consider each individual itinerary –that is, each line of action– as a strictly random walk: the direction that each line takes will be conditioned by the position and intensity of the rest. On the other hand, a fundamental factor must be taken into account, and it is the possibility of a rejection, which will then force the process to be rearmed at least from the previous level.



Figures 8a y 8b: Scheme of one of the solutions integrated in the wall (left) and isolated (right)

Pearson's classic formulation for a random path does not fit this problem. The Markovian process, however, is more open and contemplates the two singularities mentioned above. The fact that some itineraries have an influence on others falls within the usual considerations of stochastic systems in discrete time, though in the case of the proposed model, of course, there is no time factor in a literal sense, but rather the advance in the vertical load. On the other hand, the possibility of rejection is contemplated in one of the classic continuous time processes, known as the Markov process –as is its implementation from the probabilistic point of view.

With regard to the programming used, the implementation of these systems through graphic tools, as in the one presented in this paper, might limit the range of responses. For the designer it is ideal that the digital tools for the analysis of the local behaviour of this type of structures can be implemented in the same tools that make parametric design possible, as this would allow to contrast the local problems of the structure directly during the design process, and thus anticipate possible stress concentrations in the model prior to its construction, without the need to export the geometry to another tool for its treatment. However, the performance of the standard hardware with which these design tools are usually handled is limited, and that determines the scope of the analysis.

5 CONCLUSIONS

Masonry structures, and in particular those of buildings such as the neomudéjar façades studied here, are constructive elements of high complexity, whose indented brick bonds are interesting examples of the possibilities offered of combinatorial theory applied to architecture. The façade of the Aguirre Schools has a bond full of these recessions, which produce discontinuities in the volume of the masonry that is capable of conveying the loads paths. It is not, as detailed, a structure consisting of strictly dry-stack blocks, but its complexity makes it an ideal case study to implement the method described above.

These structures often present with local problems, which cannot be studied with the tools commonly used for surveying masonry structures –as its objective is the analysis of global behaviour. The proposed method allows evaluating at least one of the possible load paths compatible with the boundary conditions and the material requirements, also taking into account the geometric discontinuities of this type of masonry elements. The development of this method, and its application in a number of iterations that enable its statistical analysis, can allow detecting the areas where stress concentrations are most likely to occur. This numerical model needs to be complemented by physical tests –which are currently in progress– and statistically contrasted to establish its validity.

The implementation of this model through graphic programming languages –the same ones that make parametric design possible– entails limitations for large-scale statistical analysis. To achieve a sufficient number of iterations it is necessary, of course, to use analytical tools. However, and although these analytical tools are necessary for a general study at the theoretical level, graphic tools can also be useful in the practical one, since problems of local behaviour are frequently due to irregularities, be they geometric or material, in the nearby area.

REFERENCES

[1] del Real Mateos, A. *Estudio Histórico-Constructivo de las Escuelas Aguirre de Madrid.* (2018) Master Thesis. ETS Arquitectura (Universidad Politécnica de Madrid)

[2] Camino Olea, M. S. *Construcción y ornamentación de las fachadas de ladrillo prensado, al descubierto, en la ciudad de Valladolid.* (2001) PhD Thesis. Universidad de Valladolid

[3] Drescher, A., de Josseling de Jong, G. 1972. Photoelastic verification of a mechanical model for the flow of a granular material. Journal of the Mechanics and Physics of Solids, 20(5): 337–340.

[4] Mencías-Carrizosa, D., García, J. and Magdalena, F. Qualitative and quantitative approaches to highly local behaviours of historical masonry structures. In: Van Balen & Verstrynge (Eds.) *Structural Analysis of Historical Constructions – Anamnesis, diagnosis, therapy, controls,* (2016) Taylor & Francis Group

[5] Mencías-Carrizosa, D. *La geometría analítica como herramienta de análisis estructural de fábricas históricas*. (2017) PhD Thesis. ETS Arquitectura (Universidad Politécnica de Madrid). doi:10.20868/UPM.thesis.47167.

[6] Rajchenbach, J. Stress transmission through a cohesionless material. *Material Physics Mechanichs* (2001) 3: 1–4.
[7] Bigoni, D. & Noselli, G. 2010. Localized stress percolation through dry masonry walls. Part I – Experiments & Part II - *Modelling European Journal of Mechanics A/Solids*, 29, 291–298 and 299–307.
[8] Baig, I., Ramesh, K. ans Hariprasad, M. P. Analysis of stress distribution in dry masonry walls using three fringe photoelasticity. *International Conference on experimental Mechanics 2014*. Cambridge: International Society for Optics and Photonic. 93022P–93022P.
[9] Magdalena-Layos, F., García-Muñoz, J. & Mencias-Carrizosa, D. Estructuras de fábrica: enfoques «antiguos» y «modernos» para fenómenos muy locales. *Informes de la Construcción*, 68(542): e150 (2016) doi: 10.3989/ic.16.032.
[10] Magdalena Layos, F. & García Muñoz, J. 2018. Approaches to strongly local phenomena in dry masonry structure. *Int. J. Masonry Research and Innovation*, (2018) Vol. 3, No. 2, doi:10.1504/IJMRI.2018.092460-4
[11] Magdalena, F. Aznar, A. de la Torre and J. F. Hernando, J. I. Recent Results on Sliding Collapse for Masonry Structures Under Static Load Test. *Engineering and Applied Sciences*. Vol. 1, No. 4, 2016, pp. 99-106. doi: 10.11648/j.eas.20160104.14
[12] Shahriar, A. Waldmann, D. Scholzen, F. and Louge, A. Numerical analysis for the determination of stress percolation in dry-stacked wall systems. 2016. Masonry International. 29. pp 27-38
[13] Grimmett G. Hiemer P. Directed Percolation and Random Walk. In: Sidoravicius (ed) *In and Out of Equilibrium. Progress in Probability*, Vol 51, (2002) Birkhäuser, Boston
[14] Pearson, K. The Problem of the Random Walk. *Nature* (1905) 72:294 doi:10.1038/072294b0
[15] Van Kampen, N.G. S*tochastic Processes in Physics and Chemistry*. North-Holland Personal Library. (2007) doi:10.1016/B978-0-444-52965-7.X5000-4
[16] McNeel, Robert. Rhinoceros: *NURBS modeling for Windows* v.5. (2016)
[17] Ruten, D. and McNeel, R. Grasshopper: Generative Modeling with Rhino v.0.9.0076 (2016).