EUROMECH Colloquium 656
Data-driven mechanics and physics of materials
21-23 May 2025, Gothenburg, Sweden

Physics informed holomorphic neural networks for fracture mechanics

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Introduction

In recent years, there has been a growing interest in leveraging scientific machine learning – a blend of computational methods and advanced machine learning techniques – to solve complex problems in solid mechanics. Particularly, physics-informed neural networks (PINNs) have attracted attention due to their remarkable potential in integrating experimental data and physical models [1].

In the simplest form of the PINN approach, a neural network is used as the ansatz function to solve a boundary value problem. The network weights are optimized during the training stage by minimizing a loss function that includes the residuals of the governing partial differential equations (PDEs), of the boundary conditions (BCs), as well as any deviations from available experimental data [2].

However, a major drawback of the PINN approach is the long training time. In this respect, the recent development of physics-informed holomorphic neural networks (PIHNNs) has greatly accelerated the training process for problems where the solution can be represented via holomorphic functions [3]. The reason is that PIHNNs can automatically satisfy the underlying PDEs by enforcing the network output to be holomorphic by construction. Consequently, the goal of the training process reduces to solely finding the network parameters that allow fulfilling the BCs and any available experimental data within a user-defined tolerance for the training loss set as objective.

Plane linear elasticity is particularly suited for the PIHNN approach, as the Kolosov-Muskhelishvili representation [4] guarantees that any linear elastic solution can be expressed in terms of two holomorphic functions. On the other hand, a limitation of PIHNNs is their inability to learn discontinuous and singular solutions, as their output, generated by the combination of holomorphic functions, is inherently continuous and smooth everywhere [3]. As a result, the approach is currently unsuitable for fracture mechanics analyses involving cracks.

To extend the applicability of PIHNNs to fracture mechanics, we investigate in this work the incorporation of enrichment functions in the network representation using two strategies [5]. The first consists in enriching the holomorphic neural networks with the square root term from Williams' series that provides the correct asymptotic profile near the crack tip. The second leverages Rice's exact global representation of the solution for a straight crack, which effectively decouples the holomorphic part of the solution from the singular, non-holomorphic terms. To demonstrate the potential of the method, stress intensity factors are computed for several case studies, showing superior accuracy and speed compared to the classic PINN approach. Moreover, it is demonstrated that transfer learning makes the method potentially well-suited for simulating crack propagation.

References

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