15th World Congress on Computational Mechanics (WCCM-XV)
8th Asian Pacific Congress on Computational Mechanics (APCOM-VIII)
31 July – 5 August 2022, Yokohama, Japan

A Discrete Exterior Calculus Based Framework for CFD

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Key Words: *Discrete exterior calculus, vortex dynamics, multiphase flows, Solar Convection.*

Exterior calculus is a generalization of vector calculus to manifolds of arbitrary dimension. Discrete exterior calculus (DEC) [1] is the discrete version of exterior calculus. In DEC, physical fields are discretely expressed as *k*-forms representing the integration of the physical quantity on k-dimensional primal/dual mesh objects. DEC ensures that vector calculus properties are discretely satisfied. In addition to preserving the mathematical properties it allows for conservation of secondary quantities, such as kinetic energy and vorticity for an inviscid flow [2]. DEC is coordinate independent, and convenient for investigating flows over curved surfaces.

We present a framework for computing flows on surfaces based on a DEC discretization of Navier-Stokes (N-S) equations on simplicial meshes. The framework incorporates rotational and primitive formulations of the NS equations and allows for different time integration methods including those which feature energy conservation [3]. Other features of the framework consist of the inclusion of the Coriolis force term to investigate flows on rotating surfaces, and phase-field based interface tracking methods for multiphase flows. Moreover, we have enhanced our DEC framework to build hybrid DEC-FD and DEC-FFT discretizations which are useful for the investigation of convection in spherical shells and aerodynamic flows, respectively. The method is second order accurate on structured triangular meshes, and first order on otherwise unstructured meshes. It demonstrates the conservation of inviscid invariants such as kinetic energy and enstrophy over an extended period of time [4].

We present simulation results for a variety of physical configurations including the flow past a circular cylinder and airfoils, multiphase flows, vortex dominated flows on surfaces with and without rotation, and convection in spherical shells.

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