IX International Conference on Particle-Based Methods
PARTICLES 2025
J. Bonet, M. Cremonesi, A. Franci, A. Gil (Eds)

POWER DISTRIBUTION OF VARIOUS ROTOR CONFIGURATIONS IN A VERTICAL STIRRED MILL DETERMINED WITH DEM

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Key words: discrete element method, mining, mineral processing, comminution, grinding, HIGmill

Abstract. Vertical stirred milling is a well-recognized technology for fine grinding applications in the mineral processing industry. It is increasingly used due to the higher energy efficiency compared to horizontal mills. Vertical arrangement leads to high grinding intensity and power draw in the bottom grinding chambers, thus bottom rotors may wear out more than upper rotors. Modelling power draw is of great interest proven by the effort in previous work. An analytical power model for this type of mills based on measurement data was developed by Heath et al. [1] and a particle-based approach was presented by Larsson et al. [2].

In this contribution a DEM model to determine the torque distribution along the shaft for every grinding compartment in a vertical stirred mill with castellated rotors [3] is presented. Different rotor configurations are evaluated, whereby the rotors vary in diameter, alignment and spacing. An optimized configuration with a combination of different rotors could be developed, which leads to a more even power distribution and a significant reduction of load on the bottom rotors. Simulations also reveal particle dynamics, which explain field experience in terms of wear. Furthermore, the influence of grinding media filling level on power draw is investigated. Higher filling levels increase power draw following an exponential trend. This model allows to optimize the rotor configuration along the shaft, which has a strong impact on rotor and liner wear, critically influencing operational costs.

1 INTRODUCTION

Comminution is the most energy-intensive process in the mineral processing industry and accounts for a large part of operating costs for mineral production [4]. Approximately 50% of the electric energy at mine site and 4% of the global energy is consumed by comminution [5]. This allows the mineral processing industry to significantly contribute to a sustainable future and a reduction of emissions by implementing more efficient comminution solutions. Tumbling mills are frequently used for comminution, especially for coarse grinding applications. In a tumbling mill different types of grinding media like balls, rods or pebbles are lifted together with the material in a rotating horizontal drum and particle breakage is achieved mainly by impacts. A significant share of the energy is hereby absorbed in low-impact contacts, which do not lead to particle breakage or further size reduction [4].

Stirred media mills are an attractive alternative to tumbling mills, especially for fine and ultra-fine grinding applications due to the high induced shear energy. The specific breakage energy increases with decreasing initial particle size [6][7]. Thus, more energy is required when grinded finer. This energy is proportional to the grind size to the power of a constant, generally ranging from −1 to −3 [8]. This makes energy efficient fine and ultra-fine grinding even more important. Compared to ball mills, stirred media mills are more energy efficient [9] and are increasingly considered for applications previously performed by ball mills [10].

In a wet stirred media mill, comminution is obtained by pumping slurry consisting of feed material and water through a grinding chamber, which is filled with grinding media. Slurry and grinding media are set in motion by a rotating shaft with attached agitator elements. This induces attrition between many grinding media contacts, resulting in particle size reduction of the feed material through shear and compression forces.

In a vertical stirred mill, rotating grinding rotors apply energy only in radial direction, thus no power is used for lifting the grinding media resulting in high energy efficiency. Additionally, the vertical arrangement leads to high grinding intensity in bottom grinding compartments due to gravity. These advantages make the vertical stirred wet milling technology a well-recognized technology for tertiary, regrind and fine grinding applications and are increasingly replacing ball mills also in coarser grinding applications in the secondary and tertiary stages [11].

Modelling comminution processes is of great interest because it allows to optimize and adjust to a large set of mutually influencing parameters. This is beneficial not only for research and development, but also in the aftermarket and service allowing to properly react to changes in customer requirements and eventual failures. Especially, modelling power draw is of great interest, proven by extensive work in developing an analytical power model based on measurement data for fine grinding HIGmills™ with castellated rotors by Heath et al. [1].

The Discrete Element Method (DEM), originally proposed by Cundall and Strack [18], is a powerful tool for investigating phenomena occurring at the scale of particle diameter and simulating bulk behavior. Thus, DEM has become widely established as an efficient method addressing a variety of engineering problems with granular and discontinuous materials like granular flows, powder mechanics or comminution [19].

Depending on particle size, number of particles, computational power and required accuracy different approaches have been developed to describe breakage embedded in DEM. In the Bonded Particle Model (BPM) [20] the progeny fragments are resolved throughout the DEM simulation, which is well suitable for agglomerates containing of cemented granular materials [21] and has also been used for ball mills [22], the wheat milling process [23] and crushers [24].

In the Particle Replacement Method (PRM), originally proposed by Cleary [25], the particles are replaced by several progeny particles when the criterion for failure is met. Various shapes are used for progeny particles, commonly spheres [26][27] or polyhedral cells [28][29]. Based on the probabilistic approach by Tavares et al. [27], a probabilistic replacement strategy with Voronoi tessellated fragments was developed by Denzel et al. [29][30]. Originally used to prevent breakage in conveying [31], storage [32] and mixing processes [33] mainly with iron ore sinter [34], this approach is also suitable for comminution processes ensuring mass and volume consistency allowing to simulate multiple breakage embedded in DEM and predict the resulting particle size distribution.

In some cases, especially when small particles are involved, the number of particles exceeds nowadays computational limits and other numerical methods such as Computational Fluid Dynamics (CFD) or Smoothed Particle Hydrodynamics (SPH) may play a leading role in simulating some comminution processes [35]. SPH is particularly useful for two-phase and free surface fluid flows, as occurring in tumbling mills, and can be coupled with DEM [36]. CFD is ideal for single-phase flows including laminar and turbulent flows with various fluid rheologies and is able to describe particle-fluid interactions and media flow accurately when coupled with DEM [37].

As DEM breakage models are more suitable for coarser comminution applications due to still limited computational power, Population Balance Models (PBM) are additionally used to predict product fineness in grinding applications [38] and can speed up DEM breakage simulations [39]. The UFRJ (Universidad Federal de Rio de Janeiro) mechanistic mill model [40], originally proposed by Tavares and Carvalho to describe size reduction in tumbling mills [41], was applied to describe comminution in a vertical stirred mill [42]. This approach was also used by Carvalho et al. [37] to simulate media and slurry motion in a continuous vertical stirred mill using a DEM-CFD coupled model assuming a Newtonian fluid with a viscosity exclusively depending on solids volume fraction. This was further developed [43] with an improved rheology model considering slurry concentration and particle size.

A different approach for modelling a wet vertical stirred media mill (HIG5L) is presented by Larsson et al. [2] using the particle finite element method (PFEM) for the fluid including non-Newtonian behavior, DEM for the grinding media and the Finite Element Method (FEM) for the mill structure.

2 HIGMILL TECHNOLOGY

The HIGmill^{TM} is a high-speed vertical wet stirred media mill, designed for highly efficient comminution targeting fine and ultrafine particle size distributions [12] offered in sizes from 75kW and 200L to 12.5MW and 75,000L [13]. Its configuration consists of a stationary shell with rotating grinding rotors on a shaft to stir the media charge against a series of stator rings.

Extensive studies were performed to optimize rotor design in terms of energy efficiency using DEM, laboratory, pilot scale and industrial size mills resulting in a castellated rotor design [3]. The castellations hold a layer of grinding beads, reducing slippage between beads and rotors compared to flat discs. This leads to reduced rotor wear [14] due to increased bead-to-bead shear further away from the rotor more inside the bulk of the bead volume [15].

The feed slurry is introduced from the mill bottom, travels upwards through a media bed and eventually discharges at the top (Figure 1a). The process is typically a single pass with no external classification required.

Particle breakage in the HIGmill^{TM} occurs primarily through attrition mechanism, combining compression and shear forces. The special design encourages attrition breakage, particularly in high intensity grinding zones. The rotating grinding rotors create centrifugal forces which push coarser particles and grinding media into the high intensity grinding zones on the periphery of the grinding chamber, while finer particles travel upwards closer to the mill shaft (Figure 1b). This design feature also prevents overgrinding and ensures the energy is applied mainly to coarser particles. Gravity and the vertical arrangement of the HIGmill^{TM} ensure that the media is kept compact during operation, leading to intense inter-bead contacts and maximized energy transfer through the mill volume. These effects lead to high grinding efficiency and minimize energy consumption [12]. In combination with other process equipment, the HIGmill^{TM} also contributes to minimizing water consumption, further reducing carbon footprint and OPEX [16][17].

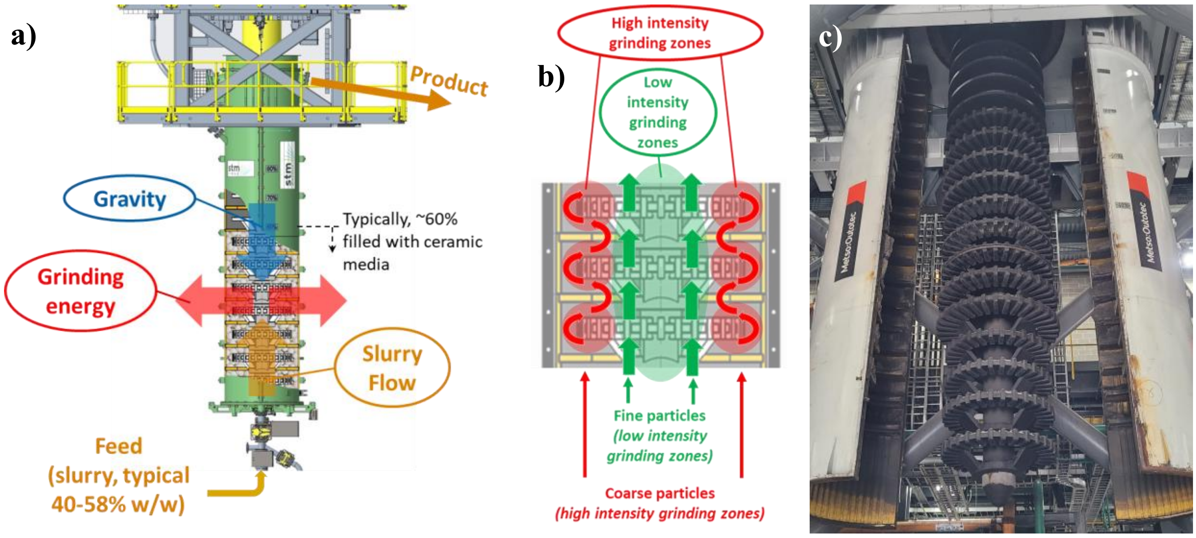


Figure 1: a) HIGmill^{TM} operating principle, b) Grinding zones, c) 50,000L HIGmill^{TM} with modified rotor configuration

3 MODELLING AND SIMULATION

In this contribution the torque distribution along the vertical shaft of a 50,000L HIGmill^{TM} is evaluated for different rotor configurations as shown in Figure 2. Depending on process conditions and special requests, different rotor configurations for the same mill type are in operation. Retrospective changes in operational conditions like feed material or power consumption require modified shaft configurations in some cases (Figure 1c). Another reason for modified shaft configurations is the intentional transfer of wear to parts which are more accessible for logistical reasons or changes in supply chains.

A comparative study with DEM for a variety of rotor configurations is performed. The torque relative to a reference configuration is evaluated by keeping all other operational parameters constant including shaft speed and grinding media. Accordingly, power can be calculated from torque by multiplying with rotational speed. Thus, mainly rotor geometry and alignment influence overall torque and its distribution. The number of rotors submerged in grinding media, projected area in circumferential direction combined with shape effects and induced pressure on beads due to media above play major roles in this case.

In the simulation the grinding media is placed in the HIGmill^{TM} with a volumetric fill feature, which instantly fills a volume with particles around a seed considering boundary geometries. Different rotor configurations create different voids inside the mill, which lead to different filling levels when filled with the same number of grinding media particles. The rotor configurations are compared with equal grinding media mass of mono-sized upscaled spherical particles. Due to the high mill volume and particle number, a scale up is necessary to reduce computational effort to reasonable calculation times within days. Particle scale up affects absolute torque in simulation but has a low influence on relative torque when comparing torque results relative to a reference configuration. The standard rotor configuration with same sized rotors and same alignment is used as a reference, which refers to configuration A in Figure 2. Material and interaction parameters from previous studies [44] are used.

To evaluate the torque distribution along the shaft, a frame enveloping each rotor is defined. The torque for each frame and rotor is then plotted separately. Thus, the torque share of each rotor can be calculated in relation to the torque sum of all frames (Figure 4).

Due to high peaks during start-up, torques are only evaluated after a quasi-steady state. Duration to reach this quasi-steady state is dependent on rotor configuration and increases with higher grinding media filling levels.

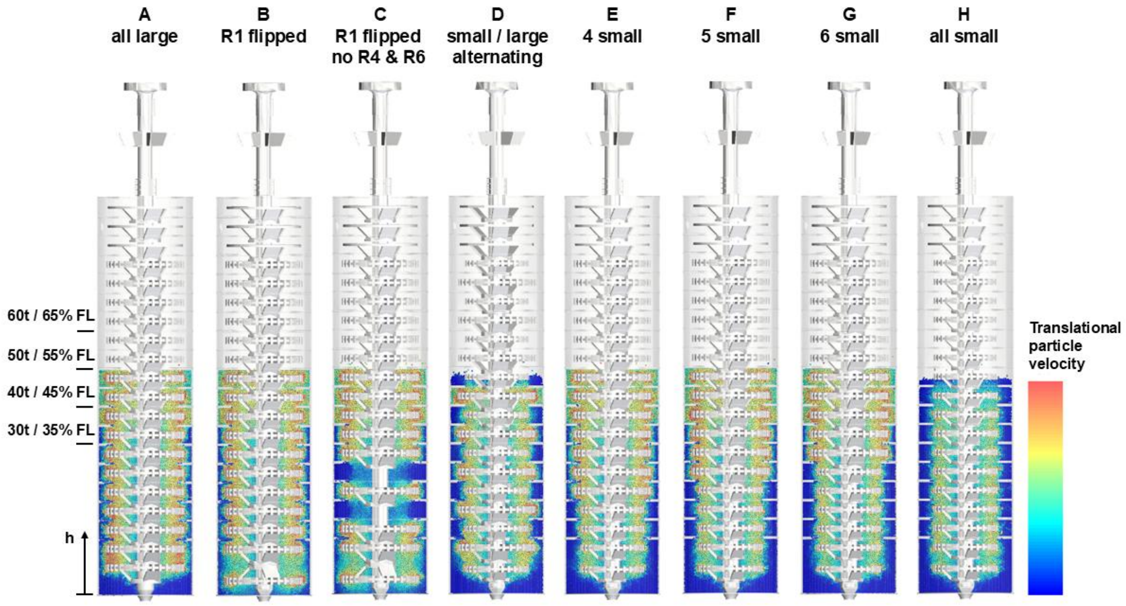


Figure 2: DEM simulation of different rotor configurations for a 50,000L HIGmill^{TM} with 50t grinding media, corresponding to 55% filling level in rotor configuration A.

4 FLUIDIZATION

Fluidization decreases torque and power draw due to buoyancy and lubrication effects. On the other hand, cohesion, friction and sedimentation effects may increase torque in some cases, especially in start-up scenarios. Various effects influence each other and depend on media filling level and shaft speed as apparent slurry viscosities drop with increasing shear rate following a non-Newtonian behavior [2][43]. Thus, fluid parameters like flow rate, density, viscosity and solids content of slurries significantly influence torque and power draw. This makes it a challenging task to generalize fluidization effects, especially when slurries are involved.

In this contribution, water-fluidization is considered using a calculation method based on data from previous studies by Probst [44], where trials and DEM-CFD coupled simulations with and without water-fluidization were performed for a laboratory 5L HIGmill^{TM}. Further evaluation of this data shows that water-fluidization decreases torque and power draw by a reduction factor f_w which is inversely proportional to grinding media filling level. Thus, the torque-decreasing effect of water-fluidization increases with filling level. Due to limited data, the reduction factor f_w could only be determined for three different filling levels and was inter- and extrapolated for further calculation assuming a linear trend following Equation (1). Relative torque curves considering water-fluidization in Figure 3b were constructed from DEM results reduced by f_w. This procedure allows a simple estimation of water-fluidizing effects but needs to be confirmed in further investigations using comparative experiments and coupled DEM-CFD simulations.

$$f_w(FL) = -0.0034 \cdot FL + 0.8578 \tag{1}$$

f_W	…	Reduction factor due to water-fluidization	$[-]$
FL	…	Grinding media filling level	[%]

5 RESULTS AND CONCLUSION

DEM simulations in Figure 2reveal particle dynamics and show high velocity gradients with respect to the distance to rotor castellations. This indicates high relative movement between particles which is necessary for efficient grinding. The highest utilization of grinding chamber volume can be achieved with a flipped bottom rotor (configuration B), creating the least zones without particle movement, which do not contribute to grinding. Comparing the bottom grinding chamber of configuration A, which has no stator ring between rotors R1 and R2 with the adjacent grinding chambers above proves the beneficial effect of stator rings by increasing velocity gradients and grinding efficiency. The same effect is demonstrated for castellated shell liners from rotor R1 up to R8. This matches field experience where upper shell liners and stator rings wear out first.

Overall shaft torque relative to rotor configuration A is plotted in Figure 3a for direct DEM results and with considered water-fluidization calculated with Equation (1) in Figure 3b. The plots show the torque increase due to a higher filling level following an exponential trend described in Equation (2). Fluidization reduces overall torque and torque increase due to higher filling levels indicated by curve inclines. This is quantified in Table 1, where the fitted exponents q for water-fluidized torque curves in Figure 3b are significantly lower than for the non-fluidized torque curves in Figure 3a. Rotor configuration also affects the incline of the curves, showing rotor configurations A and B with the highest inclines and configuration H with the lowest incline due to the highest number of small rotors.

The average torque of each rotor configuration relative to configuration A over the investigated filling levels is stated in Figure 3 in the diagram legends in brackets. Thus, a flipped bottom rotor R1 increases average shaft torque by 2% due to its lower position and higher pressure on grinding media. Removing rotors R4 & R6 reduces shaft torque by 26% and replacing all large rotors with smaller diameter rotors reduces torque by 37% in this case. There is no significant difference in overall shaft torque between rotor configuration G (6 small) and configuration C (R1 flipped, no R4 & R6). Thus, reducing torque and power draw is recommended by replacing rotors with smaller diameter rotors rather than removing rotors.

Figure 4 shows the torque distribution along the shaft. The diagrams show the torque share of each single rotor depending on rotor position height h relative to the mill shell bottom (indicated in Figure 2) for different rotor configurations and filling levels. This provides essential data to optimize shaft hub connections in terms of durability and operational safety.

Furthermore, this data is used to predict wear rates and service intervals for rotor replacements. The results match field experience, where bottom rotors wear out first and determine service intervals. Comparing rotor configurations A (Figure 3a) and B (Figure 3b) clearly shows the advantage of a flipped bottom rotor as this reduces the torque share significantly at low and medium filling levels. Configuration B also shows a clear decreasing linear trend for torque shares with increasing rotor position height at medium and high filling levels. Figure 3c shows disadvantageous torque peaks on rotors adjacent to empty grinding chambers. The most-even torque distribution is achieved with rotor configuration G with 6 small rotors (Figure 3g) in this case.

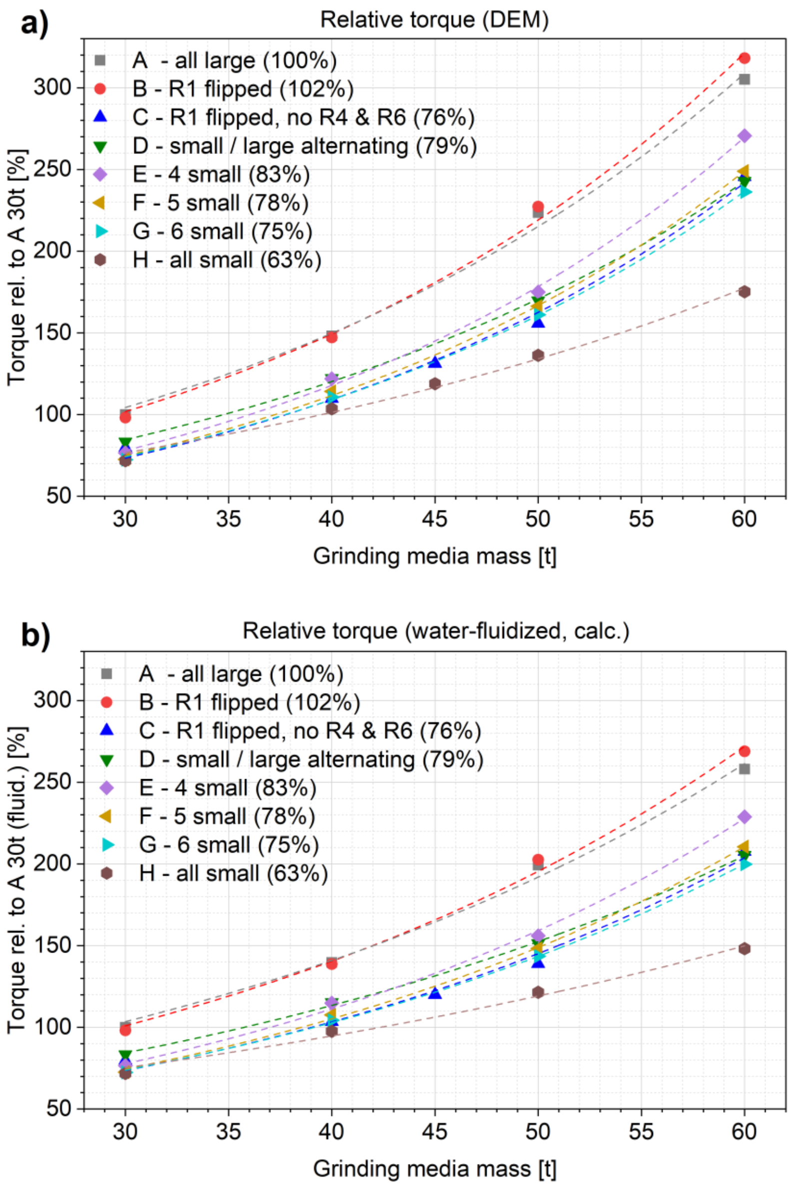


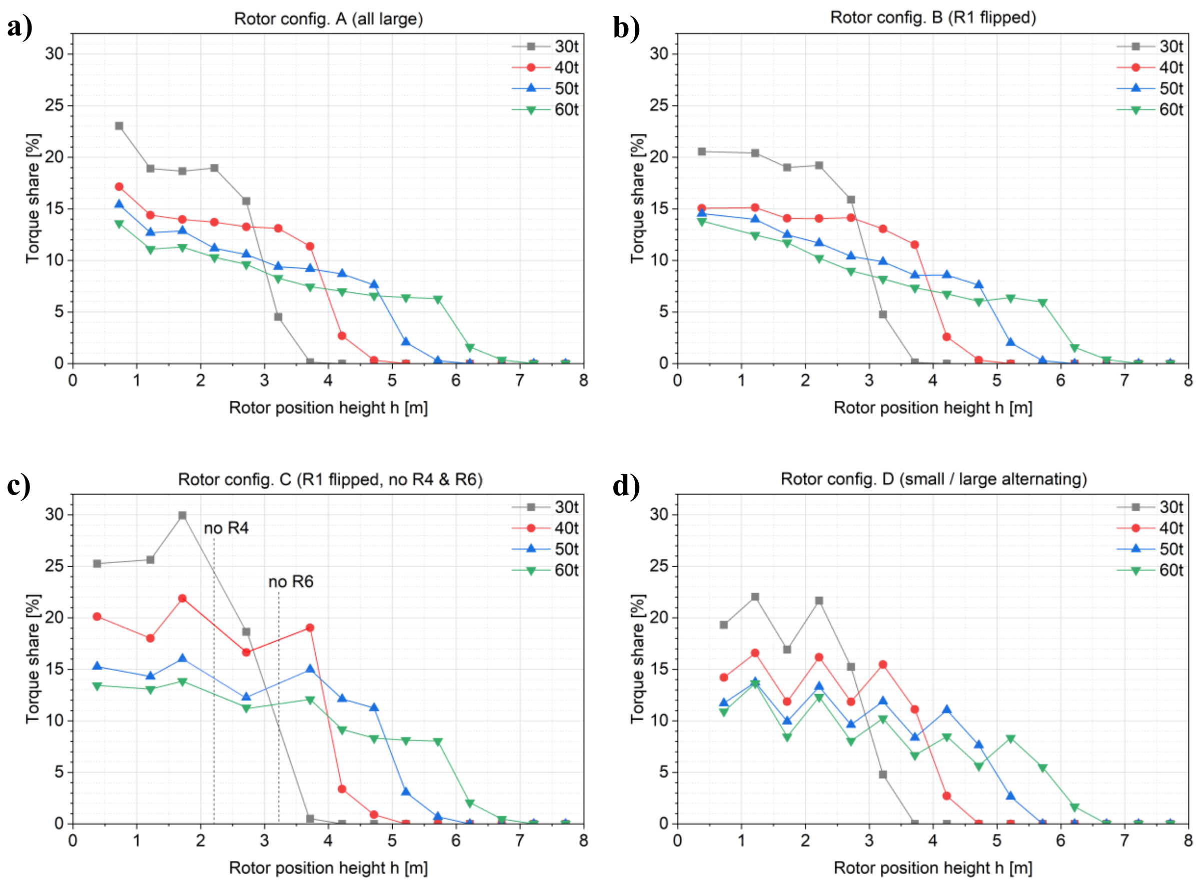
Figure 3: Shaft torque relative to rotor configuration A with 30t grinding media mass. The average offsets to the curve correlated to configuration A are stated in brackets in the legend.

$$T_{rel}(m_{GM}) = K \cdot e^{q \cdot m_{GM}} \tag{2}$$

T_{rel}	…	Torque relative to config. A	$[-]$
m_{GM}	…	Mass of grinding media	$[t]$
K	…	Fitting factor	$[-]$
q	…	Fitting exponent	$[t^{-1}]$

Table 1: Fitting parameters for relative torque curves in Figure 3. R^2 represents the coefficient of determination.

Rotor configuration	K	q	R^2
A – all large (DEM)	0.352	0.036	0.996
H – all small (DEM)	0.331	0.028	0.993
A – all large (water-fluid.)	0.410	0.031	0.994
H – all small (water-fluid.)	0.378	0.023	0.991



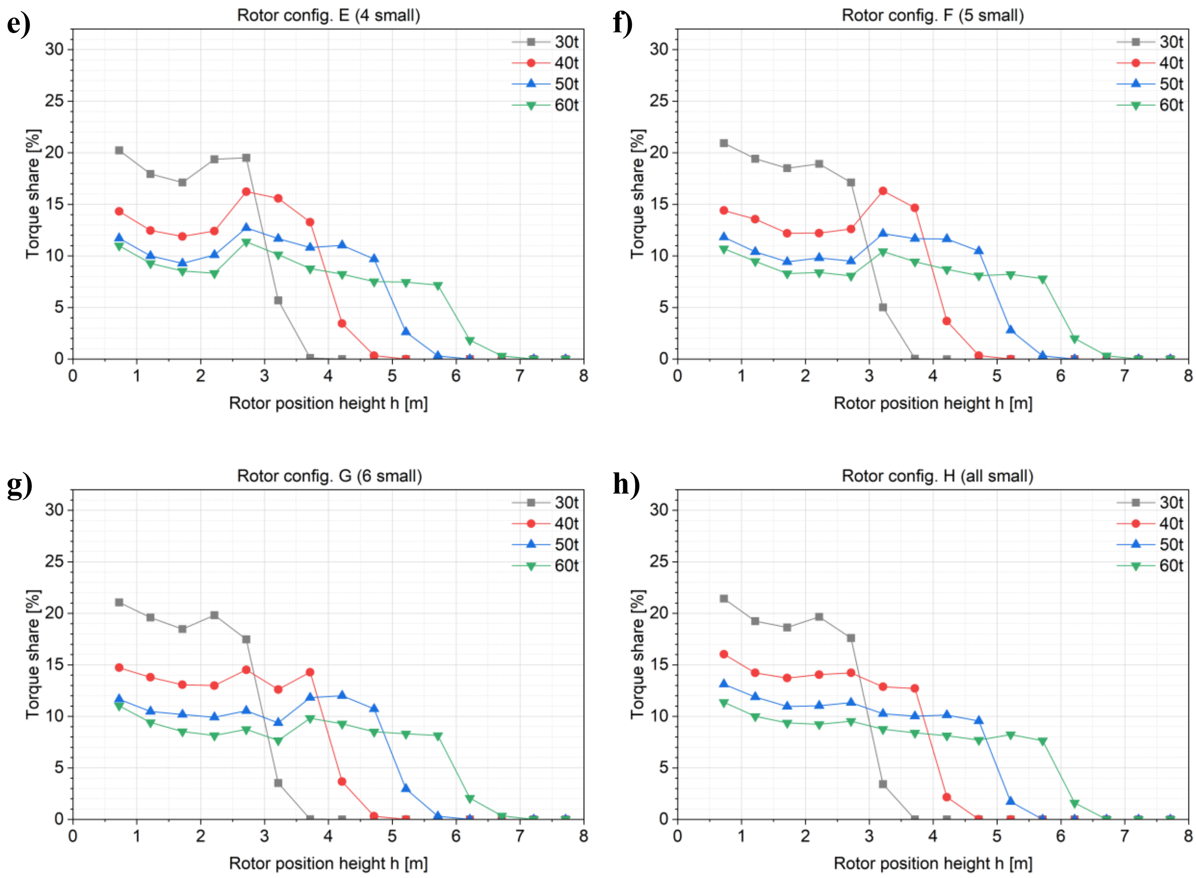


Figure 4: Torque distribution along the shaft. Torque shares single rotors depending on rotor position heights for different rotor configurations and filling levels.

OUTLOOK

Laboratory and pilot-scale trials are planned in the near future to validate the findings in this contribution. Furthermore, a DEM-CFD coupled model is being developed to predict torque and power draw of vertical stirred mills under different operating conditions. These include grinding media filling level, shaft speed and fluidization as well as changes in grinding compartment configuration. An extensive test program on an especially equipped test mill will be performed to validate the model.

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