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NOVEL CONSTITUTIVE MODELLING APPROACH FOR SHAPE MEMORY ALLOYS VIBRATION CONTROL DEVICES

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Abstract. *One of the main fields of shape memory alloy application in civil engineering is oriented on mitigation of earthquake effects on structures. Vibration isolators that incorporate elements made of SMA take advantage of its characteristic phenomenon of nonlinear hysteretic response, also known as superelasticity. In this work, authors presents an approach to phenomenological modelling of SMA by using rheological schemes. One of the advantages of this approach is a possibility of formulation of constitutive relationships as a set of explicit differential equations. As an illustration of validity of the formulation, authors present the response of single degree of freedom oscillator that incorporates SMA elements modelled by different existing SMA models. The response obtained based on the model that uses rheological schemes is compared with Lagoudas thermodynamic constitutive SMA model and simplified material model. All of the compered models are found to match well and show important reduction in displacement transmissibility.*

1 INTRODUCTION

Shape memory alloys (SMA) belongs to the family of smart materials being comprehensively studied in recent decades. One of the main fields of its application in civil engineering is oriented on mitigation of earthquake effects on structures. Vibration control devices that incorporate elements made of SMA take advantage of its characteristic phenomenon of nonlinear hysteretic response, also known as superelasticity [1]. Such devices are especially promising for retrofitting and protection of architectural heritage what was verified in several application for strengthening of cultural heritage structures.

In this work, authors present a novel approach to modelling of SMA by using rheological schemes. As an illustration of validity of the formulation, authors present the response of a single degree of freedom oscillator that incorporates SMA elements modeled by different existing SMA models. The response obtained based on the model that uses rheological schemes is compared with thermodynamic constitutive SMA model developed by Lagoudas et al. [2, 3] and simplified material model also presented by Lagoudas et al. [4].

2 CHARACTERISTICS OF SMA

The first observations of shape memory specific characteristic behaviour were done during the first half of 20th century. First in 1932, Ölander observed “rubber like effect”, later named as superelasticity, in an alloy of gold and cadmium [5]. Further studies of this alloy led to observation of “shape recovery”, later named as shape memory effect, by Chang and Read in 1951 [6]. The breakthrough event for SMA studies was the discovery of Nitinol, an alloy of nickel and titanium, in 1963 by Buehler and co-workers from U.S. Naval Ordnance Laboratory [7]. Since then it has been discovered that alloys such as copper-zinc-aluminium alloy [8], copper-aluminium-nickel alloy, iron-platinum alloy, iron-palladium or iron-manganese-silicon alloy [9] presents similar unique properties. However, the Nitinol is considered as the alloy that shows the best shape memory properties [10]. SMAs find applications in various fields of engineering such as aeronautics [11], biomedical engineering [12] (especially cardiovascular applications [13]) and structural engineering [14, 15].

The main characteristic of SMAs that benefits for the application in the structural engineering field is superelasticity. This phenomenon can be described based on the graph of the response for the uniaxial tension test of the SMA specimen (Figure 1). After the initial, close to linear, response the superelastic plateau occurs. This kind of response is similar to the one observed during yielding but in this case it is related to stress-induced phase transformation from austenite to martensite (arrow 1 in Figure 1). When the phase transformation completes the further elastic transformation follows the rise of tensile stresses. Upon the stress removal, the reverse phase transformation, from martensite to austenite (arrow 2 in Figure 1) is induced. It results in almost complete strain recovery.

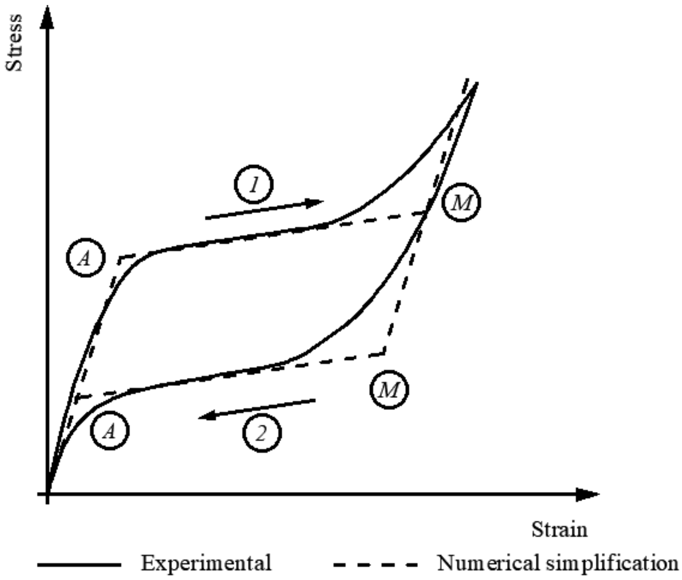

Figure 1: Shape of hysteresis loop of SMA and its numerical simplification

3 SMA IN RETROFITTING OF HISTORICAL STRUCTURES

Applications of SMA in strengthening of cultural heritage structures base on the traditional tying techniques increasing the box-type behaviour of the building. The performance of ties that incorporate SMA elements is enhanced due to the phenomenon of the superelasticity. In case of ground motions with low intensity or other horizontal loads the SMA elements work in the first, elastic branch (Figure 1) what results in behaviour similar to classic (e.g., steel) ties. In case of strong ground motion, SMA elements undergo phase transformation what effects in reduction of stiffness and allowance for controlled displacement of tied members. This phenomenon permits the structure to dissipate part of the energy through a microcracking instead of a dangerous macrocracking. The displacement is later recovered upon the load removal. The noticeable fact is a lower force transmitted to the historical material. Finally, in case of severe ground motion or other extraordinary horizontal load the stiffness of SMA elements increase what helps preventing from excessive displacements [16].

The reinforcement technique for historical structures that incorporates SMA was studied and developed within the ISTECH Project [17]. It based on a device, equipped with wires made of SMA, that reduce the risk of collapse of masonry buildings in case of strong earthquake actions. The trial intervention was implemented in 1999 during restoration of bell tower of San Giorgio church in Trignano after the earthquake in October 1996 which struck the Reggio Emilia and Modena Districts (Italy) [18]. SMA devices were installed as a parts of vertical post-tensioned ties that improve bending and shear resistance (Figure 2a). Despite the numerical and experimental campaign, the application was also verified by another earthquake in June 2000, after which the structure showed no damage of any type. That proved the effectiveness of such a technique.

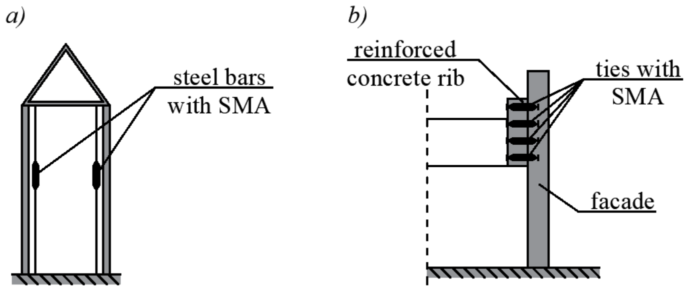

Figure 2: Schemes of the SMA devise arrangements for retrofitting of historical structures – (a) vertical ties in slender structures [18] and (b) horizontal ties for facades [16]

The successful results of the research and exploitation of ISTECH project led to the further application of SMA in restoration of the Basilic of San Francisco in Assisi and the Cathedral of San Feliciano in Foligno [16]. In those cases the strengthening was intended to improve the connection between perpendicular elements (Figure 2b), such as the transept tympanum (in the Basilic of San Francisco) and the façade (in the Cathedral of San Feliciano). Such an intervention aim at the prevention of a out-of-plane collapse of tied elements.

Another investigation was performed by Cardone et al. [19] who examined a behavior of SMA implemented in ties of timber roof trusses. The experimental campaign, that included thermal behavior and shaking table tests, led to the installation of prototype device in the San Paolo Eremita church in Brindisi.

Presented applications of SMA in historical structures show their several advantages. The improvement of box-type behaviour of buildings, originated from traditional ties, is followed by possibility of controlled displacement that leads to energy dissipation. Moreover, the features of SMA, allow to control the forces in ties not only related to the seismic event but also to changes of temperature, corrosion, and relaxation due to deformation of masonry (creep) [18–20].

4 NUMERICAL MODELLING OF SMA

According to Cisse et al. [21] the constitutive models of SMA could be divided in terms of modelling scale and mathematical structure in the following groups:

- microscopic thermodynamic models,
- micro-macro models and
- macroscopic models.

The models from the first category describe microstructural features in SMA behaviour (e.g., phase nucleation, interface motion, martensite twin growth) at the lattice or grain-crystal levels. The models from the following category rely on micromechanics to describe the material behaviour at the micro or meso scales and then based on a scale transition macroscopic constitutive equations are derived. The last group consists of the models that describe the behaviour of polycrystalline SMAs based on phenomenological considerations, simplified micro-macro thermodynamics or direct experimental data fitting [21].

In case of the analysis of the applications in the field of structural engineering the main goal is to study the influence of SMA elements incorporated in the structure to its response for the excitation. In such a case the most efficient models are the phenomenological macroscopic one.

4.1 SMA vibration control devices modelling with rheological schemes

The concept of modelling SMA as a certain set of rheological elements was earlier discussed by authors in [22–26]. This approach belongs to the group of macroscopic models of SMA and allows to analyse behaviour of structural members without a detailed analysis of the phenomena at the microcrystal level.

The model of a vibration control device can be represented as a single degree of freedom system (Figure 3a) where the mass displacement is given by $X(t)$ and base displacement by $Y(t)$. In this configuration the SMA element is given as a “black box” that expresses the particular set of rheological elements. By the proper configuration of linear elastic springs (characterized by the stiffnesses k_1 and k_2 in Figure 3b), perfectly plastic body (characterized by the force T_0 in Figure 3b) and perfectly elastic body (characterized by the force P_0 in Figure 3b) the hysteretic loop given in Figure 3c can be obtained. In such a configuration the perfectly plastic body is responsible for energy dissipation of the structure, while the perfectly elastic body, along with linear elastic springs, are responsible for energy accumulation. A detailed explanation as well as one-dimensional characteristics of elements are wider discussed in [26].

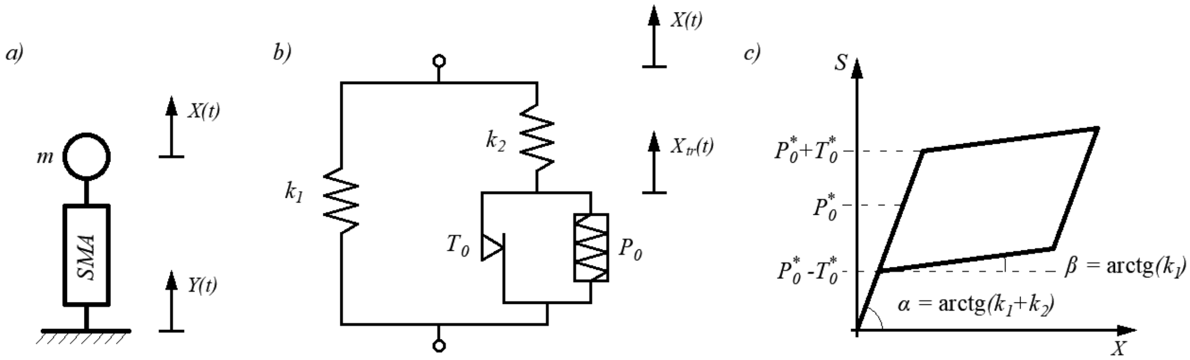

Figure 3: Schematic representation of SMA vibration control device, SMA rheological scheme and the graph of the hysteretic loop

The resultant hysteretic loop (Figure 3c) is characterized by the modified values of forces P_0^* and T_0^*. The correction is a result of kinematic strengthening which is associated with the presence of the k_1 spring in the model. This implicates the increase of the initial value of the internal force at the beginning of the martensitic phase transformation. The correction is given by the equations

$$P_0^* = \frac{k_1 + k_2}{k_2} P_0, \quad T_0^* = \frac{k_1 + k_2}{k_2} T_0 \tag{1}$$

In the structural analysis of buildings subjected to the earthquakes the typical excitation applied to the structure is a ground acceleration related to ground motion. The following equations will be presented following such a kind of excitation. Taking it into consideration the equation of motion of the system shown in Figure 3a is given by the following differential equation

$$m\ddot{X}(t) + S(t) = -m\ddot{Y}(t) \tag{2}$$

where $S(t)$ is a force response of SMA rheological model given by the equation

$$\begin{aligned} S(t) &= S_1(t) + S_2(t) \\ S_1(t) &= k_1 X(t) \\ S_2(t) &= k_2 \left(X(t) - X_{tr}(t) \right) \end{aligned} \tag{3}$$

The rate of deformation of the phase transformation $\dot{X}_{tr}$ is given by

$$\dot{X}_{tr} = f_{SMA}\left(\dot{X}, X, X_{tr}\right) \tag{4}$$

where

$$f_{SMA} = \begin{cases} \dot{X} & \text{if} \quad |S_2| = P_0 + T_0 \ \text{and} \ S_2 \dot{X} > 0 \\ \dot{X} & \text{if} \quad |S_2| = P_0 - T_0 \ \text{and} \ S_2 \dot{X} < 0 \ \text{and} \ S_2 X_{tr} > 0 \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

The above presented set of explicit non-linear differential equations enable to model a behaviour of SMAs in isothermal conditions in a certain temperature. In different temperatures values of P_0 and T_0 will vary. The generalization of the model for different thermal conditions is possible by determination of the relation between forces P_0 and T_0 and temperature. However this generalization does not influence the character of the presented equations.

5 RESULTS COMPARISON

In order to compare results of the discussed approach to constitutive modelling let consider a SDOF system presented earlier in Figure 3a. The damping of the mass displacements takes place due to stress induced phase transformation of SMA and its hysteretic response.

The considered excitation is a harmonic acceleration applied to the mass. Details of the excitation and material characteristics are given in Table 1. In order to make a comparison those values are based on the numerical example from the work of Lagoudas et al. [4].

Table 1 Design and material parameters

Design parameters		Material model parameters	
Mass	500 kg	k_1	527 N/mm
Amplitude of base excitation	153,4 m/s^2	k_2	14620 N/mm
Frequency of base excitation	50 Hz	P_0	6519 N
		T_0	5761 N

The discussed equations (2) - (5) were implemented in Wolfram Mathematica software. The set of differential equations was numerically solved applying *NDSolve* function by incorporation of forward Euler (*ExplicitEuler*) method. The results are presented below as a set of graphs showing the time history of the mass displacement (Figure 4) as well as the hysteretic loop (Figure 5).

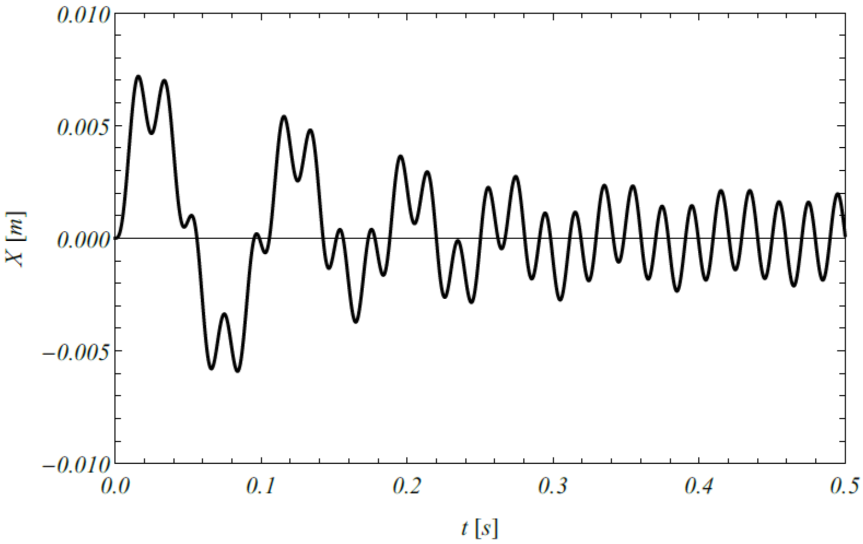

Figure 4 Time history of the mass displacement

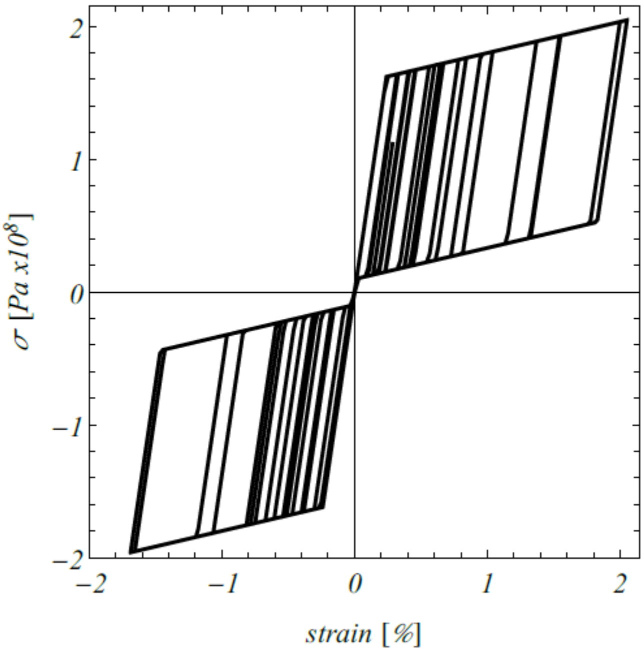

Figure 5 Hysteretic loop of the analyzed SMA device

The graph of the mass displacement history presents noticeable reduction of the amplitudes in the following cycles. The hysteretic loop has a shape characteristic for a superelasticity behaviour of SMA.

Results obtained from basic polynomial model [2] and simplified model [4] obtained by Lagoudas et al. [4] are presented for comparison in Figure 6 and Figure 7. Despite the dissimilarities of maximal and minimal values compared results the character of all of the graphs is coherent. This implicates that the presented formulation stays in agreement with compared models and the differences may result from assumed material parameters.

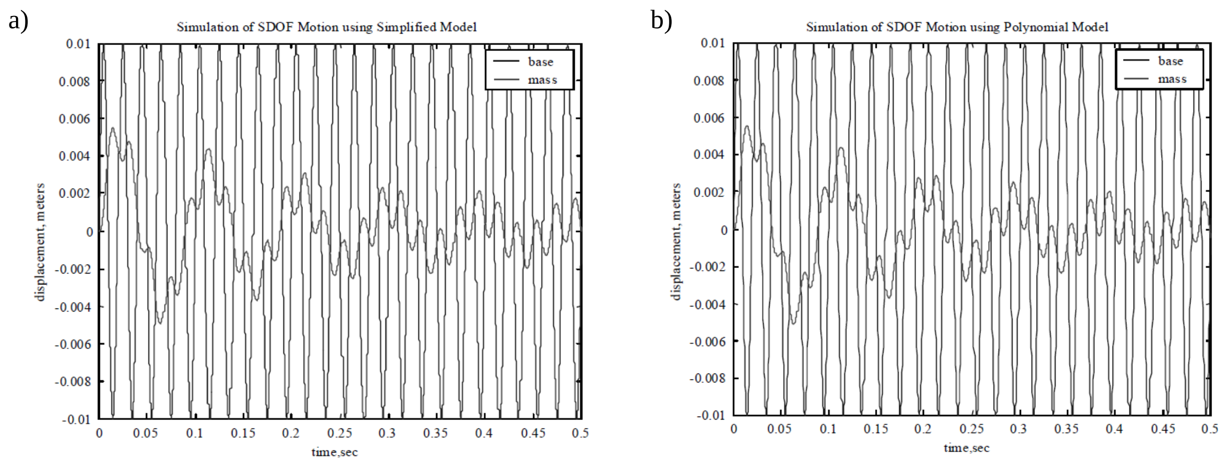

Figure 6 Maps of the displacements for different models of SMA [4] – a) simplified model and b) basic polynomial model

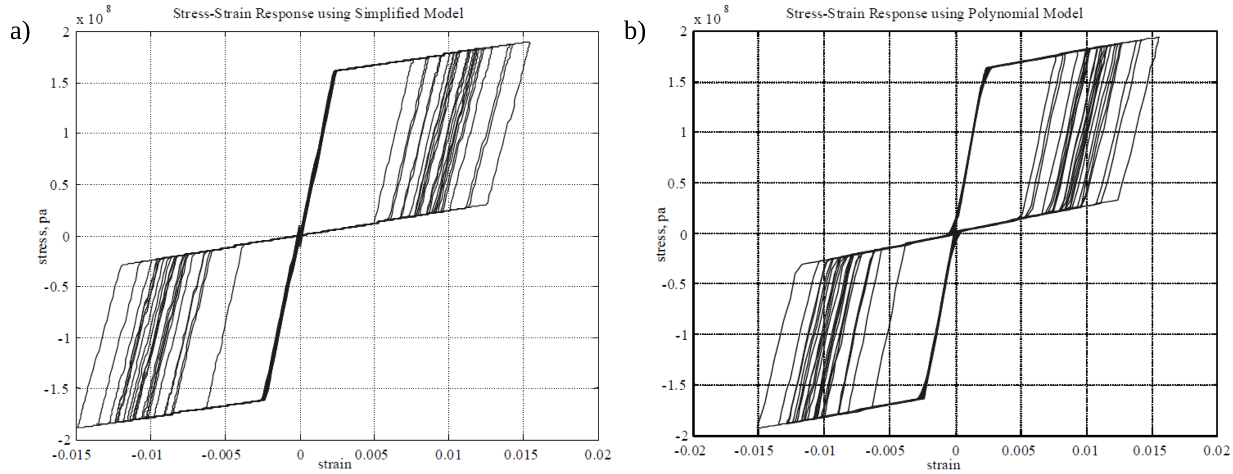

Figure 7 Hysteretic loops for different models of SMA [4] – a) simplified model and b) basic polynomial model

6 CONCLUSIONS

Results of the presented comparison of numerical example show the validity of the formulation. The important fact is that the proposed methodology of the formulation of SMA constitutive equations is based on original rheological schemes. It results in explicit form of differential equations of the presented model. Such a system of equations can be easily implemented in mathematical software. The equations could be directly implemented in the commercial FEM codes as a user's subroutines (e.g. UMAT and VUMAT subroutines in Abaqus [22–24]). Such an implementation may facilitate the analysis of complex structures, such as historical masonry, retrofitted with SMA devices.

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