Adaptive Control of Uncertain Coupled Mechanical Systems: Application to Base-isolated Buildings

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ABSTRACT

A problem of feedback stabilization is addressed for a class of uncertain nonlinear mechanical systems with n degrees of freedom and $n_c < n$ control inputs. Each system of the class has the structure of two coupled subsystems with n_c and n_r degrees of freedom, respectively: a prototype being an uncertain base isolated building structure with n degrees of freedom actively controlled via actuators applying forces to specific degrees of freedom of the base movement, $n_c < n$ in number. A nonlinear adaptive feedback strategy is described, which, under appropriate assumptions on the system uncertainties, guarantees a form of practical stability of the zero state. Numerical simulations are also presented to illustrate the application of the control strategy to a base isolated building.

1 INTRODUCTION

The problem of actively controlling structures has been extensively studied in the last two decades. Whilst in many cases controllers have been designed under the assumption of having a perfectly known structural model, there has also been considerable interest in questions of robustness. Among the approaches proposed in the literature for the control of uncertain systems, there is one in which the systems are described by differential equations, the uncertainties are modelled deterministically and Lyapunov techniques are used constructively to design feedback controllers to render the system "practically stable" (see [1] for an overview and extensive bibliography). This approach has been adopted for active control of structures in previous work [2–4].

[1] Department of Applied Mathematics III, Technical University of Catalonia, Barcelon, Spain

[2] School of Mathematical Sciences, University of Bath, England

[3] Department of Structural Mechanics, Technical University of Catalonia, Barcelon, Spain

Although much of the literature has dealt with fixed-parameter controllers, the design of adaptive control laws has also been considered [5,6]. In this context, the word *adaptive* means that the control law is parameterized by a variable gain whose value is autotuned according to some appropriately designed law. The present paper essentially falls into this category and focusses on the construction of adaptive controllers for a class of uncertain nonlinear coupled mechanical systems that can be decomposed into two subsystems with feedback control acting on one of them. Although the control law is developed in the context of this general class of systems, the problem of actively controlling a base–isolated building structure is our prototype.

2 THE CLASS OF SYSTEMS

We consider a class of uncertain mechanical systems Σ with n degrees of freedom and $n_c < n$ control inputs. Each system of the class has the structure of two coupled subsystems, Σ_c and Σ_r, with n_c and n_r degrees of freedom, respectively, $n = n_c + n_r$, and described by equations of motion of the following form.

$$\Sigma_r : \qquad M_r(q_r(t))\ddot{q}_r(t) + g_r(q_r(t), \dot{q}_r(t)) = h(q_c(t), \dot{q}_c(t)),$$

$$(q_r(t_0), \dot{q}_r(t_0)) = (\overset{0}{q}_r, \overset{0}{v}_r)$$

$$\Sigma_c : \qquad M_c(q_c(t))\ddot{q}_c(t) + g_c(t, q_r(t), \dot{q}_r(t), q_c(t), \dot{q}_c(t)) = u(t),$$

$$(q_c(t_0), \dot{q}_c(t_0)) = (\overset{0}{q}_c, \overset{0}{v}_c).$$

(1)

Here, $q_r(t) \in \Re^{n_r}$, $q_c(t) \in \Re^{n_c}$ are vectors of generalized coordinates and $u(t) \in \Re^{n_c}$ is the vector of control forces; the matrix-valued functions M_r and M_c represent inertias, and the (nonlinear) functions g_r, h, g_c model damping, stiffness, coupling and Coriolis effects, as well as extraneous inputs and disturbances acting on the overall system.

Assumptions 1 to 6 below complete the description of the system class Σ.

Assumption A1. *The function M_r is continuous with uniformly bounded inverse: that is, for some (unknown) positive scalar $\tilde{m}$, $\|M_r^{-1}(q_r)\| \leq \tilde{m}$ for all $q_r \in \Re^{n_r}$.*

Assumption A2. *The function M_c is continuous and such that, for some (unknown) positive scalars $\overline{m}$, $\underline{m}$ and known continuous function μ, the following hold for all $q_c \in \Re^{n_c}$: (i) $\|M_c^{-1}(q_c)\| \leq \overline{m}\mu(q_c)$, and (ii) $M_c^{-1}(q_c) \geq \underline{m}I$ (in the sense that $\langle v, M_c^{-1}(q_c)v \rangle \geq \underline{m}\|v\|^2$ for all $v \in \leq pr^{n_c}$.*

Assumption A3. *The function g_r is continuous.*

Assumption A4. *The function h is continuous, with $h(0,0) = 0$.*

Assumption A5. *With $h \equiv 0$, the subsystem Σ_r is quadratically asymptotically stable, in the sense that there exists an (unknown) positive definite quadratic form v_r*

on $\Re^{n_r}$ such that, for some (unknown) positive scalar c

$$\frac{d}{dt}V_r(q_r(t), \dot{q}_r(t)) \leq -cV_r(q_r(t), \dot{q}_r(t))$$

Assumption A6. *The function g_c is of Carathéodory class and such that, for some known continuous function γ, the following holds for some (unknown) scalar α:*

$$\|g_c(t, q_r, v_r, q_c, v_c)\| \leq \alpha\gamma(q_r, v_r, q_c, v_c)$$

for almost all $t \in \Re$ and all $(q_r, v_r, q_c, v_c) \in \Re^{2n}$.

Thus, the only *a priori* system information available to the controller is the pair of continuous functions γ and μ: in particular, we stress that the uncertainty bounding parameters $\tilde{m}$, $\underline{m}$, $\overline{m}$ and α are unknown.

The question to be addressed can be posed as follows:

Does there exist an adaptive feedback strategy, parameterized by $\lambda > 0$, which, for every system (unknown to the controller) of class Σ, every solution of the feedback-controlled initial-value problem (1) is asymptotic to a ball centred at zero in $\Re^n$ of radius $\rho(\lambda)$, where $\rho(\lambda) \longrightarrow 0$ as $\lambda \longrightarrow 0$? In Section 3, we answer this question affirmatively by explicit construction of one such feedback strategy.

2.1 Example: active control of base-isolated structures

In the field of civil engineering structures there exist a great interest in reducing the structural response produced by seismic ground motions. In recent years, as one of the possibilities to achieve this objective, hybrid control systems have been proposed, which combine base isolators with active control systems.

Base isolators attempt to uncouple the structure from the seismic ground motion by means of replaceable devices, placed between the building and the foundation, capable of absorbing part of the energy induced by earthquakes [7]. The base isolation component can reduce by itself both the inter-story drift and the absolute accelerations of the structure. Thus the structure tends to behave like a rigid body, the price paid being a significant displacement of the base. Another drawback of such systems is the dependence of their effectiveness on the frequency of the excitation. Moreover, they cannot be applied in the case of tall or heavy structures, due to the size of the dynamic forces involved and to the risk of endangering the global stability of the structure. The objective of the active control component is to reduce the base displacement by means of forces applied on the base. From a practical point of view, this hybrid scheme is appealing since it is possible to achieve the above mentioned objective by means of a single force which, moreover, does not exceed some acceptable limits due to the high flexibility of the base isolators. Moreover, the active control action essentially does not depend on the frequency content of the dynamic excitation. From a theoretical point of view, the development of a control law to calculate the

active force involves difficulties associated with the nonlinear behaviour of the base isolators and to the uncertainties in the models describing the structure–base isolator system and in the seismic excitation.

A robust control law for linear systems has been proposed in a previous work [2]. Also for linear systems, the application of predictive control has been considered [8] as well as a form of bang–bang control [9]. The nonlinearity of the isolators has been considered in reference [10], assuming no uncertainties in the structure–base model. Some experimental works with small-scale hybrid systems have been recently reported [11,12].

The hybrid control problem we are dealing with can be cast within the framework of the class of systems defined by equations (1). In the remaining of this section the equations of motion governing this problem will be presented.

The dynamic behaviour of the structure with the hybrid control system can be described by means of a model composed of two coupled systems : Σ_r (the building) and Σ_c (the base). It is assumed that the structure has a linear behaviour due to the effect of the base isolation. The behaviour of the isolator may be nonlinear. The vector q_r represents the horizontal displacements of the n degrees of freedom respect to an inertial frame, while the displacement of the structural base is described by a single degree of freedom with horizontal displacement q_c relative to the above-mentioned frame. The dynamic excitation is produced by a horizontal seismic ground motion, characterized by a displacement $d(t)$ and its velocity $v(t)$. A single horizontal control force $u(t)$ acts upon the structural base. Thus, the equations of motion are

$$\Sigma_r : \qquad M\ddot{q}_r + C\dot{q}_r + Kq_r = CJ\dot{q}_c + KJq_c$$

$$\Sigma_c : \qquad \begin{aligned} &m_b\ddot{q}_c + [c_b + J^T C J]\dot{q}_c + [k_b + J^T K J]q_c \\ &-J^T C\dot{q}_r - J^T Kq_r - c_b v - k_b d + f(q_c, \dot{q}_c, d, v) = u \end{aligned} \qquad (2)$$

where M, C and K are the mass, damping and stiffness matrices of the structure, respectively. The vector J expresses the rigid body motion according to the degrees of freedom of the model (in this case it is an unit vector). m_b, c_b and k_b are the mass, damping and stiffness of the base. The last two parameters correspond to the elastic and damping forces which appear on the base due to the linear effects of the isolator; f is an additional horizontal force produced on the structural base by nonlinearities in the isolator.

Assumptions A1–A5 hold as M is invertible, $m_b > 0$ and C and K are positive definite. Assumption A6 holds under the following conditions:

$$\|c_b v(t) + k_b d(t)\| \leq \nu \qquad (3)$$

$$\|f(q_c, \dot{q}_c, d, v)\| \leq \alpha' \gamma'(q_c, \dot{q}_c) \qquad (4)$$

for almost all t and all $(q_c, \dot{q}_c) \in \Re^2$, ν and α' being unknown scalars and γ' a known continuous function.

4

3 THE ADAPTIVE STRATEGY

Throughout this section, we assume $\lambda > 0$. We first introduce some notation. Let d_λ denote the function defined (on $\Re^{n_c}$, $\Re^{n_r}$, $\Re^{2n_c}$ or $\Re^{2n_r}$ as context dictates) by

$$d_\lambda : v \mapsto \begin{cases} \|v\| - \lambda, & \|v\| \geq \lambda \\ 0, & \|v\| < \lambda \end{cases}$$

Let s_λ denote the function defined on $\Re^{n_c}$ by

$$s_\lambda : v \mapsto \begin{cases} \|v\|^{-1} v, & d_\lambda(v) > 0 \\ \lambda^{-1} v, & d_\lambda(v) = 0 \end{cases}$$

The proposed adaptive strategy, parameterized by $\lambda > 0$, is given by

$$
\begin{aligned}
u(t) &= -k(t) U_\lambda(q_r(t), \dot{q}_r(t), q_c(t), p_c(t)) \\
p_c(t) &= \dot{q}_c(t) + \eta q_c(t) \\
\dot{k}(t) &= K_\lambda(q_r(t), \dot{q}_r(t), q_r(t), p_r(t)) \\
k(t_0) &= k^0
\end{aligned}
\tag{5}
$$

where $\eta > 0$ (a design parameter) is open to choice, and the functions U_λ and K_λ are defined as follows

$$
\begin{aligned}
U_\lambda &: (q_r, v_r, q_c, p_c) \mapsto p_c + \gamma_\mu(q_r, v_r, q_c, p_c) s_\lambda(p_c) \\
K_\lambda &: (q_r, v_r, p_c) \mapsto d_\lambda(p_c) \left[\|p_c\| + \gamma_\mu(q_r, v_r, q_c, p_c) \right] \\
\gamma_\mu &: (q_r, v_r, q_c, p_c) \mapsto \mu(q_c) \gamma(q_r, v_r, q_c, p_c - q_c)
\end{aligned}
\tag{6}
$$

3.1 Stability analysis

The overall controlled system representation on $\Re^N$, $N = 2(n_r + n_c) + 1$ now becomes

$$
\begin{aligned}
M_r(q_r(t)) \ddot{q}_r(t) + g_r(q_r(t), \dot{q}_r(t)) &= h(q_c(t), p_c(t) - q_c(t)) \\
\dot{q}_c(t) &= -\eta q_c(t) + p_c(t) \\
\dot{p}_c(t) &= P_\lambda(t, q_r(t), \dot{q}_r(t), q_c(t), p_c(t), k(t)) \\
\dot{k}(t) &= K_\lambda(q_r(t), \dot{q}_r(t), q_c(t), p_c(t)) \\
(q_r(t_0), \dot{q}_r(t_0), q_c(t_0), p_c(t_0), k(t_0)) &= (q_r^0, v_r^0, q_c^0, p_c^0, k^0) =: x^0 \in \Re^N
\end{aligned}
\tag{7}
$$

where the function P_λ is given by

$$P_\lambda(t, q_r, v_r, q_c, p_c, k) := p_c - \eta q_c - M_c^{-1}(q_c)[g_c(t, q_r, v_r, q_c, p_c - \eta q_c)$$

$$+ k U_\lambda(q_r, v_r, q_c, p_c)]$$

Equivalently, writing $x(t) = (q_r(t), \dot{q}_r(t), q_c(t), p_c(t), k(t))$,

$$\dot{x}(t) = F_\lambda(t, x(t)), \quad x(t_0) = x^0 \tag{8}$$

where
$$F_\lambda : \ x = (q_r, v_r, q_c, p_c, k) \mapsto$$

$$\left(v_r, \ M_r^{-1}[h(q_c, p_c - \eta q_c) - g_r(q_r, v_r)], \ -\eta q_c + p_c, \ P_\lambda(t, x), \ K_\lambda(x) \right).$$

This system satisfies the classical Carathéodory conditions and so, for every $(t_0, x^0) \in \Re \times \Re^N$, the above initial-value problem has a solution and every solution can be extended into a maximal solution.

On $[0, \infty)$, define

$$\hat{h} : \lambda \mapsto \sup\{\|h(q_c, p_c - \eta q_c)\| \mid d_\lambda(q_c) = 0 = d_\lambda(p_c)\},$$

which, by virtue of Assumption A4, is continuous with $\hat{h}(\lambda) \longrightarrow 0$ as $\lambda \longrightarrow 0$.

Theorem 1.
Let $\lambda > 0$ and $(t_0, x^0) \in \Re \times \Re^N$.
For every maximal solution $x(\cdot) = (q_r, \dot{q}_r, q_c, p_c, k)(\cdot) : [t_0, \omega) \longrightarrow \Re^N$ of the initial-value problem (7) (equivalently (8)),
(i) $\omega = \infty$;
(ii) $\lim_{t \longrightarrow \infty} k(t)$ exists and is finite;
(iii) $d_\lambda(q_c(t)), \ d_\lambda(p_c(t)) \longrightarrow 0$ as $t \longrightarrow \infty$;
(iv) for some positive scalar c, $d_{c\hat{h}(\lambda)}(q_r(t), \dot{q}_r(t)) \longrightarrow 0$ as $t \longrightarrow \infty$.

Proof. Let V denote the C^1 function

$$V : \ p_c \mapsto \frac{1}{2} d_\lambda^2(p_c).$$

Then, for almost all $t \in \Re$ and all $x = (q_r, v_r, q_c, p_c, k) \in \Re^N$, we have

$$\langle \nabla V(p_c), P_\lambda(t, x) \rangle \leq d_\lambda(p_c) \left[\|q_c\| + (\alpha \overline{m} - \underline{m}k)\gamma_\mu(q_r, v_r, q_c, p_c) + (1 - \underline{m}k)\|p_c\| \right]$$

Defining $k^* := \underline{m}^{-1}(1 + \alpha \overline{m})$, it follows that

$$\frac{d}{dt} V(p_c(t)) \leq -\underline{m}(k(t) - k^*)\dot{k}(t) + d_\lambda(p_c(t))\|q_c(t)\|$$

for almost all $t \in [t_0, \omega)$. Integration now yields, with $c_0 := V(p_c^0) + \frac{1}{2}\underline{m}(k^0 - k^*)^2$,

6

$$0 \leq V(p_c(t)) \leq c_0 - \frac{1}{2}\underline{m}(k(t) - k^*)^2 + \int_{t_0}^{t} d_\lambda(p_c(s))\|q_c(s)\|ds$$

which is valid for all $t \in [t_0, \omega)$.

We briefly digress to prove a technicality.

Proposition. *For some positive scalar c_1,*

$$\int_{t_0}^{t} d_\lambda(p_c(s))\|q_c(s)\|ds \leq c_1 \int_{t_0}^{t} [d_\lambda(p_c(s)) + d_\lambda^2(p_c(s))]ds$$

for all $t \in [t_0, \omega)$.

Proof. First observe that

$$\|q_c(s)\| \leq \|q_c^0\| + \int_{t_0}^{s} e^{-(s-\sigma)}\|p_c(\sigma)\|d\sigma \leq \|q_c^0\| + \int_{t_0}^{s} e^{-(s-\sigma)}[d_\lambda(p_c(\sigma)) + \lambda]d\sigma$$

$$\leq \|q_c^0\| + \lambda + \int_{t_0}^{s} e^{-(s-\sigma)} d_\lambda(p_c(\sigma))d\sigma.$$

Therefore,

$$\int_{t_0}^{t} d_\lambda(p_c(s))\|q_c(s)\|ds$$

$$\leq (\|q_c^0\| + \lambda) \int_{t_0}^{t} d_\lambda(p_c(s))ds + \int_{t_0}^{t} d_\lambda(p_c(s)) \int_{t_0}^{s} e^{-(s-\sigma)} d_\lambda(p_c(\sigma))d\sigma \, ds$$

Applying Hölder's inequality to the second term on the right,

$$\int_{t_0}^{t} d_\lambda(p_c(s)) \int_{t_0}^{s} e^{-(s-\sigma)} d_\lambda(p_c(\sigma))d\sigma \, ds$$

$$\leq \left(\int_{t_0}^{t} d_\lambda^2(p_c(s))ds \right)^{\frac{1}{2}} \left(\int_{t_0}^{t} e^{-2s} \left(\int_{t_0}^{s} e^{\sigma} d_\lambda(p_c(\sigma))d\sigma \right)^2 ds \right)^{\frac{1}{2}}.$$

Integrating by parts in the last term on the right

$$\int_{t_0}^{t} e^{-2s} \left(\int_{t_0}^{s} e^{\sigma} d_\lambda(p_c(\sigma))d\sigma \right)^2 ds \leq \int_{t_0}^{t} d_\lambda(p_c(s)) \int_{t_0}^{s} e^{-(s-\sigma)} d_\lambda(p_c(\sigma))d\sigma \, ds.$$

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We may now conclude that

$$\int_{t_0}^{t} d_\lambda(p_c(s)) \int_{t_0}^{s} e^{-(s-\sigma)} d_\lambda(p_c(\sigma)) d\sigma \, ds \leq \int_{t_0}^{t} d_\lambda^2(p_c(s)) ds,$$

whence the claim.

Returning to the proof of the theorem, we now have

$$0 \leq V(p_c(t)) \leq c_0 - \frac{1}{2}\underline{m}(k(t) - k^*)^2 + c_1(1 + \lambda^{-1})(k(t) - k^0)$$

for all $t \in [t_0, \omega)$. Therefore, we see that the monotone increasing function $k(\cdot)$ is bounded. This, in turn, implies boundedness of $V(p_c(\cdot))$ and so $p_c(\cdot)$ is bounded. It immediately follows that $q_c(\cdot)$ is bounded. By assumptions A4 and A5, we see that $(q_r, \dot{q}_r)(\cdot)$ is bounded. We have now shown that the solution $x(\cdot)$ is bounded and so $\omega = \infty$. Assertion (ii) of the theorem is now a consequence of boundedness and monotonicity of $k(\cdot)$.

To prove Assertion (iii), we argue as follows. Observe that

$$d_\lambda(p_c(t))\|q_c(t)\| \leq d_\lambda(p_c(t))\lambda^{-1}\|p_c(t)\|\|q_c(t)\| \leq \lambda^{-1}\|q_c(t)\|\dot{k}(t)$$

and so, by boundedness of $q_c(\cdot)$, there exists positive scalar c_2 such that

$$d_\lambda(p_c(t))\|q_c(t)\| \leq c_2\dot{k}(t)$$

for almost all $t \geq t_0$. Writing $c_3 := \underline{m}(k^* - k^0) + c_2$, we conclude that

$$\frac{d}{dt}V(p_c(t)) \leq -\dot{k}(t) + (c_3 + 1)\dot{k}(t)$$

for almost all $t \geq t_0$. Therefore the function

$$W : (p_c, k) \mapsto V(p_c) - (1 + c_3)k$$

is such that

$$\frac{d}{dt}W(p_c(t), k(t)) \leq -\dot{k}(t) \leq -d_\lambda(p_c(t))\|p_c(t)\| \tag{9}$$

for almost all $t \geq t_0$. Boundedness of the solution $x(\cdot)$ ensures that it has non-empty ω-limit set Ω. Since the solution approaches its ω-limit set, we first prove that

$$d_\lambda(p_c(t)) \longrightarrow 0 \quad \text{como} \quad t \longrightarrow \infty$$

by showing that $d_\lambda(\bar{p}_c) = 0$ for all $\bar{x} = (\bar{q}_r, \bar{v}_r, \bar{q}_c, \bar{p}_c, \bar{k}) \in \Omega$. Suppose otherwise. Then there exists $\bar{x} = (\bar{q}_r, \bar{v}_r, \bar{q}_c, \bar{p}_c\bar{k}) \in \Omega$ and $\epsilon > 0$ such that $d_\lambda(\bar{p}_c)\|\bar{p}_c\| > 2\epsilon$. By continuity, there exist s $\delta > 0$ such that

8

$$\|\xi - \bar{p}_c\| < \tilde{\delta} \implies d_\lambda(\xi)\|\xi\| > \epsilon.$$

Since $\bar{x}$ is an ω-limit point, there exists a sequence (t_j) with $t_j \longrightarrow \infty$ and

$$x(t_j) = (q_r(t_j), \dot{q}_r(t_j), q_c(t_j), p_c(t_j), k(t_j)) \longrightarrow \bar{x}$$

as $j \longrightarrow \infty$. By Assumptions A1–A6, it is readily verified that there exist $\bar{\delta} > 0$ and $R > 0$ such that

$$\|x - \bar{x}\| < \bar{\delta} \implies \|F_\lambda(t, x)\| < R. \tag{10}$$

We may assume $\bar{\delta} < \tilde{\delta}$. By continuity of W,

$$W(p_c(t_j), k(t_j)) - W(\bar{p}_c, \bar{k}) < \frac{\epsilon\bar{\delta}}{4R} \tag{11}$$

for all j sufficiently large. Let j^* be such that

$$\|x(t_j) - \bar{x}\| < \frac{1}{2}\bar{\delta} \quad \forall\, j > j^*.$$

By (7) and (9), it follows that

$$\|p_c(t_j) - \bar{p}_c\| < \bar{\delta} \quad \forall\, t \in [t_j, t_j + (\bar{\delta}/(3R))]$$

which holds for all $j > j^*$. Therefore, using (8), we have for all $j > j^*$

$$W(p_c(t_j), k(t_j)) - W(\bar{p}c, \bar{k}) \geq \int_{t_j}^{t_j + (\bar{\delta}/3R)} d_\lambda(p_c(t))\|p_c(t)\|dt \ \geq\ \frac{\epsilon\bar{\delta}}{3R}$$

which contradicts (10). Therefore, $d_\lambda(p_c(t)) \longrightarrow 0$ as $t \longrightarrow \infty$.

Since $\dot{q}_c(t) = -q_c(t) + p_c(t)$, we also have $d_\lambda(q_c(t)) \longrightarrow 0$ as $t \longrightarrow \infty$. This establishes Assertion (iii).

Finally, Assertion (iv) is a direct consequence of Assumptions A4 and A5.   Q.E.D.

Remarks. By the above theorem, we see that the proposed adaptive feedback strategy ensures a form of practical stability for the system class Σ. In essence, for any prescribed $\lambda > 0$, the subsystem state $(q_c(t), p_c(t))$ is asymptotic to that ball (centred at zero in $\Re^{2n_c}$) of radius λ, the remaining subsystem state $(q_r(t), \dot{q}_r(t))$ is asymptotic to a ball (centred at zero in $\Re^{2n_r}$) of radius $c\lambda$ – however, the scale factor $c > 0$ depends on the unknown function h and so is not computable from *a priori* system information.

4 ILLUSTRATIVE EXAMPLE

Consider a 10-story base isolated shear building as shown in Figure 1 and described by equations (2). The masses of the base and of each floor of the building are 6×10^5 Kg. The stiffness of the base is $k_b = 7 \times 10^8$ N/m and its damping ratio is 0.1. The stiffness of the building varies in 5×10^7 N/m between floors, from 9×10^8 N/m the first one to 4.5×10^8 N/m the top one, while the damping ratio is 0.05. The nonlinear force f produced by the base isolation device on the base has elastoplastic hysteretic or frictional characteristics (Figure 2).
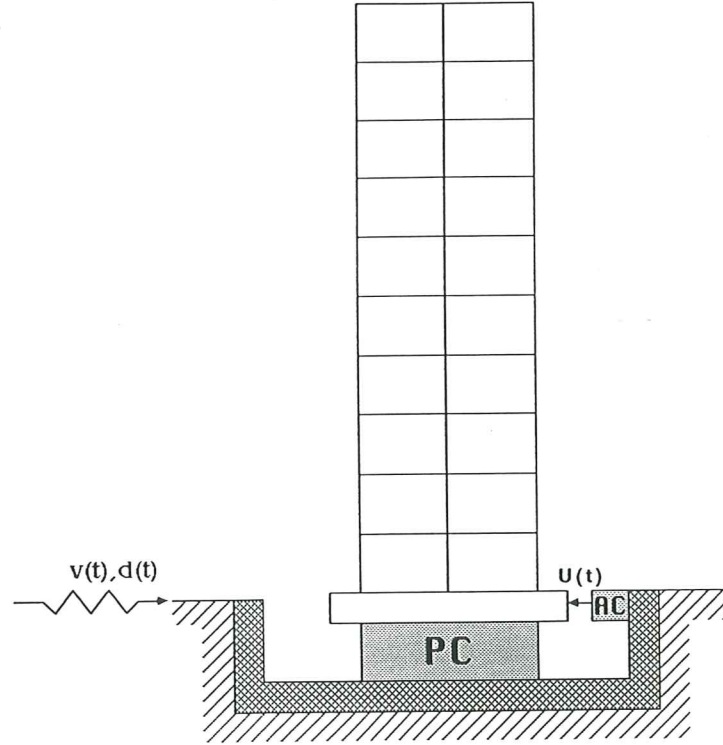
Figure 1 Building structure with a hybrid control system. **PC** – passive control system. **AC** – active control system.

The purpose of this example is to show the effectiveness of the control law (5) and (6) when applied to the above described structural system. To do this, the first step is to identify the function γ_μ appearing in (6). In this case, $\mu(q_c) = 1$. To obtain the function γ, according to assumption A6, we first need to identify function g_c for the case of equations (2). Comparing subsystems Σ_c in equations (1) and (2), it is observed that

$$g_c(t, q_r(t), \dot{q}_r(t), q_c(t), \dot{q}_c(t)) = [c_b + J^T C J]\dot{q}_c + [k_b + J^T K J]q_c$$

$$- J^T C \dot{q}_r - J^T K q_r - c_b v - k_b d + f(q_c, \dot{q}_c, d, v)$$

(12)
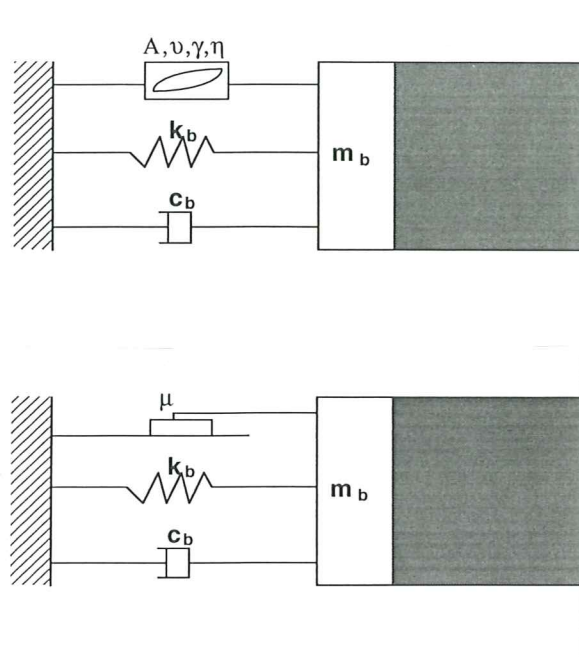
Figure 2 Base isolation systems: (a) histeretic); (b) frictional.

According to the elastoplastic or the friccional behaviour considered for the isolator, the nonlinear force f remains always bounded. Thus, condition (4) reduces to

$$\|f(q_c, \dot{q}_c, d, v)\| \leq \xi \tag{13}$$

ξ being an unknown scalar. Using now inequalities (3) and (13) in equation (12), it can be readily written

$$\|g_c(t, q_r, \dot{q}_r, q_c, \dot{q}_c)\| \leq \epsilon \gamma(q_r, \dot{q}_r, q_c, \dot{q}_c) \tag{14}$$

where ϵ is an unknown scalar and

$$\gamma(q_r, \dot{q}_r, q_c, \dot{q}_c) = [q_c^2 + \dot{q}_c^2 + q_{r_1}^2 + \cdots, + q_{r_n}^2 + \dot{q}_{r_1}^2 + \cdots + \dot{q}_{r_n}^2 + 1]^{1/2} \tag{15}$$

Using this function, the control law (5) and (6) is now applied to compute the active control u for all time t.

The application of the control law has been numerically simulated and some of the results are included in this section. In all the tests, parameter η has been chosen equal to 1. In Figures 3-5, the case of the structure of Figure 1 with a hysteretic base isolation is considered. Figures 3 and 4 show the time histories of the absolute displacement and the displacement of the 10th floor of the structure relative to the base. The seismic excitation has been that of the El Centro (1940) earthquake. In both figures the responses for the passive and hybrid cases are compared. It can be observed that, for the hybrid case, the displacement response rapidly enters within a

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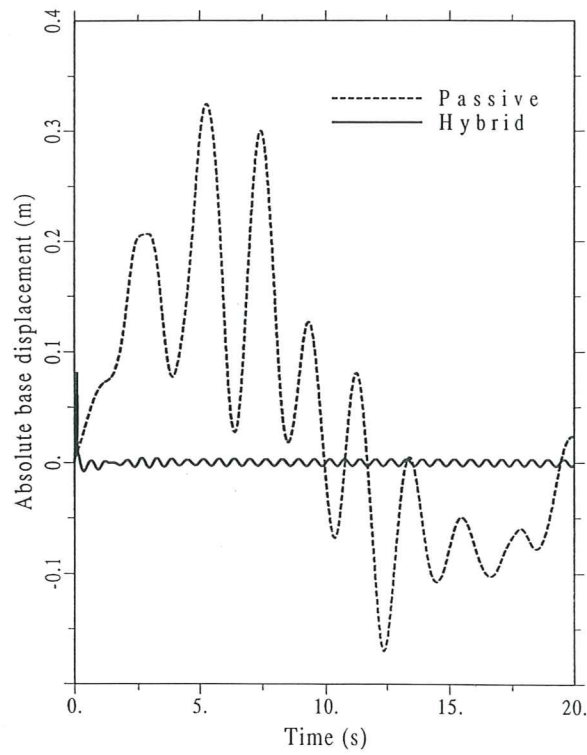
Figure 3 Absolute base displacement response for passive and hybrid cases —hysteretic base isolator—.
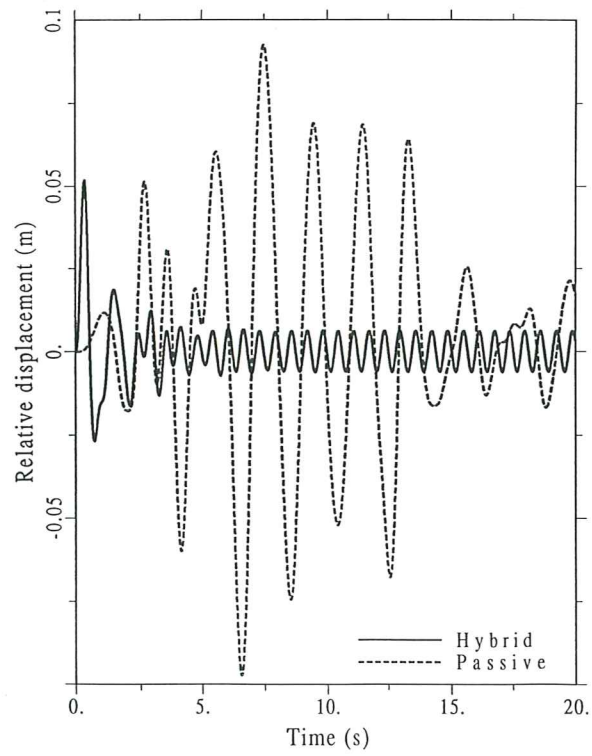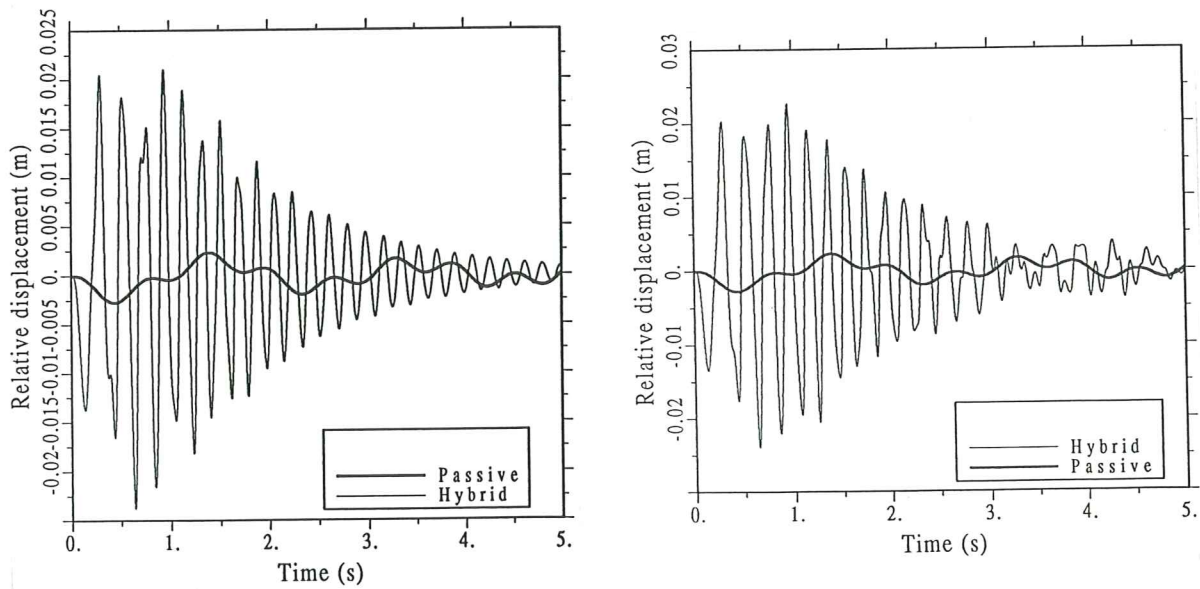

Figure 4 Displacement of the 10th floor relative to the base for passive and hybrid cases —hysteretic base isolator—.

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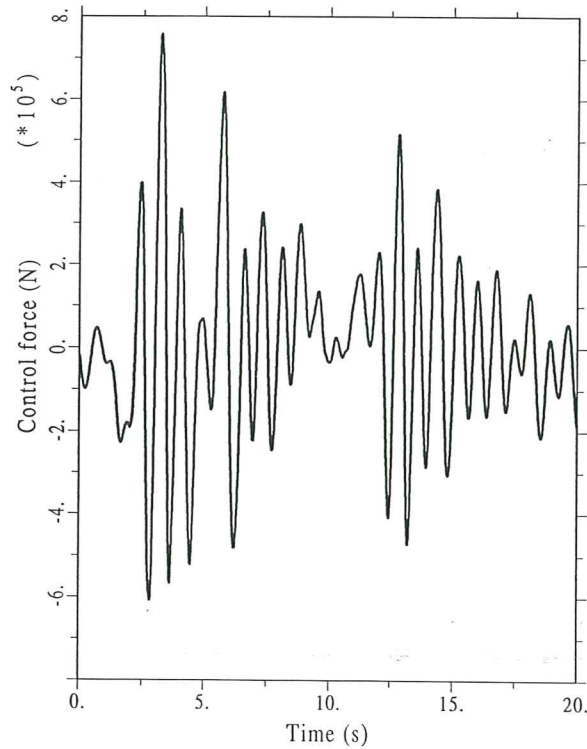
Figure 5 Active control force —hysteretic base isolator—.

bounded region around zero. This shows a behaviour as expected from the stability analysis performed in previous section 3.1. The corresponding control force, plotted in Figure 5, remains within an acceptable range. The responses of Figures 3-5 have been simulated for a value $\lambda = 0.8$.

λ is the most significant parameter in the implementation of the adaptive control law, since it defines the size of the stability region. The control forces calculated according to this law assure the accomplishment of the control objective, that is they maintain the values of the base response within a ball whose size is λ. It can be observed that even in the case in which the base response lies within this ball, the adaptive control law generates continuously control forces aimed to avoid that the response goes out of the ball. In this last case an interesting condition from a practical point of view can be added to the control strategy: if the value of the base response is smaller than λ, the control force is set equal to zero. In the following examples, the standard use of the adaptive control law will be denoted as case (a) while the situation in which the control forces are calculated by applying the mentioned practical condition will be denoted as case (b). Comparative results for these two cases can be seen in Figures 6-9 for the same hysteretic base isolation system used in the example of Figures 3-5.

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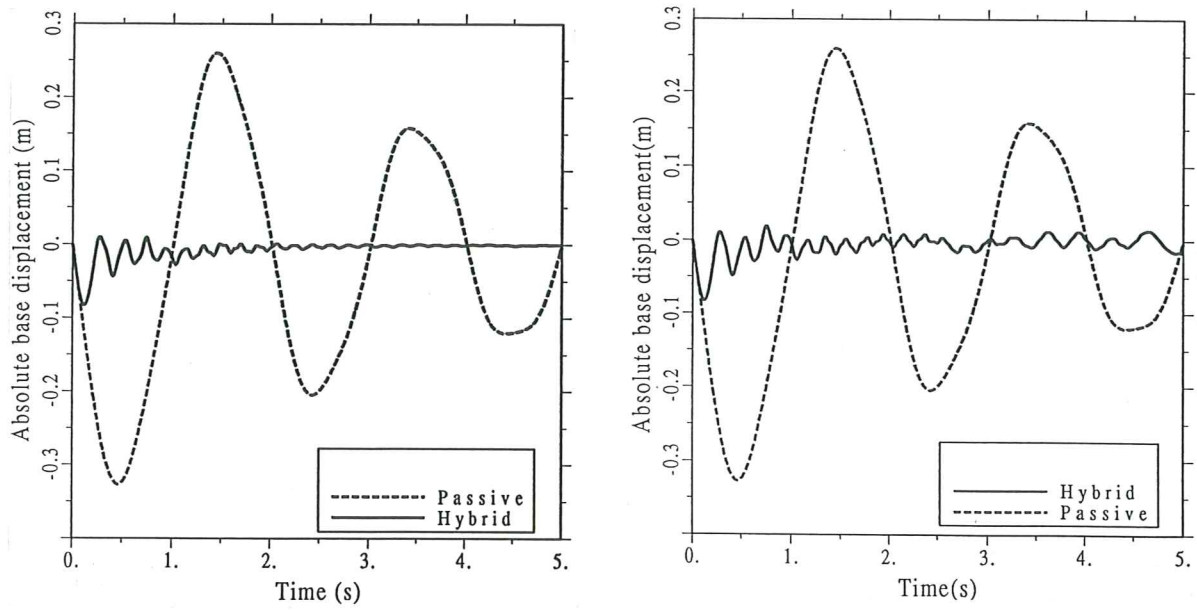
Figure 6 Absolute base displacement response for passive and hybrid cases –hysteretic base isolator–. (a) standard adaptive control law. (b) zero force if response within the ball.

Figure 7 Displacement of the 10th floor relative to the base for passive and hybrid cases —hysteretic base isolator—. (a) standard adaptive control law. (b) zero force if response within the ball.
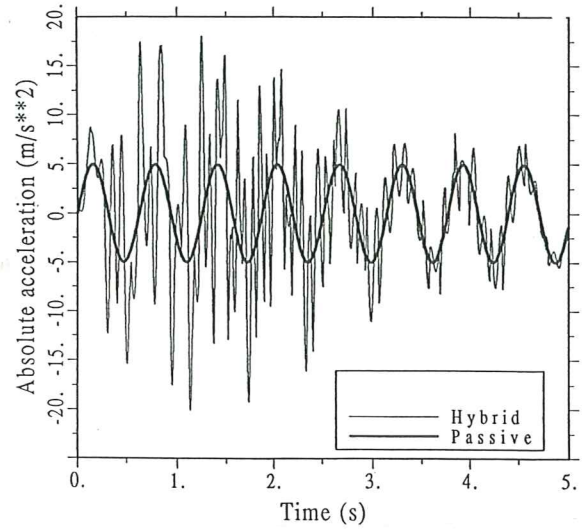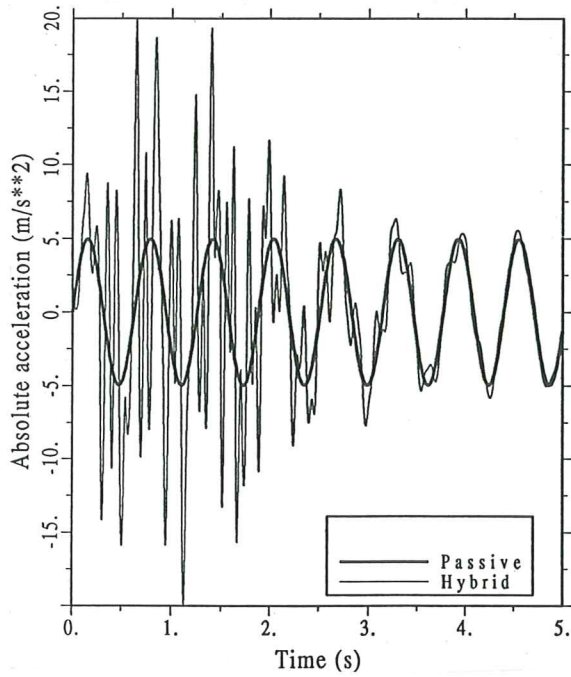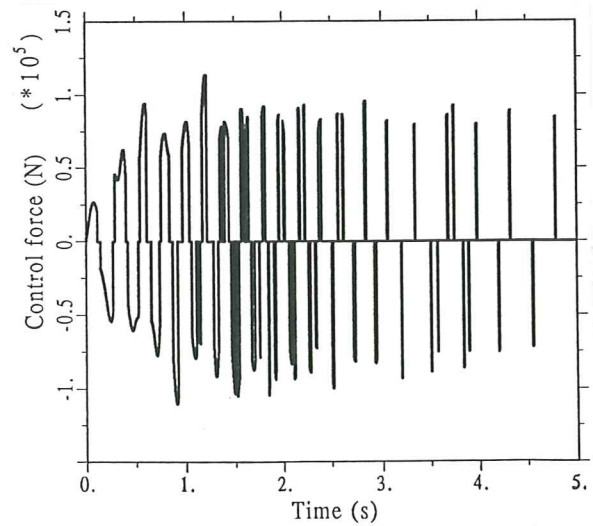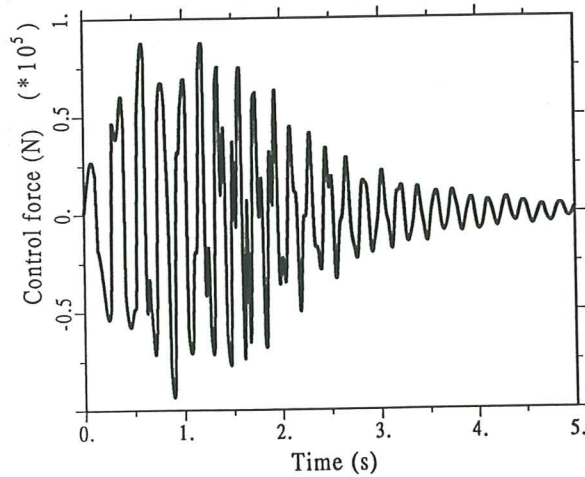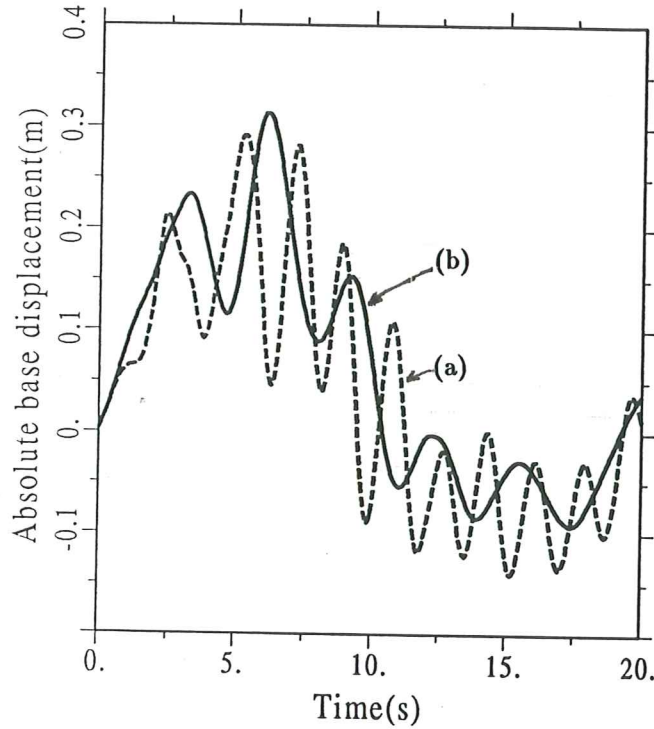
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Figure 8 Absolute acceleration response of the 10th floor for passive and hybrid cases —hysteretic base isolator—. (a) standard adaptive control law. (b) zero force if response within the ball.

Figure 9 Active control force —hysteretic base isolator—. (a) standard adaptive control law. (b) zero force if response within the ball.

An extreme case is that of Figures 10-12 in which λ has been set to 1.2, a high value for which the structural response lies always within the ball. As a consequence, algorithm (b) produces zero control forces, the simulated structural response being identical to that of the passive case. It can be observed from Figure 11 that the relative displacement is higher if the standard adaptive control law is used, due to the effect of the non-zero control forces of Figure 12.

Figure 10 Absolute base displacement response for the hybrid case
—hysteretic base isolator—. (a) standard adaptive control law. (b) zero force if response within the ball.

In order to assess the influence of parameter λ on the effectiveness of the control law, a single degree of fredom model has been considered, having the same mass and stiffness as the first floor of the previously studied 10-story building. The stiffness of the base is $k_b = 7 \times 11.843525^7$ N/m and its damping ratio is 0.2. Figures 13-16 display the maximum values of the absolute base displacement, displacement of the 10th floor relative to the base, absolute acceleration of the top floor and control force, respectively, as a function of λ. Five harmonic ground motions have been considered, each of them having a different frequency and the maximum responses are plotted for both the pure friction and hysteretic cases. The general observation can be made that smaller the value of λ is, smaller controlled displacements are obtained, since it implies a more demanding control objective. The final, constant, zones of the plots correspond to the passive base isolated building, as the before mentioned zero force condition has been used for the case in which the response lies within the ball.
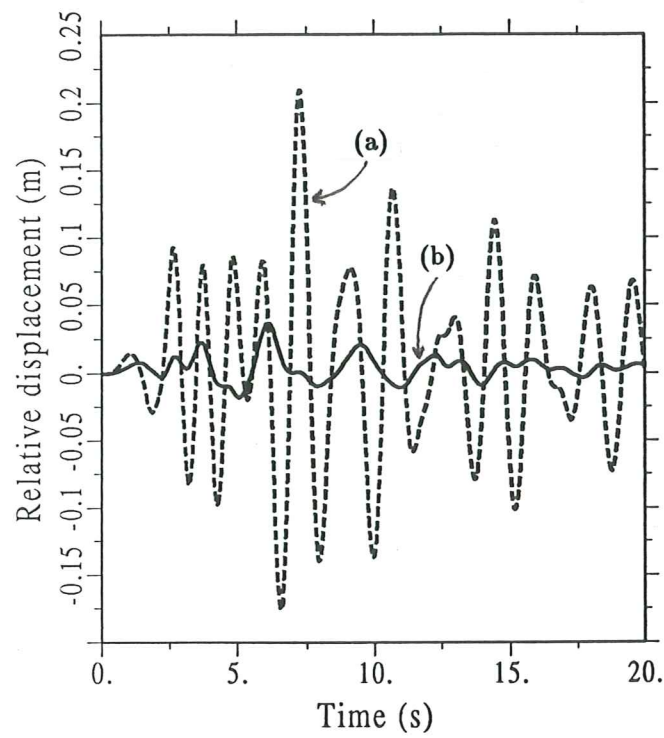
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Figure 11 Displacement of the 10th floor relative to the base for the hybrid case —hysteretic base isolator—. (a) standard adaptive control law. (b) zero force if response within the ball.
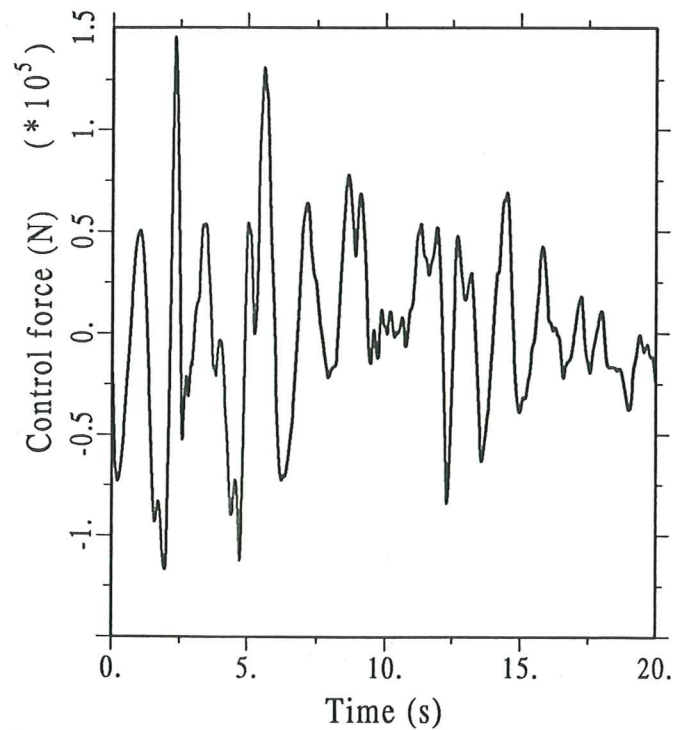

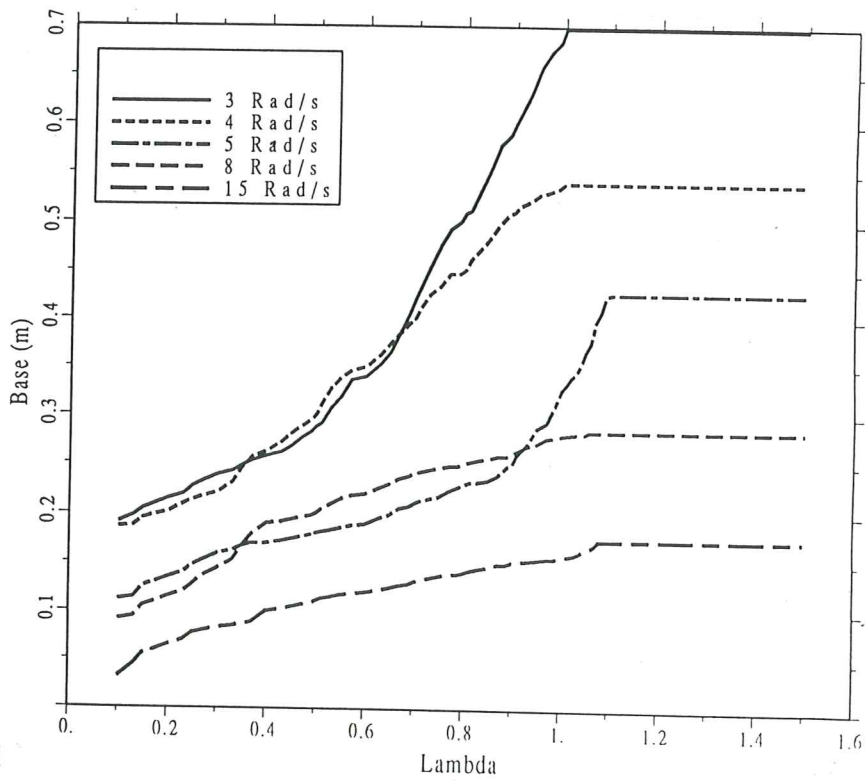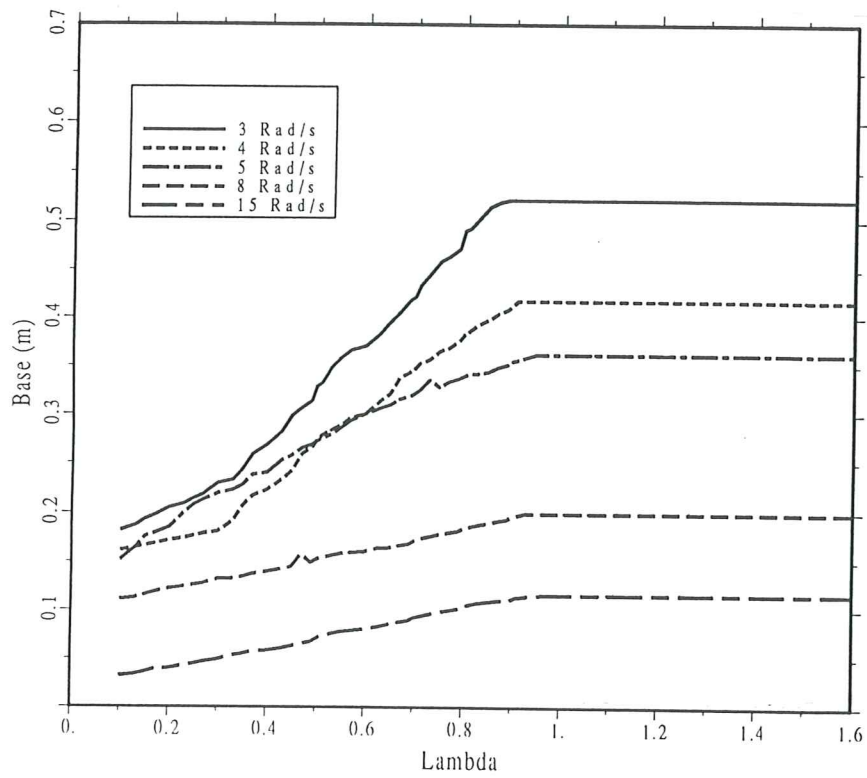**Figure 12** Active control force —hysteretic base isolator—. Standard adaptive control law.

Figure 13 Plot of the absolute base displacement against λ. (a) pure friction base isolator. (b) hysteretic base isolator.
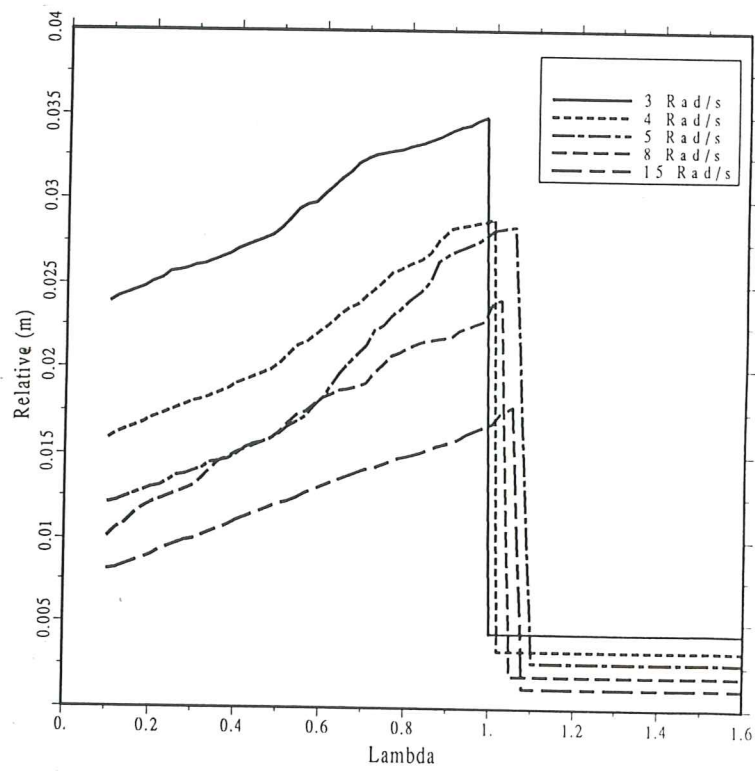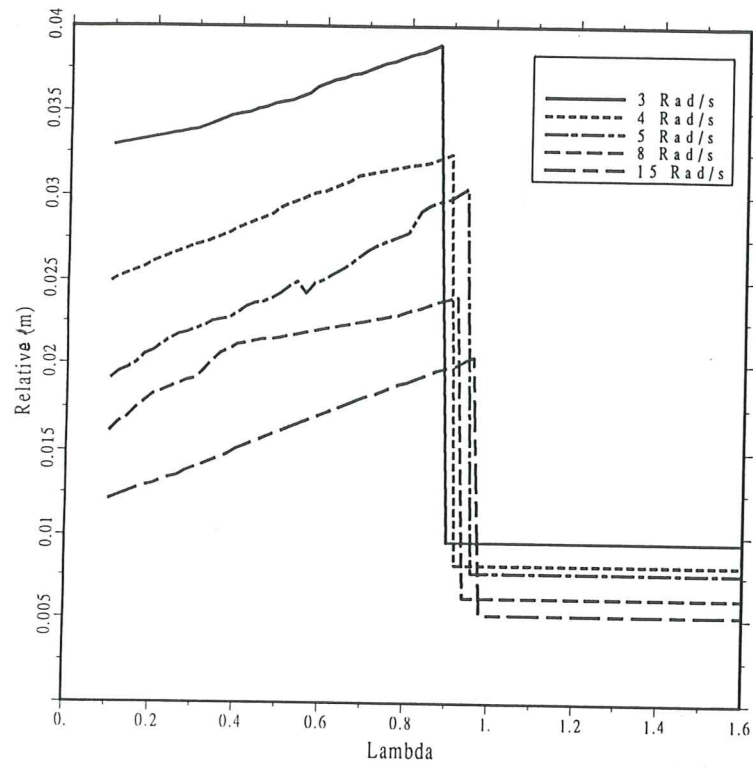
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Figure 14 Plot of the displacement of the model relative to the base against λ. (a) pure friction base isolator. (b) hysteretic base isolator.
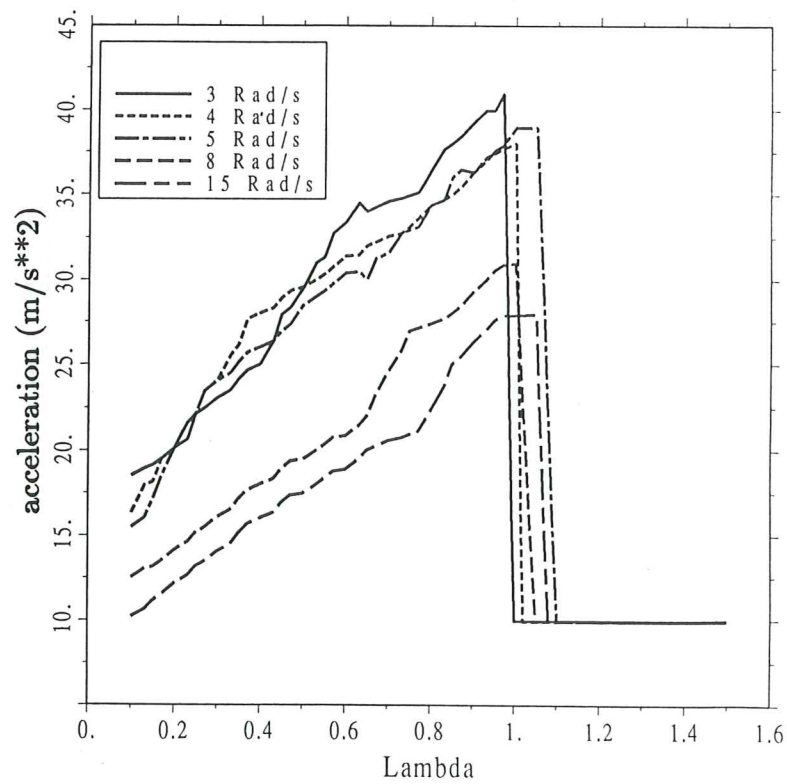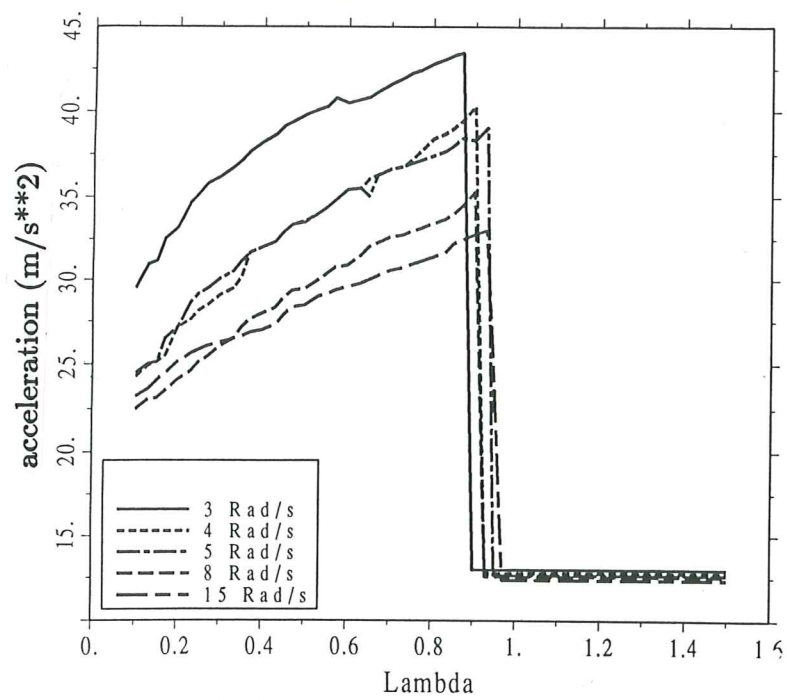
Figure 15 Plot of the absolute acceleration of the model against λ. (a) pure friction base isolator. (b) hysteretic base isolator.
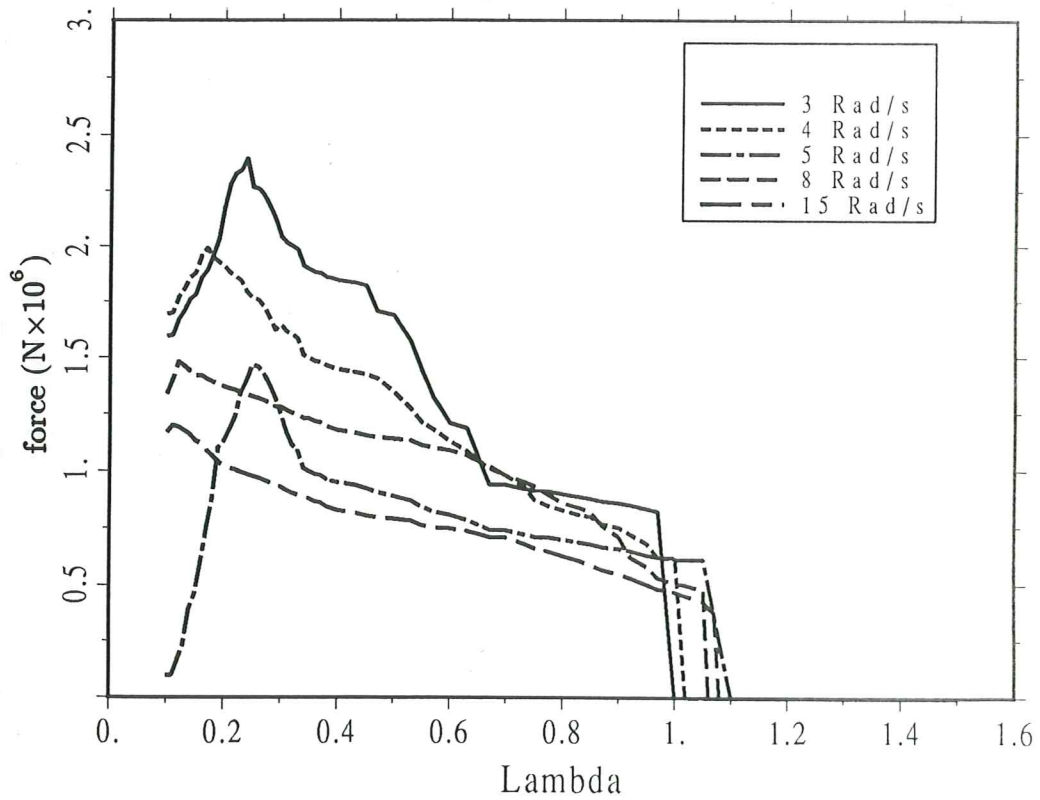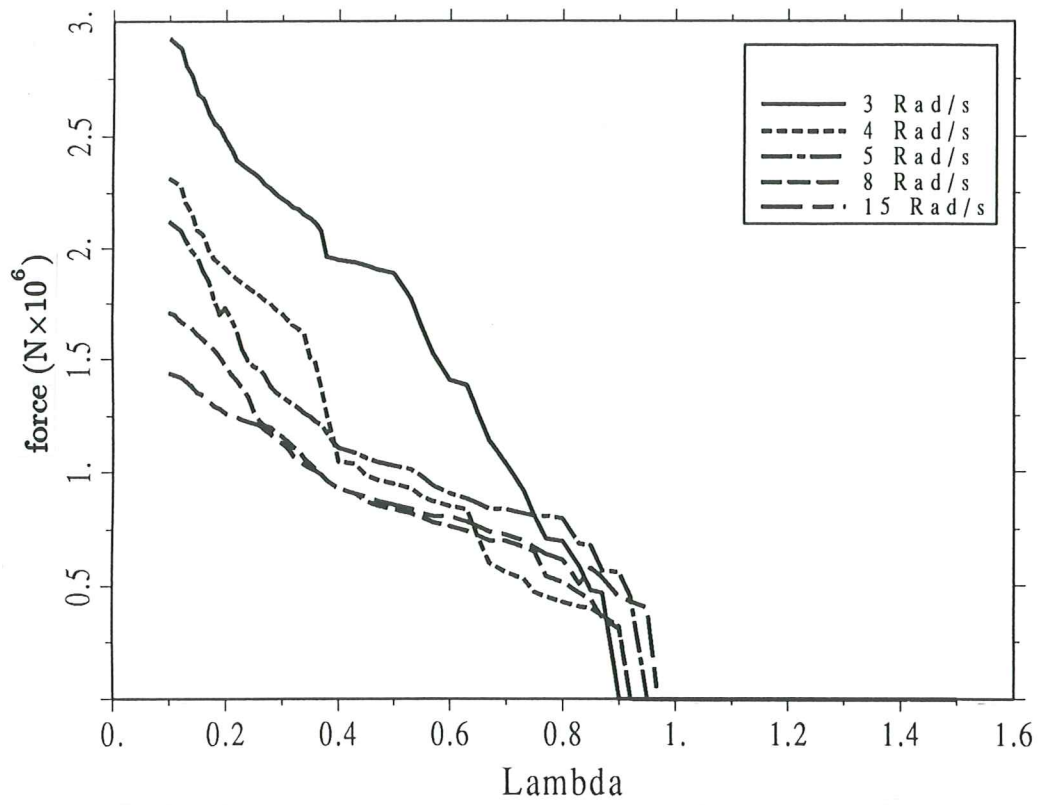
Figure 16 Plot of the active control force against λ. (a) pure friction base isolator. (b) hysteretic base isolator.

ACKNOWLEDGEMENTS

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