Research Article

A Method for Multiple Attribute Decision Making Based on the Fusion of Multisource Information

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We propose a new method for the multiple attribute decision making problem. In this problem, the decision making information assembles multiple source data. Two main advantages of this proposed approach are that (i) it provides a data fusion technique, which can efficiently deal with the multisource decision making information; (ii) it can produce the degree of credibility of the entire decision making. The proposed method performs very well especially for the scenario that there exists conflict among the multiple source information. Finally, a traffic engineering example is given to illustrate the effect of our method.

1. Introduction

In the decision-making theory, many methods and their applications have been extensively studied. Recently, multiple attribute decision making (MADM) problems [1, 2], whose decision making information comes from multiple source data, receive more and more attention. Among these problems, the MADM problems which have the subjective and the objective information [1–5] at the same time, and the multiple attribute group decision making (MAGDM) [6–9] problems are the two hot topics in this research field.

The key to the two kinds of problems is to fuse various pieces of information [10]. For example, the following literature is to solve the first kind of problems. The literature [3] has proposed an optimization model to deal with the MADM problems with preference information on alternatives, which were given by decision maker in a fuzzy relation. With respect to the MADM problems with intuitionists fuzzy information, the literature [4] has proposed an optimization model based on the maximum deviation method. By this model, we can derive a simple and an exact formula for determining the completely unknown attribute weights. The literature [5] has proposed a linguistic weighted arithmetic averaging operator to solve the MADM problems, where there is linguistic preference information and the preference values take the form of linguistic variables and so forth.

In the respect of MAGDM problems, the literature [6] has researched the MAGDM problem with different formats of preference information on attributes; the literature [7] has researched the 2-tuple linguistic MAGDM problems with incomplete weight information and established an optimization model based on the maximizing deviation method; the literature [8] has presented a new approach to the MAGDM problems, where cooperation degree and reliability degree are proposed for aggregating the vague experts' opinions; the literature [9] has developed a compromise ratio methodology for fuzzy MAGDM problems and so forth.

Through these literatures, we could find that most of the solutions have used some subjective attitudes or information [10, 11], which were not provided by the problem itself. This is seriously out of line with the social needs. In order to overcome this defect, this paper presents two methods for the above two kinds of problems. The proposed methods are based on strong calculation and combined with the optimization theory [12] or the variation coefficient method [13].

The highlights of this new method could be summarized into two points. The first, it can efficiently deal with the

multisource decision making information; the second, it could provide the credibility degree of the final decisional results.

The rest parts of this paper will be organized as follows. In Section 2, we introduce the problems which the article would explore; in Section 3, we introduce the main tool of our research; in Section 4, we put forward two new decision methods; in Section 5, an application example is presented to illustrate the new method; in Section 6, we make some conclusions and present some further studies.

2. Two Problems

2.1. The MADM Problems under the Condition of Information Conflict. We will introduce this problem as follows. Let $X = \{x_1, x_2, \ldots, x_m\}$ be a discrete set of m feasible alternatives, let $F = \{f_1, f_2, \ldots, f_n\}$ be a finite set of attributes, and let $y_{ij} = f_j(x_i)$ $(i = 1, 2, \ldots, m; j = 1, 2, \ldots, n)$ be the values of the alternative x_i under the attribute f_j. In this paper, we only consider the situation that y_{ij} is given in real numbers. The decision matrix of attribute set F with regard to the set X is expressed by the matrix

$$Y = \begin{pmatrix} y_{11} & y_{12} & \cdots & y_{1n} \\ y_{21} & y_{22} & \cdots & y_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ y_{m1} & y_{m2} & \cdots & y_{mn} \end{pmatrix}. \tag{1}$$

For convenience, we suppose that the decision matrix Y has been normalized and denote $M = \{1, 2, \ldots, m\}$, $N = \{1, 2, \ldots, n\}$. For specific details of standardization, please see the literature [3, 14].

The experts have provided the subjective preference information for the alternative set X. We denote the information as $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_m)^T$, in which $\lambda_i \in [0, 1]$, $i \in M$.

Based on the above conditions, the problem is to select and rank the alternatives. In this paper, we mainly consider the situation where there are serious conflicts between the subjective information and the objective information [15].

2.2. The MAGDM Problems with Interval Vectors. In this subsection, we will introduce a kind of MAGDM problems with interval vectors. The basic concepts are the same as the above subsection, and we use the mathematical symbols, such as $X = \{x_1, x_2, \ldots, x_m\}$, $F = \{f_1, f_2, \ldots, f_n\}$, $y_{ij} = f_j(x_i)$, $W = (w_1, w_2, \ldots, w_n)^T$, and the matrix Y directly.

Here, the same as the above subsection, we suppose that the decision matrix Y has been normalized, and we only consider the situation that y_{ij} is given in real numbers.

Unlike the above subsection, here, the experts do not provide the subjective preference information for the alternative set X but provide the weight information directly. Consider $D = \{d_1, d_2, \ldots, d_t\}$ as the collection of experts, and denote the weight vectors which are provided by D as

$$W_1 = ([a_{11}, b_{11}], [a_{12}, b_{12}], \ldots, [a_{1n}, b_{1n}])^T,$$

$$W_2 = ([a_{21}, b_{21}], [a_{22}, b_{22}], \ldots, [a_{2n}, b_{2n}])^T,$$

$$\ldots\ldots\ldots\ldots\ldots$$

$$W_t = ([a_{t1}, b_{t1}], [a_{t2}, b_{t2}], \ldots, [a_{tn}, b_{tn}])^T. \tag{2}$$

Here, $0 \leq a_{kj} \leq b_{kj} \leq 1$, $k = 1, 2, \ldots, t$, $j = 1, 2, \ldots, n$.

The problem is to solve the MAGDM problem with the above conditions.

3. Main Tool of Our Research

The common character of the two problems is that they all involve the operation of interval numbers. In addition, we must point out that the situation we have no weight information equals to the situation where the weight is a variable located in the interval $[0, 1]$. In the following, we would give a new method for operating the interval numbers.

The new method originates from the basic of strong calculation by modern computer.

Without loss of generality, we take calculating the distance between $([a_{11}, b_{11}], [a_{12}, b_{12}], \ldots, [a_{1n}, b_{1n}])$ and $([a_{21}, b_{21}], [a_{22}, b_{22}], \ldots, [a_{2n}, b_{2n}])$ as example. The detailed procedure is illustrated as follows.

Step 1. Divide each $[a_{pq}, b_{pq}]$ ($p \in \{1, 2\}, q \in \{1, 2, \ldots, n\}$) into n^* parts. The value n^* depends on the demand of the decision makers. Then, we will get a set of segmentation points as

$$\widetilde{S} = \left\{ a_{pq}, a_{pq} + \frac{1}{n^*} (b_{pq} - a_{pq}), a_{pq} \right.$$
$$\left. + \frac{2}{n^*} (b_{pq} - a_{pq}), \ldots, b_{pq} \right\}. \tag{3}$$

We represent each interval $[a_{pq}, b_{pq}]$ ($p \in \{1, 2\}$, $q \in \{1, 2, \ldots, n\}$) by $\widetilde{S}$. Then, we represent the two vectors $([a_{11}, b_{11}], [a_{12}, b_{12}], \ldots, [a_{1n}, b_{1n}])$ and $([a_{21}, b_{21}], [a_{22}, b_{22}], \ldots, [a_{2n}, b_{2n}])$ by two sets of real-valued vectors. Denote the two sets as P and Q.

Step 2. Take any element p from P and take any element q from Q; according to the formula

$$\left| W_i - W_j \right|$$
$$= \sqrt{(w_{i1} - w_{j1})^2 + (w_{i2} - w_{j2})^2 + \cdots + (w_{in} - w_{jn})^2}, \tag{4}$$

we could calculate the distance. By doing so, we could get $(n^*)^{2n}$ distances. Then, we calculate the average value of these distances and denote the average value as d.

Step 3. Increase the value n^* gradually and repeat the above steps. When the value d holds steady to two digits after the decimal point, end the procedure and see the final result d^* as the distance between $([a_{11}, b_{11}], [a_{12}, b_{12}], \ldots, [a_{1n}, b_{1n}])$ and $([a_{21}, b_{21}], [a_{22}, b_{22}], \ldots, [a_{2n}, b_{2n}])$.

Obviously, the main advantage of this method is that the calculation procedure is in an objective, consistent way, and

there is no subjective information involved in the calculation procedure.

4. Decision Methods

At the beginning of this section, we would introduce a method called the simple additive weighting method [1]. Now, we consider a problem in hypothetical situation, where we have known the weight vector $W = (w_1, w_2, \ldots, w_n)^T$ and the attribute values. In this situation, we could get the comprehensive attribute value Z_i $(i = 1, 2, \ldots, m)$ by

$$Z_i(W) = \sum_{j=1}^{n} w_j y_{ij} \quad (i \in M, j \in N). \tag{5}$$

Obviously, the bigger $Z_i(W)$ leads to the more excellent x_i. Therefore, we could accomplish the process of getting the best alternative and ranking all of the alternatives by (5), and we could see that the determination of the weight vector $W = (w_1, w_2, \ldots, w_n)^T$ is the core of the MADM problem in general conditions.

4.1. Decision Method 1. In this paper, we only consider the situation where there is significant difference among these weight vectors; that is, the Kendall consistence [13] of the above weight vectors is imperfect.

The following, we present a new decision making method for solving the problems of subsection 2.1. The characteristic of our method is that it could provide the credibility of the decision maker for the subjective information as well as the entire decisional results. Specific decision steps are as follows.

Step 1. Solve the single objective programming model

$$\max \quad Z_i = \sum_{j=1}^{n} w_j y_{ij},$$
$$\text{s.t.} \quad w_j \geq 0, \quad j \in N, \quad \sum_{j=1}^{n} w_j = 1, \tag{6}$$

and denote the result of model (6) as $Z_i^{\max}$ $(i \in M)$, and $Z_i^{\max}$ is the ideal value of the comprehensive attribute value of x_i $(i \in M)$.

Solve the single objective programming model

$$\min \quad Z_i = \sum_{j=1}^{n} w_j y_{ij},$$
$$\text{s.t.} \quad w_j \geq 0, \quad j \in N, \quad \sum_{j=1}^{n} w_j = 1, \tag{7}$$

and denote the result of model (7) as $Z_i^{\min}$ $(i \in M)$, and $Z_i^{\min}$ is the negative ideal value of the comprehensive attribute value of x_i $(i \in M)$.

Step 2. Denote

$$\lambda_i^* = \frac{Z_i - Z_i^{\min}}{Z_i^{\max} - Z_i^{\min}}, \tag{8}$$

and establish one single objective optimal model

$$\min \quad |\lambda^* - \lambda|^2 = \sum_{i=1}^{m} |\lambda_i^* - \lambda_i|^2,$$
$$\text{s.t.} \quad w_j \geq 0, \quad i \in M, j \in N, \quad \sum_{j=1}^{n} w_j = 1. \tag{9}$$

Step 3. Solve the model (9), and we would get the weight vector $W^* = (w_1^*, w_2^*, \ldots, w_n^*)^T$. Up to this point, we could calculate the comprehensive attribute values of each x_i $(i \in M)$ by (5). Then, we could rank the alternatives and get the optimal alternative x^*.

Step 4. If the optimal solution x^* is consistent with the subjective decision information λ, we consider it as the final optimal solution of the entire decision making process, and consider

$$\eta_1 = 1 - \sqrt{\left(w_1^* - \frac{1}{n}\right)^2 + \left(w_2^* - \frac{1}{n}\right)^2 + \cdots + \left(w_n^* - \frac{1}{n}\right)^2}$$
$$\cdot \left(\sqrt{\frac{n-1}{n}}\right)^{-1} \tag{10}$$

as the degree of believing for the subjective information.

In (10), the value $\sqrt{(n-1)/n}$, which is obtained by optimization theory is the max Euclidean distance between $((1/n), (1/n), \ldots, (1/n))^T$ and any possible weight vector W^*. The value η_1 reflects the similarity scale of W^* and $((1/n), (1/n), \ldots, (1/n))^T$. From the aspect of set-valued statistics, the bigger the η_1 is, the more support would be got from the data of the objective information.

Because there is coordination between the subjective and objective information, and they all support the optimal alternative x^*, we set the credibility of the entire decision as 1.

Step 5. If the optimal solution x^* is inconsistent with the subjective decision information λ, we would believe that the subjective information has got no support from the objective information. Here, we correct the value η_1 and set η_1 as zero. Define

$$\tilde{\lambda}_p = \max \{\lambda_1, \lambda_2, \ldots, \lambda_m\}, \tag{11}$$

and define $\tilde{x}$ as the alternative corresponding to the index $\tilde{\lambda}_p$. Obviously, the alternative $\tilde{x}$ could represent the subjective information to some extent.

Step 6. We use the parameter η_2 to represent the credibility of the entire decisional results. In this step, we assume that the MADM problems have the alternative set of $\{x^*, \tilde{x}\}$ only. Because the weight information is unknown, we consider the weight vector as random element in weight space

$$V = [0, 1] \times [0, 1] \times \cdots \times [0, 1], \tag{12}$$

and the random element follows a uniform distribution.

Step 7. By using the main tool of our research, which has been introduced in Section 3, we compare the advantages of the alternative x^* with the alternative $\tilde{x}$ and calculate the credibility of them. By (5), every element of V would support one optimal alternative. Based on this, each alternative would be supported by a region of hypercube V, and the ranking of all alternatives could be solved by comparing the regions of hypercube V. The result of this regions comparison could be got by the technique of numerical simulation [16].

It's worth mentioning that if the sum of the weight vector is not one, by normalization, it is equivalent to a weight vector with the sum one.

4.2. Decision Method 2. Now, we present a new decision making method for solving the problem of subsection 2.2. In this paper, we only consider the situation where there is significant difference among these weight vectors $W_1, W_2, \ldots, W_t$; that is, the Kendall consistence [13] of the above weight vectors is imperfect.

For convenience, we denote the expert weight vector of set D as

$$\widetilde{W}^* = (\widetilde{w}_1^*, \widetilde{w}_2^*, \ldots, \widetilde{w}_t^*). \tag{13}$$

Obviously, the relative attribute weights of the set F could be got by

$$\begin{aligned}
\overline{W} = {} & (\widetilde{w}_1^*, \widetilde{w}_2^*, \ldots, \widetilde{w}_t^*) \\
& \times \begin{pmatrix}
[a_{11}, b_{11}] & [a_{12}, b_{12}] & \cdots & [a_{1n}, b_{1n}] \\
[a_{21}, b_{21}] & [a_{22}, b_{22}] & \cdots & [a_{2n}, b_{2n}] \\
\vdots & \vdots & \ddots & \vdots \\
[a_{t1}, b_{t1}] & [a_{t2}, b_{t2}] & \cdots & [a_{tn}, b_{tn}]
\end{pmatrix}.
\end{aligned} \tag{14}$$

The result of the formula [13] is one interval number column vector; we denote it as

$$\overline{W} = \left([\overline{a}_1, \overline{b}_1], [\overline{a}_2, \overline{b}_2], \ldots, [\overline{a}_n, \overline{b}_n] \right)^T. \tag{15}$$

In the following, we would present the new decision making method.

Step 1. Denote the weight vectors which are provided by experts d_i and d_j as

$$\begin{aligned}
W_i &= (w_{i1}, w_{i2}, \ldots, w_{in}), \\
W_j &= (w_{j1}, w_{j2}, \ldots, w_{jn}).
\end{aligned} \tag{16}$$

The distance between W_i and W_j would be got by the main tool of our research, which has been introduced in Section 3. The computation follows the formula [5].

Step 2. By formula [5], denote

$$\begin{aligned}
A_1 &= \sum_{k=1}^{t} |W_1 - W_k| \\
&= \sum_{k=1}^{t} \sqrt{(w_{k1} - w_{11})^2 + (w_{k2} - w_{12})^2 + \cdots + (w_{kn} - w_{1n})^2}, \\
A_2 &= \sum_{k=1}^{t} |W_2 - W_k| \\
&= \sum_{k=1}^{t} \sqrt{(w_{k1} - w_{21})^2 + (w_{k2} - w_{22})^2 + \cdots + (w_{kn} - w_{2n})^2}, \\
&\qquad \cdots\cdots\cdots\cdots \qquad \cdots\cdots\cdots\cdots \\
A_t &= \sum_{k=1}^{t} |W_t - W_k| \\
&= \sum_{k=1}^{t} \sqrt{(w_{k1} - w_{t1})^2 + (w_{k2} - w_{t2})^2 + \cdots + (w_{kn} - w_{tn})^2}.
\end{aligned} \tag{17}$$

Step 3. Take

$$W^* = \left(\frac{1}{A_1}, \frac{1}{A_2}, \ldots, \frac{1}{A_t} \right)^T, \tag{18}$$

then standardize W^*. We would get the weight vector for experts of set D. Denote $W^* = (w_1^*, w_2^*, \ldots, w_t^*)^T$.

Step 4. Divide each $[\overline{a}_j, \overline{b}_j]$ ($j \in \{1, 2, \ldots, n\}$) into $\overline{n}^*$ parts. The value $\overline{n}^*$ depends on the demand of decision makers. Then, we will get a set of segmentation points as

$$\overline{S}_j = \left\{ a_j, a_j + \frac{1}{\overline{n}^*}(b_j - a_j), a_j + \frac{2}{\overline{n}^*}(b_j - a_j), \ldots, b_j \right\}. \tag{19}$$

Step 5. Represent each interval $[\overline{a}_j, \overline{b}_j]$ ($j \in \{1, 2, \ldots, n\}$) by set $\overline{S}_j$, and represent the vectors $\overline{W}$ by one set of real-valued vectors, which would be denoted as $\check{W}^*$ here. It is easy to see that the element number of the set $\overline{W}$ is $(\overline{n}^*)^n$.

Step 6. Take any element W from $\check{W}^*$, take any i from M, and denote

$$Z_i = (y_{i1}, y_{i2}, \ldots, y_{in}) \cdot W. \tag{20}$$

According to comparing each Z_i ($i \in M$), any element W from $\check{W}^*$ will support one alternative. Thus, the set $\check{W}^*$ would be divided into i subsets. We denote the element number of these subsets as $n_1, n_2, \ldots, n_m$ and denote the support degree for each x_i ($i \in M$) as $\eta_i = (n_i / \overline{n}^*)$.

Step 7. Increase the value $\overline{n}^*$ gradually and repeat the above steps. When the value η_i ($i \in M$) holds steady to two digits

after the decimal point, the procedure is ended and the final result η_i $(i \in M)$ is seen as the support degree for each x_i $(i \in M)$.

Step 8. By comparing each η_i $(i \in M)$, we could sort and select the optimal alternatives.

5. An Application Example

In this section, we would present an example to illustrate our proposed methodology. Because the first method is relatively complex, and the two methods are similar, we only give an example to verify the first method.

This example is coming from the traffic engineering. In this example, $X = \{x_1, x_2, \ldots, x_5\}$ is the set of alternatives, $F = \{f_1, f_2, f_3, f_4\}$ is the set of attributes, and all attributes are of benefit types. Consider $W = (w_1, w_2, w_3, w_4)^T$ as the weight vector of all attributes. Here, we have no information about W. The standardized decision matrix is given as

$$\begin{bmatrix} 0.50 & 0.80 & 1.00 & 0.50 \\ 0.70 & 1.00 & 0.70 & 0.90 \\ 0.60 & 0.90 & 0.60 & 1.00 \\ 0.30 & 0.90 & 0.30 & 0.70 \\ 1.00 & 0.80 & 0.40 & 0.80 \end{bmatrix}. \tag{21}$$

The evaluation vector, which is given towards the set $\{x_1, x_2, \ldots, x_5\}$ and determined by the decision maker, is

$$\lambda = (0.45, 0.34, 0.27, 0.25, 0.25). \tag{22}$$

Now we seek to rank these alternatives and find the most desirable one.

Firstly, according to the method proposed in this paper, we build one optimal decision model as follows:

$$\begin{aligned} \min \quad & W^* = \left(\lambda_1^* - 0.45\right)^2 + \left(\lambda_2^* - 0.34\right)^2 + \left(\lambda_3^* - 0.27\right)^2 \\ & \quad + \left(\lambda_4^* - 0.25\right)^2 + \left(\lambda_5^* - 0.25\right)^2 \\ \text{s.t.} \quad & w_j \geq 0, \quad \sum_{j=1}^{4} w_j = 1, \end{aligned} \tag{23}$$

in which

$$\lambda_1^* = \frac{(0.50w_1 + 0.80w_2 + 1.00w_3 + 0.50w_4 - 0.50)}{(1.00 - 0.50)},$$

$$\lambda_2^* = \frac{(0.70w_1 + 1.00w_2 + 0.70w_3 + 0.90w_4 - 0.70)}{(1.00 - 0.70)},$$

$$\lambda_3^* = \frac{(0.60w_1 + 0.90w_2 + 0.60w_3 + 1.00w_4 - 0.60)}{(1.00 - 0.60)}, \tag{24}$$

$$\lambda_4^* = \frac{(0.30w_1 + 0.90w_2 + 0.30w_3 + 0.70w_4 - 0.30)}{(1.00 - 0.30)},$$

$$\lambda_5^* = \frac{(1.00w_1 + 0.80w_2 + 0.40w_3 + 0.80w_4 - 0.40)}{(1.00 - 0.40)}.$$

Next, by solving the model (23), we would get

$$W = (0.0955, 0.0319, 0.5276, 0.3450)^T. \tag{25}$$

Afterwards, by using the simple additive weighting method [1], the vector of comprehensive attribute values of each alternative could be obtained and it is

$$(0.7734, 0.7786, 0.7476, 0.4571, 0.6081)^T. \tag{26}$$

Obviously we can conclude that x_2 would be the optimal alternative and it is inconsistent with the subjective decision information λ. Thus, we set the degree of believing for the subjective information as 0.

Then, we use the method of hypercube segmentation [16] as a tool to compare the advantages of alternative x_1 and alternative x_2. Results show that x_2 comes to be the best alternative, and its credibility is 98.7654%.

In a word, there are conflicts between the subjective and objective decision-making information in this case. According to our method, x_2 is the best alternative, with the reliability 0 for the subjective information and the reliability 98.7654% for the entire decision.

6. Conclusions

Firstly, from the above example, it could be found that the uncertainty of the multisource decision making information has been studied and fused, and the process of the given method is objective, with no subjective factors. This is the highlight of our new method.

Secondly, though our two methods are all based on the new algorithms of interval numbers, they also have the diversity. The first method combines with the optimization theory, and the second method combines with the principle that the minority is subordinate to the majority.

Thirdly, from the example it can be seen that it is easy and convenient to use the two new methods, and the numerical example illustrates that our proposed method can deal with the multisource decision-making information well.

So, the proposed method may have a higher availability and a better application prospect.
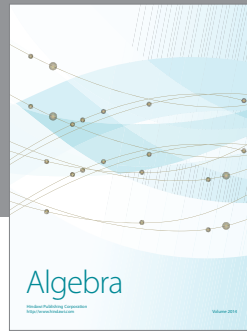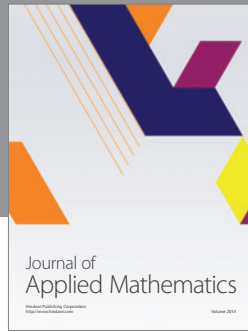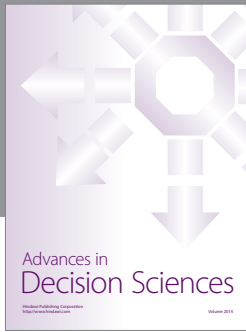
Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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