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Research Article

Intergreen Time Calculation Method of Signalized Intersections Based on Safety Reliability Theory: A Monte-Carlo Simulation Approach

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In China, around ninety percent of the traffic accidents at signalized intersections occur within the signal change intervals, especially during signal change from green to red. Hence, intergreen time (IGT), that is, yellow change interval plus red clearance interval, is of great significance to the safety at signalized intersections. The conventional calculation method of IGT ignores the randomness of drivers' behaviors, which we believe is an important factor in calculation of IGT. Therefore, the purpose of this research is to investigate a new approach to calculate the IGT based on safety reliability theory. Firstly, a comprehensive literature review concerning the conventional calculation methods of IGT is conducted. Secondly, a theoretical calculation method of IGT based on safety reliability theory is put forward; different from the conventional methods, this model accounts for the uncertainty of driving behavior parameters. Thirdly, a Monte-Carlo simulation is employed to simulate the interactive process of perception-reaction time (PRT) and vehicular deceleration and solve the proposed model. Finally, according to the Monte-Carlo simulation results, the curve clusters describing the relationship between IGT, safety reliability (50%-90%), and intersection width (15-35m) are drawn. Results show that the IGT of a signalized intersection, obeying the normal distribution, is influenced by multiple factors and most sensitive to the PRT and vehicular deceleration. Our method thus successfully incorporates the probabilistic nature of driving behavior. Taking the safety reliability into consideration can provide a more reasonable method to calculate the IGT of signalized intersections.

1. Introduction

China still maintains a relatively poor traffic safety record. According to the latest statistics provided by the World Health Organization [1], in 2013, 261,400 people in China were estimated to be unfortunately killed by road traffic accidents, accounting for 21.64% of global road traffic fatalities. Furthermore, a study [2] showed that 16~22% of China's traffic accidents occurred in the urban-road intersections, and 90% of the above accidents happened within the signal change interval, especially during the change of signal lights from green to red. The above statistics illustrate that during a signal cycle, the signal phase change interval is the most dangerous. Therefore, calculating the intergreen time (IGT) of signalized intersections more reasonably is crucial to enhance the safety at signalized intersections.

IGT is designed to avoid traffic conflicts during the signal change interval and in general equals yellow change interval plus red clearance interval [3]. The purpose of setting yellow light in intersections is to alert drivers that the green light is going to turn red. Similarly the all-red time is designed to clear the intersection before giving the right of way to the cross traffic flow [3]. Obviously, to enhance the safety of signalized intersections, a relatively longer IGT is needed; however, this may lead to an increase in lost time and a decrease in efficiency.

However, accurately calculating an appropriate IGT for a signalized intersection is of considerable complexity, which is

Table 1: Variable notations in this research.

Variable notations	Description	Unit
D_s	Safe stopping distance(SSD)	m
D_c	Critical cross distance (CCD)	m
t_{pr}	Perception-reaction time (PRT)	s
v_{oy}	Vehicular approaching velocity at the onset of yellow light	m/s
a	Vehicular deceleration	m/s^2
I_g	IGT of signalized intersection	s
μ_{I_g}	Mean value of I_g	s
δ_{I_g}	Standard deviation of δ_{I_g}	s
w_i	Width of signalized intersection	m
l_v	Length of vehicle	m
I_g'	An setting IGT value in the signal controller for a given signalized intersection	s
$f(I_g)$	Distribution function of I_g	
ρ	Safety reliability of signalized intersection	%
$g(t_{pr})$	Distribution function of t_{pr}	
$\mu_{t_{pr}}$	Mean value of t_{pr}	s
$\delta_{t_{pr}}$	Standard deviation of t_{pr}	s
$h(a)$	Distribution function of a	
μ_a	Mean value of a	m/s^2
δ_a	Standard deviation of a	m/s^2

mainly caused by the randomness and complexity of drivers' driving behaviors. For example, when a vehicle approaches in one direction into a signalized intersection at the end of green light or at the onset of yellow light, the driver needs to make a "stop/go" decision [4, 5]. This decision is influenced by multiple factors including driver characteristics [6–8], signal change modes [9], traffic volume [8–10], etc. On the other hand, in the crossing direction the vehicles approaching the intersection at the onset of green light have various velocities which are affected by many factors, not simply the design speed.

The laws governing driver response to yellow light vary across countries. Drivers are taught to appropriately respond to the yellow light through laws which can be classified into three types. Type 1 is that the vehicles can enter the intersection at any point during yellow light. This type is called permissive yellow law. Type 2 is that vehicle cannot enter or be in the intersection on red light interval and finally in Type 3, vehicles should stop during the yellow light, but it is legal to proceed with caution through the intersection if it is not possible to do safely. The 2nd and 3rd types are termed as restrictive yellow light law by prior work [11]. *Law on Road Traffic Safety of the People's Republic of China* adopts the 1st type, which in other words mean that the yellow light in China is permissive.

In the conventional calculation method of IGT of signalized intersection, the relevant parameters, e.g., the PRT, vehicular approaching velocity, and vehicular deceleration, are usually considered as a constant value. These conventional methods ignore the randomness of drivers' driving behavior and therefore, a method considering the randomness will be introduced in this research. This paper aims at putting forward a safety-reliability-based approach to calculate the IGT. The remainder of this paper is structured as follows. Section 2 makes a comprehensive summary of the conventional calculation method of IGT of signalized intersections with a special focus on the concept of dilemma zone (DZ). An IGT calculation model based on safety reliability is put forward in Section 3, followed by a Monte-Carlo simulation in the Section 4 to solve the proposed model. In Section 5, the curve clusters describing the relationship between IGT, safety reliability, and intersection width are drawn. Finally, Section 6 concludes this study.

2. Conventional Calculation Method of IGT

To facilitate the presentation, key variable notations used hereafter in this research are summarized in Table 1.

Gazis et al. [3] in 1961 built a fundamental theory on the calculation method of IGT at signalized intersections. The core idea of that paper was to put forward the concept of dilemma zone (DZ). The DZ is an area at the approach of the signalized intersection where drivers travelling at the legal speed threshold can neither stop before the stop line nor proceed through the stop line safely. When a driver is on an approach to a signalized intersection and yellow light turns on, if he/she chooses to stop before the stop line, a minimum safety stopping distance (SSD) is needed. The SSD is obviously associated with vehicular velocity at the onset of yellow light, the driver's perception-reaction time (PRT), and vehicular deceleration. On the contrary, if the driver decides to proceed through the stop line at the onset of yellow light, he/she cannot be too far from the stop line. This critical cross distance (CCD) depends on the approaching velocity

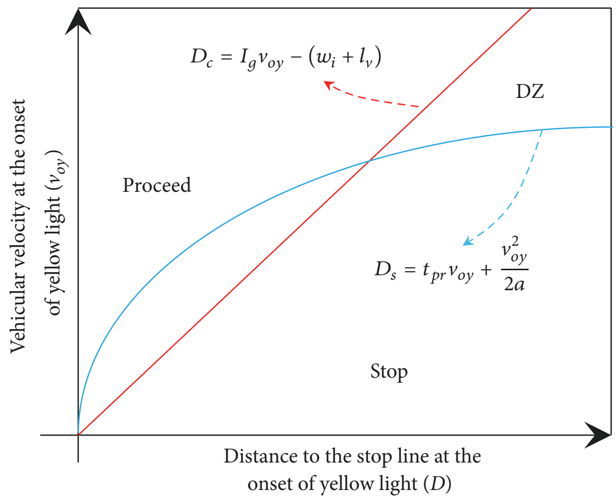

Figure 1: Relationship between vehicular velocity and distance to the stop line at the onset of yellow light and DZ.

at the onset of yellow light and the IGT. Based on the basic knowledge of dynamics, the SSD (D_s) and CCD (D_c) are given by the following equations, respectively:

$$D_s = t_{pr} v_{oy} + \frac{v_{oy}^2}{2a} \tag{1}$$

$$D_c = I_g v_{oy} - (w_i + l_v) \tag{2}$$

As shown in Figure 1, if $D_c < D_r$, a DZ will exist. It is possible to eliminate the DZ only if $D_c \geq D_r$. Therefore the shortest IGT, in which $D_c = D_r$, can be determined as follows:

$$I = I_{D_s = D_r} = t_{pr} + \frac{v_{oy}}{2a} + \frac{w_i + l_v}{v_{oy}} \tag{3}$$

The above calculation method of IGT is also known as G.H.M. method, which is widely used and is the fundamental theory on IGT calculation. This method has also been developed by researchers with the consideration of other factors, such as the grade of approach [13]. In addition, scholars have investigated the calibration of the driving behavior parameters involved in (3), e.g., the PRT [14] and vehicular deceleration.

It should be noted that, at present, there are two kinds of methods for IGT calculation: clearance-based method and conflict-avoid-based method [15], although with minor differences. The former is utilized in the signal timing manual in the US [13] and Japan [16]. The operational logic is that when the green light turns on at one phase, to ensure the safety, the vehicles entering the intersection at the last phase should have already completely driven out of the intersection. The conflict-avoid-based method has also been introduced by the signal timing manual in the UK [17] and Germany [18]. Its operational logic is that when the head car of one phase arrives at the conflict point, the vehicles entering the intersection at the previous phase should have already passed the conflict point. Obviously, compared with the conflict-avoid-based method, the clearance-based method is more conservative in terms of safety but less efficient due to longer IGT.

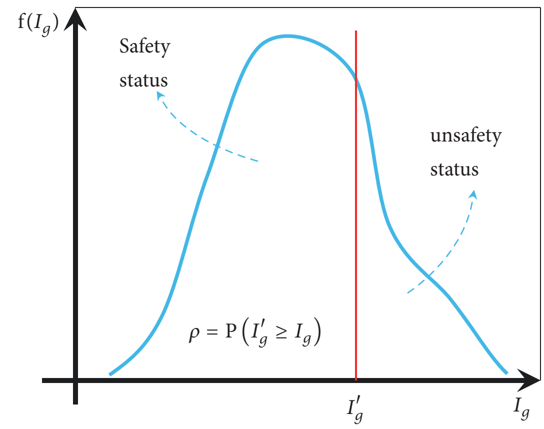

Figure 2: The safety and unsafety status of a signalized intersection.

Despite the above researches, the DZ will still exist because the two kinds of methods ignore the randomness of driver behavior parameters, e.g., the PRT, vehicular approaching velocity at the onset of yellow light, vehicular deceleration, and lengths of vehicles. Hence, it is not easy to calculate the IGT accurately based on the deterministic traffic flow theory. As a result, a probabilistic method reflecting the randomness of driver behavior parameters needs to be investigated.

3. Methodology

3.1. Safety-Reliability-Based Calculation Method of IGT. For a given signalized intersection, suppose we have set an IGT value I_g' in the signal controller, and let the probability density function (PDF) of I_g considering the randomness of driver behavior parameters be $f(I_g)$. From the viewpoint of IGT, if $I_g' \geq I_g$, the intersection is in a safety status; otherwise, the intersection is in an unsafety status as demonstrated in Figure 2. It should be noted that, in previous researches such as Amer et.al [19, 20], the objective function is that the drivers will not encounter a DZ, which also essentially equals $I_g' \geq I_g$.

According to the safety reliability theory, the safety reliability of a signalized intersection ρ can be calculated as follows:

$$\begin{aligned}\rho &= P\left(I_g' \geq I_g\right) = \int_0^{I_g'} f\left(I_g\right) dI_g \\ &= \iint \iiint \left(t_{pr} + \frac{v_{oy}}{2a} + \frac{w_i + l_v}{v_{oy}}\right) dt_{pr} dv_{oy} da dw_i dl_v\end{aligned} \tag{4}$$

In order to simplify (4), we assume that the vehicles pass through the intersection at the speed limit (40km/h), and the car length in China is taken as 6m. For a given signalized intersection, the width is assumed to be a constant value (e.g., $w_i = 20m$). Hence, (4) can further be simplified as (5), which

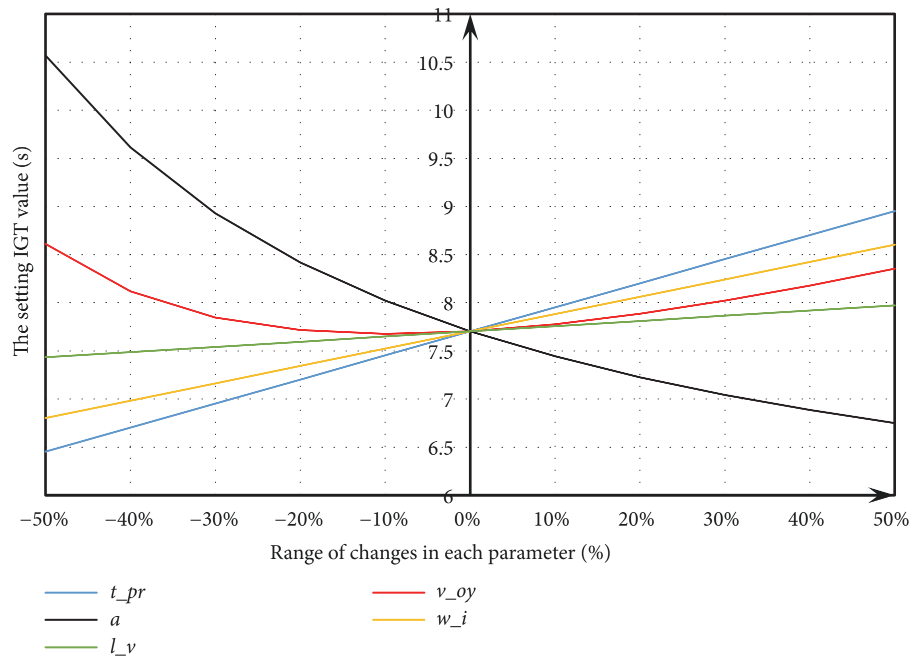

Figure 3: The sensitivity analysis diagram of vehicular IGT in intersections.

can be used to calculate the safety reliability of signalized intersections with different IGTs.

$$\rho = \mathrm{P}\left(I_g' \geq I_g\right) = \int_0^{I_g'} \mathrm{f}\left(I_g\right) dI_g = \iint \left(t_{pr} + \frac{v_{oy}}{2a} + \frac{w_i + l_v}{v_{oy}}\right) dt_{pr} da \quad (5)$$

3.2. Sensitivity Analysis. The purpose of sensitivity analysis is to look into the most significant factor influencing the IGT among the 5 variables in (3). Sensitivity analysis is commonly used in the major of transportation and economics [21, 22]. The first step of sensitivity analysis is to calibrate the parameters to set a benchmark. Based on the previous discussion, we have set the following benchmarks: $v_{oy} = 40km/h$, $w_i = 20m$, and $l_v = 6m$. In this section, emphasis will be placed on the calibration of PRT and vehicular deceleration.

It is not easy to directly observe the PRT and vehicular deceleration in engineering practice; however, a driving simulator experience was conducted earlier by the authors [12] to investigate the PRT and deceleration. The results in that experience show that drivers' PRTs follow a normal distribution with a mean and standard deviation of 2.50s and 1.30s, respectively. Vehicular decelerations are also distributed normally with mean and standard deviation $1.94\mathrm{m/s}^2$ and $0.76\mathrm{m/s}^2$. This observation result has a high degree of agreement with the conclusions in relevant researches. In NCHRP report 600 [23], it concludes that the PRT value in good and poor visibility condition is 1.6s and 5s, respectively. A particular PRT value of 2.5s is introduced by AASHTO [24] to calculate the sight distance requirements for a given design situation, and it emphasizes that the PRT value should not be viewed as a constant, because it is influenced by many factors. In El-Shawarby et.al [25], the PRT abstracted from 2016 data records show that the PRT at 72.4km/h ranges from 1.93s to 4.69s with a mean equal to 3.59s and a standard deviation of 0.45s; additionally, the PRT at 88.5km/h ranges between 2.31s and 5.33s with a mean value equal to 3.98s and a standard deviation of 0.53s.

The sensitivity analysis diagram of vehicular IGT at signalized intersections can be drawn as Figure 3.

As can be seen in Figure 3, the PRT and vehicular deceleration have the most significant effect on the IGT. Among them, IGT is positively correlated with PRT and when the PRT increases by 50% to 3.75s from the benchmark of 2.50s, the IGT of the intersection increased by 16.23% from 7.70s to 8.95s. However, IGT is negatively related to vehicular deceleration. When vehicular deceleration is increased by 50% to 2.91s from the benchmark of 1.94s, the IGT is reduced by 12.34% from 7.70s to 6.75s. Additionally, IGT has different degrees of positive correlation with the other 3 parameters.

4. Calculation Procedure

4.1. Monte-Carlo Simulation. Now we need to find out the probability distribution function of IGT, i.e., $\mathrm{f}(I_g)$, according to (3). Since both PRT t_{pr} and vehicular deceleration a follow the normal distribution, it is not easy to find the $\mathrm{f}(I_g)$ using analytical methods. Therefore, in this research, the Monte-Carlo simulation method is employed to investigate $\mathrm{f}(I_g)$. Some other researchers such as Easa [26] and Tang [27] also introduced the Monte-Carlo simulation method into the calculation of IGT.

Table 2: The input and output data of the Monte-Carlo simulation.

Procedure	Parameters	Mean	Std. Dev	Max	Min	Distribution	Reference
input	t_{pr}(s)	2.51	1.34	3.23	1.88	normal	Literature [12]
	a(m/s^2)	1.94	0.76	2.73	1.18	normal	Literature [12]
	v_{oy}(m/s)			Constant: 11.10			
	w_i(m)			Constant: 20.00			
	l_v(m)			Constant: 6.00			
output	I_g(s)	4.24	1.51	2.78	5.39	normal	

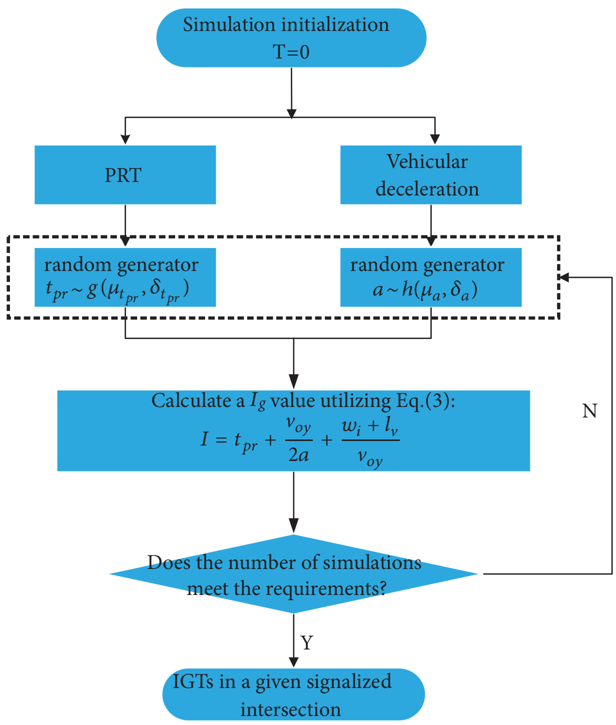

Figure 4: The key calculation procedure of the MATLAB program.

A MATLAB program was developed to calculate the probability distribution function of IGT f(I_g). The procedure of the Monte-Carlo simulation, shown in Figure 4, is as follows.

Step 1. Initialize the simulation counter T = 0.

Step 2. Set the probability distribution forms (normal distribution) and characteristic parameters ($\mu_{t_{pr}} = 2.50s$, $\delta_{t_{pr}} = 1.30s$, $\mu_a = 1.94\text{m}/m^2$, and $\delta_a = 0.76\text{m}/m^2$) of the IGT and vehicles' deceleration.

Step 3. Update the simulation counter in steps of 1, and generate a random value of t_{pr} and a.

Step 4. Calculate a value of IGT I_g using (3).

Step 5. IF T > 100000, go to **END**; else, go back to Step 1.

Figure 4 demonstrates the key calculation procedure of the developed MATLAB program.

4.2. Simulation Results. The input and output data of the Monte-Carlo simulation are shown in Table 2, and 100,000 values of IGT can be obtained by this Monte-Carlo simulation. The frequency distribution histogram and cumulative curve of the IGT are shown in Figure 5. The Jarque-Bera test [28] was performed on the simulated 100,000 values of IGT with JBSTAT=12.4774 and P=0.0048<0.05. This result shows that the IGT calculated according to (3) follows a normal distribution, i.e., $I_g \sim N(\mu_{I_g}, \delta_{I_g})$.

5. The IGT and Safety Reliability Curve Cluster

5.1. Calculating the Setting Value of IGT Based on Safety Reliability. As mentioned earlier, in the engineering practice of traffic design and signal timing, it is not easy to observe the PRT and vehicular deceleration. Generally, these two parameters are always calibrated beforehand and used as reference values in engineering manuals. Therefore, this section calculates the setting values of the IGT I_g' under different vehicular velocities v_{oy}, intersection widths w_i, and

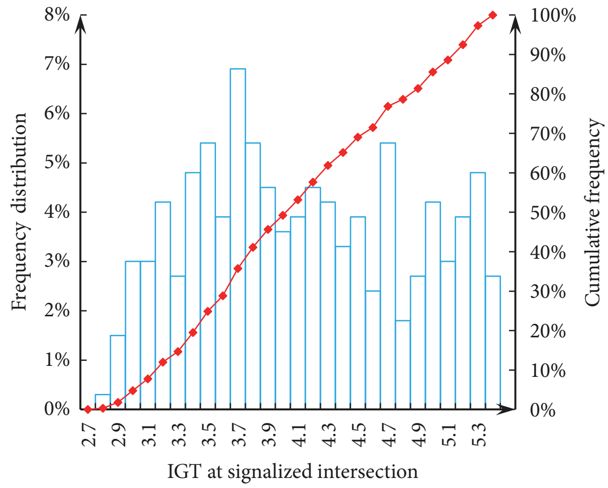

Figure 5: The frequency distribution histogram and cumulative curve IGT.

safety reliabilities ρ. Additionally, the curve clusters of them for engineering utilization will be drawn.

At this point, the problem has been translated into following: the safety reliability ρ is known, and the setting values of the IGT I_g' need to be calculated. By combining the following two equations

$$I_g \sim N\left(\mu_{I_g}, \delta_{I_g}\right)$$
$$\rho = \mathrm{P}\left(I_g' \geq I_g\right) = \int_0^{I_g'} \mathrm{f}\left(I_g\right) dI_g \tag{6}$$

it is easy to get the setting values of the IGT I_g' under different safety reliabilities ρ, as shown in

$$I_g' = \delta_{I_g}\Phi^{-1}\left(1-\rho\right) + \mu_{I_g} = \mu_{I_g}\left[1 + \frac{\delta_{I_g}}{\mu_{I_g}}\Phi^{-1}\left(1-\rho\right)\right] \tag{7}$$

Here $\Phi(\mathrm{x}) = \int_{-\infty}^{x}(1/\sqrt{2\pi})e^{-t^2/2}dt$ is the probability distribution function of the standard normal distribution; Φ^{-1} is the inverse function of $\Phi(\mathrm{x})$. The other parameters have been defined earlier.

5.2. The Engineering-Oriented Curve Clusters. In China, it is generally stipulated that the speed threshold at intersections is about half of the design speed for road sections, wherein the design speed of a typical road section in urban-road network is 30 km/h~80 km/h. Therefore, in engineering applications, v_{oy} can take the following values: 15 km/h, 20 km/h, 25 km/h, 30 km/h, 35 km/h, and 40 km/h. Additionally, typical values of w_i are 15 m, 20 m, 25 m, 30 m, and 35 m, and typical values of ρ can vary from 50%~90%, i.e., 90%, 80%, 70%, 60%, and 50%. The situation without the consideration of ρ is also illustrated as a benchmark. The setting values of the IGT I_g' calculated using the above typical values of v_{oy}, w_i, and ρ, and the engineering-oriented curve clusters describing the relationship between I_g', v_{oy}, w_i, and ρ are shown in Figure 6.

It can be seen from Figure 6 that, when the other parameters keep unchanged, the setting values of the IGT I_g' is positively correlated with the safety reliability ρ. For example, with the intersection widths w_i as 15 m and the vehicular velocity v_{oy} as 15km/h, when the safety reliability ρ is 95%, the setting values of the IGT I_g' is 7.34s; however, when ρ is 50%, the I_g' is only 5.46s.

It is worth noting that some of the setting values of the IGT I_g' in Figure 6 are relatively large, mainly for two reasons: Firstly, this research utilizes the clearance-based method rather than the conflict-avoid-based method to calculate the I_g', which is a kind of conservative method. Secondly, when calculating the I_g', the actual project conditions must be taken into consideration. For example, when the w_i is large, the v_{oy} is also relatively large in real situations; hence, we only need to consider the range of lower right corner of the curve clusters diagrams.

6. Conclusions

In this study, we proposed an engineering-oriented calculation method for IGT of signalized intersections with the consideration of safety reliability. Different from the conventional methods which ignore the randomness of driver behavior parameters, the approach proposed in this research accounts for the uncertainty of driving behavior parameters, i.e., the PRT and vehicular deceleration. In other words, this research has made an exploration of the probability method to calculate the IGT of signalized intersections. The key benefit behind this research is the introduction of the concept of "safety reliability" which is innovative to the IGT design and provides a probability measure for any selected IGT.

The IGT of signalized intersections is affected by multiple factors, among which the PRT and the vehicular deceleration are most significant. According to the Monte-Carlo simulation, the IGT of signalized intersections is normally distributed. Based on the above analysis, the setting values

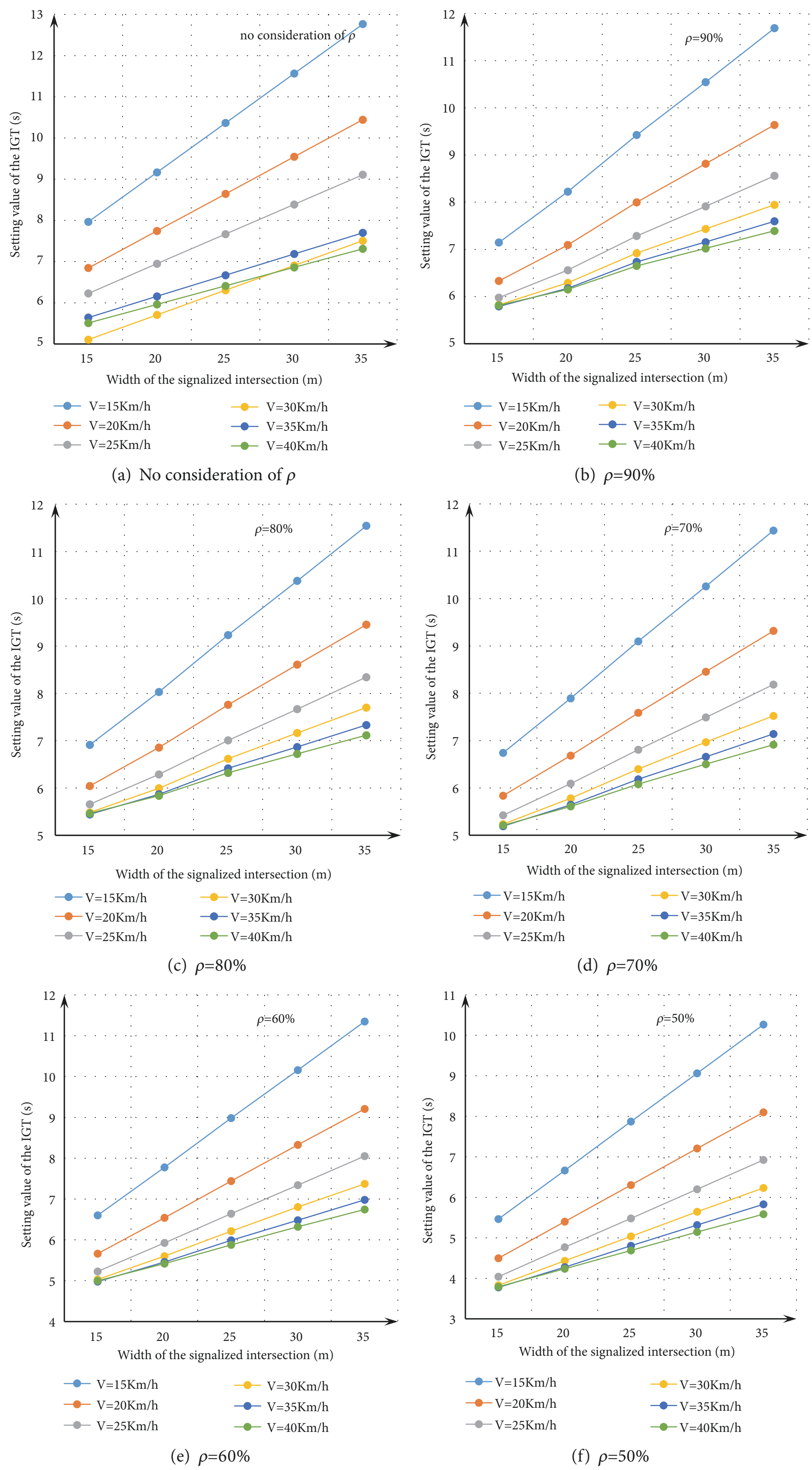

Figure 6: The curve family of IGT and safety reliability at signalized intersection.

of the IGT are calculated under different vehicular velocities, intersection widths, and safety reliabilities and finally the engineering-oriented curve clusters are drawn

In this study, in order to simplify the problem, the IGT calculation method is clearance-based rather than the conflict-avoid-based. The latter method would need further exploration in the subsequent research. In addition, how to appropriately choose values of safety ratability of signalized intersection, that is, the IGT based on the balance of efficiency and safety, is worthy of further study.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

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