Restrictions of CPTu based interpretation methods and impact thereof on limit equilibrium modelling

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ABSTRACT

There is a large reliance on piezometric cone penetration testing (CPTu) in the tailings industry to estimate strength parameters for assessing the stability of tailings storage facilities (TSFs). It is common practice to assess the post-liquefied stability of a TSF using a residual undrained shear strength ratio (USSR). In such an assessment the residual USSR is typically applied to all saturated tailings. The current methods available to analyse CPTu data, are largely based on correlations and assumptions, and therefore contain limitations. Due to the limitations of the methods, this could lead to either over- or un-conservative estimates of stability.

In this study, three methods for determining liquefaction potential and residual undrained shear strength ratio (USSR) are compared, namely: Robertson (2022), Been and Jefferies (2016), and Olsen and Stark (2002). These methods differ in the way in which they apply cone resistance, sleeve friction, and dynamic pore pressure response to estimate residual USSR. Their limitations are highlighted and discussed.

A hybrid method is proposed for applying the results to post-liquefaction stability analyses. The hybrid interpretation approach uses a combination of the methods to account for different ranges of effective overburden stress and to identify weaker and stronger layers in the tailings profile based on state parameter, residual USSR, and pore pressure response. This hybrid method was applied to a stability assessment of a decommissioned platinum tailings storage facility in South Africa. The results indicated that the hybrid interpretation resulted in a more realistic phreatic surface location and a more accurate failure plane than conventional interpretations.

Keywords: Residual USSR; tailings; hybrid interpretation method.

1. Introduction

In 2023, a CPTu testing campaign was conducted on a decommissioned platinum tailings storage facility in South Africa. The purpose of the testing campaign was to gain new data to supplement the existing knowledge database as part of a larger study to assess the stability of the facility. This study was actioned as part of the Global Industry Standard on Tailings Management compliance work.

The desired outcomes of the study were to confirm and/or supplement the knowledge data base to a point where a post-liquefied slope stability assessment could be completed at a high level of confidence. This was achieved by assessing the existing data and CPTu data to determine the extent of the capillary zone; the location of the phreatic surface; the tailings material's liquefaction potential (especially in the saturated zones); the residual USSR and extent of the material zones. A requirement for the study was that the residual USSR values, when applied in the saturated zone (considered undrained), should provide a factor of safety equal to or exceeding 1.1. Should it not achieve this, stabilisation methods were to be investigated.

When investigating various methods for obtaining specific parameters it became clear that there was disagreement between methods for determining similar properties. In many cases these differences between the methods are small. However, in certain cases the differences are significant, and have been the cause for much debate. For the practicing geotechnical engineer this poses a problem: which is the correct method to apply? This paper serves to provide examples of such discrepancies by looking at specific methods for deriving liquefaction potential and residual undrained shear strength ratio (USSR) for a tailings profile.

This paper also demonstrates how the different CPTu interpretation methods can be used in combination to provide a calibrated outcomes without defaulting to selecting only lower bound parameters based on a single interpretation method.

2. Background

2.1. Typical tailings behaviour

The facility comprises hard rock-rock flour platinum tailings. Based on the Unified Soil Classification System (USCS) the tailings material classified either as SM (silty sands and sand-mixtures) or ML (inorganic silts, very fine sands, rock flour, silty or clayey fine sands). On average the tailings are classified as ML. All samples on which Atterberg limits could be determined, tested as being non-plastic (NP) to slightly plastic (SP) with low potential expansiveness. The specific gravity of the tailings ranged between 3.1 and 3.8 with an average of 3.5. The tailings have a critical state friction angle of 34 to 36 degrees determined from triaxial compression tests.

Hydraulic deposition was utilized at the facility, prior to decommissioning, using cyclones. Due to this method of deposition, layering was expected throughout the profile of the TSF.

2.2. Study Data

The data obtained from the CPTu tests consisted of cone resistance, sleeve friction and dynamic pore pressure response. Dissipation tests were conducted at 2 m intervals where there was evidence of saturation. The dissipations were used to determine the hydrostatic pressure throughout the tailings profile. Resistivity was measured throughout the profile to provide an indication of degree of saturation. Using the above-mentioned measurements, it was possible to derive a range of parameters for use in the post-liquefied stability assessment.

2.3. Methods Considered

To determine the liquefaction potential within the facility from the CPTu data, the Been and Jefferies (2016) screening method and the Robertson (2010) methods were used. The residual undrained shear strength ratio was determined using the methods defined by Robertson (2022), Been and Jefferies (2016) and Olsen and Stark (2002).

The authors appreciate that other methods of interpretation exist, however, for the purposes of demonstrating potential discrepancies between methods, using only these methods proved sufficient. The purpose of this paper is not to promote or condemn specific methods, but to highlight the importance of surrounding the problem and considering different approaches.

To understand the level of uncertainty in these methods, a brief description of how they work is provided in the following section.

3. Methodology

3.1. Liquefaction potential

Liquefaction potential is assessed from estimates of state parameter (ψ). Generally, a state parameter greater than -0.05 generally indicates a state wet of critical while a state parameter smaller than -0.05 indicates that it is dry of critical. If the material is dry of critical state, it is considered dilative and therefore not liquefiable. If a material is wet of critical it is considered contractive and liquefiable. For this study, state parameter was determined using the Been and Jefferies (2016) screening method and the Robertson (2010) method.

The Robertson (2010) method estimates state parameter through first determining clean sand equivalent cone resistant ($Q_{tn,cs}$), as indicated in Eq.1.

$$\psi = 0.56 - 0.33 log Q_{tn,cs} \quad (1)$$

$Q_{tn,cs}$ is obtained by multiplying the measured normalized cone resistance (Q_{tn}) with a correction factor Kc, used to account for changing behaviour with increasing fines content and compressibility (Eq. 2).

$$Q_{tn,cs} = Q_{tn}.Kc \quad (2)$$

The correction factor, Kc, is dependent on the soil's behaviour type index, Ic. Robertson and Wride (1998) initially provided an equation to determine Kc. Robertson (2022) suggested that the correction factor (Kc) be modified for Ic < 3.0, to account for partial drainage during penetration. This is given in Eq.3.

$$Kc = 1.8356Ic^5 - 23.673Ic^4 + 124.02Ic^3 - 320.616Ic^2 + 405.821Ic - 199.9 \quad (3)$$

Or simplified in Eq.4.

$$Kc \approx 15 - \frac{14}{1+\left(\frac{Ic}{2.95}\right)^{11}} \; for \; Ic \leq 3.0 \quad (4)$$

When Ic < 1.7, then Kc is equal to 1, meaning that there is no correction in clean sands.

Based on the formula, the following limitations are observed:

- Sleeve friction and dynamic pore pressure response are excluded from the formulation
- Kc is calculated from the soil behaviour type index, Ic. Ic is based on CPTu measurements. Thus, direct classification of soil type is not taken into consideration.
- There is a distinct cut-off at an Ic of 1.7 and an Ic of 3.0. If a soil possesses an Ic greater than 3.0 the material falls outside the limitations of the method. This is an issue for this study due to the non-plastic classification of the tailings. Thus, classification of the tailings according to Ic will create discrepancies if the material is classified as possessing clay behaviour.

The Been and Jefferies (2016) screening method considers cones resistance and dynamic pore pressure response in the determination of state parameter. Equations 5 to 8 refer.

$$Q_p.\left(1 - B_q\right) + 1 = k.\exp\left(-m.\psi\right) \quad (5)$$

$$\frac{k}{M} = 3 + \frac{0.85}{\lambda_{10}} \quad (6)$$

$$m = 11.9 - 13.3\,\lambda_{10} \quad (7)$$

Where:

Q_p	=	Corrected Cones Resistance
B_q	=	Pore pressure ratio
ψ	=	State parameter
M	=	Critical Friction Ratio

Using the Plewes (1992) method to determine λ_{10} includes sleeve friction in the equation (Eq. 8)

$$\lambda_{10} = \frac{F}{10}\ (Plewes, 1992) \tag{8}$$

Where:
F = Friction Ratio

3.2. Residual Undrained Shear Strength Ratio

An undrained shear strength ratio (USSR) refers to the ratio in which undrained shear strength increases with increasing effective overburden stress, i.e., s_u/σ'_{vo}. Residual USSR refers to the USSR at large displacements exceeding that at which peak USSR is obtained, also referred to as large displacement or post-liquefied USSR.

For this study, residual USSR was determined using the Robertson (2022), Been and Jefferies (2016) and Olsen and Stark (2002) methods.

3.2.1. Robertson (2022)

The Robertson (2022) method uses either cone resistance or sleeve friction to determine residual USSR, dependent on the Ic of the soil

When Ic is less than 3, which implies sand-like and transitional soil behaviour, residual USSR is calculated using clean sand equivalent corrected cone resistance, $Q_{tn,cs}$, as shown in Equation 9. $Q_{tn,cs}$ is calculated using the correction factor Kc as per Equation 2.

$$\frac{su(liq)}{\sigma' vo} = 0.0007\exp\left(0.084Q_{tn,cs}\right) + \frac{0.3}{Qtn,cs} \tag{9}$$

Robertson (2022) noted that this relationship was based primarily on case histories where the effective overburden stresses at failure was less than 3 times atmospheric pressure (300 kPa) and in most cases less than 2 atmospheres (200 kPa). Robertson (2017) stated that at higher effective overburden stresses, loose sand-like soils tend to behave in a more ductile manner, with $s_{u(liq)}/\sigma'_{vo}$ moving closer to a value of 0.22 to 0.25.

Thus, without sufficient data, an estimation of residual USSR, using the Robertson (2022) method, for Ic values less than 3 is limited to effective overburden stresses less than 300 kPa. For a decommissioned facility where the water level is consistently deeper than 15 m, it leaves little data to interpret.

When Ic is greater than 3, which implies clay-like behaviour, residual USSR is determined using a ratio of friction ratio (f_s) to effective overburden stress (Eq. 10).

$$\frac{su(r)}{\sigma' vo} = \frac{fs}{\sigma' vo} = \frac{FrQtn}{100} \tag{10}$$

A drawback of classifying non-plastic tailings material as having clay-like behaviour is that the residual USSR is then determined using friction ratio. Low plasticity materials tend to have lower friction ratios which will result in over-conservative residual USSR values.

Another limitation of basing the method on Ic is that a dividing line is created at Ic values of 3. The consequence of this is that the distribution of values is skewed towards either high values or low values, with a sparse distribution of results in-between. Due to the 300 kPa effective overburden stress limit for Ic less than 3, only values with an Ic greater than 3 are included at higher overburden stresses. This skews the results towards layers which are implied to behave in a clay-like manner. This again conflicts with the non-plastic nature of the tailings being assessed.

3.2.2. Been and Jefferies (2016)

The Been and Jefferies (2016) method is formulated around the state parameter and therefore considers cone resistance, sleeve friction and dynamic pore pressure response in its formulation. A VBA function is provided in Been and Jefferies (2016) which is used to determine residual USSR. Based on the ratio of state parameter to λ_{10}, residual USSR is calculated based on either case histories or critical state theory for very loose soils.

The specifics of the formulation are not provided here; however, it is important to note that the state parameter is a key component in the formulation. The way in which state parameter is incorporated into the formulation results in an exponential decrease in residual USSR with increasing state parameter. This means that at higher state parameter values the Residual USSR moves towards zero. It is important to be aware of this limitation as it may lead to over-conservative estimates of residual USSR.

In comparison with the Robertson (2022) method, this method provides a more uniform distribution of results between the maximum and minimum values. There is also no effective overburden stress limit to the method.

3.2.3. Olsen and Stark (2002)

The Olsen and Stark (2002) method was developed based on past case histories of liquefaction failures. The method considers corrected cone resistance in its formulation as presented below in Eq.11.

$$\frac{s_{u(liq)}}{\sigma'_{vo}} = 0.03 + 0.0143(qc1) \pm 0.03$$

$$for\ qc1 < 6.5\ MPa \tag{11}$$

The method is capped at a corrected cone resistance (qc1) of 6.5 MPa. A margin of error of 0.03 is also given, which is significant considering the typical range of residual USSR values expected. The Olsen and Stark (2002) method was considered to give a lower bound residual USSR value.

3.3. General Limitations

An important point to consider when selecting a method of interpretation is the variation in data obtained between different contractors.

Prior to the 2023 CPTu testing campaign two different CPTu contractors had completed testing campaigns at the facility. A key variation in the data from all three contractors was the dynamic pore pressure responses of the probe. Specifically, it was noted how each of the contractors had varying sensitivity to pore pressure response. In such a case, using a method such as Been and Jefferies (2016), which includes dynamic pore pressure response in its formulation, would result in different estimates of state parameter between all three contractors. A method using only cone resistance could prove to provide more consistent results between the contractors.

4. Limit equilibrium slope stability analyses

4.1. Limit equilibrium modelling

Post-liquefied limit equilibrium stability analyses were completed using Slide 2 from the Rocsience software suite. The GLE-Morgenstern Price method of slices was used to obtain the global FoS as this method equilibrates both the forces and moments.

The phreatic surface was determined from the available data from the facility and included in the model. A capillary zone was included in the model as a additional layer above the phreatic level.

4.2. Heterogeneous profile of tailings

Tailings material is generally not homogenous in nature due to the variable rock mass surrounding the ore body being processed. The plant typically blends the tailings material to achieve a certain particle size distribution to prevent excessively fine material from being deposited onto the TSF. This blending is only as successful as the care given at the plant and material mined at the time. Due to this fine balance, weaker layers could develop in the TSF profile, which could be classified as possessing clay-like behaviour. These weaker layers can be prominent and continuous throughout a section to the extent that it can create preferential failure planes.

4.3. Limit equilibrium material strength

The slope stability for the project was assessed using limit equilibrium techniques. A drawback of this type of analyses is that the USSR cannot change automatically from the peak to residual value in the program under a trigger (seismic, loading, unloading etc.). The user needs to select either the peak or residual USSR for the analyses. By assessing the residual USSR values for the material below the phreatic surface the user essentially assumes that a trigger has occurred, and the tailings material has already failed. It should be noted that a TSF may be robust enough to withstand triggers and may only be prone to failure under a combination triggers. A trigger response deformation assessment should be completed to assess this. This was not included as part of this paper.

4.4. Model material zoning

The geometric section analysed was zoned into tailings underflow, tailings overflow, starter wall, waste rock surface erosion protection and norite rock mass foundation. Figure 1 shows the location and extent of these zones.

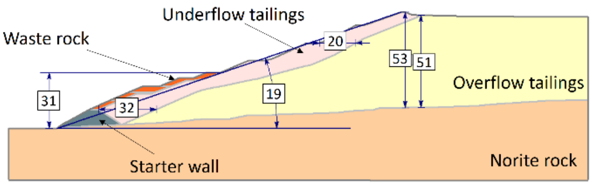

Figure 1. Geometric section material zoning

5. Conventional interpretation

5.1. RSCPTu parameters

In conventional interpretation, a statistical distribution is typically used to determine a representative residual USSR for all tailings material which is deemed saturated and liquefiable. This value is typically selected based on a percentage which accounts for a certain degree of inaccuracy or uncertainty coupled with the available data. Robertson (2022) recommends using the 30th percentile as a representative value.

Figure 2 shows the distribution determined for a representative testing line at the facility, using all three methods of determining residual USSR as discussed above.

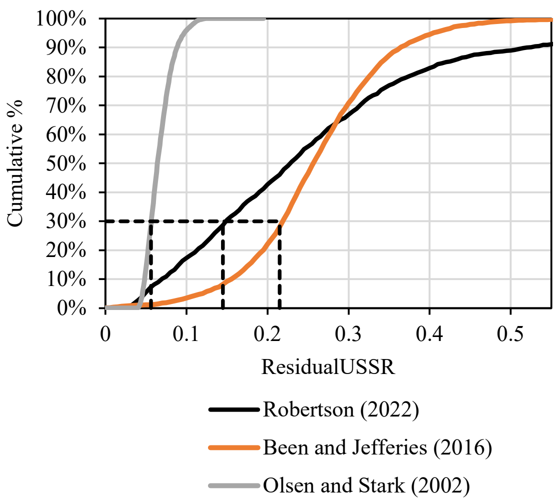

Figure 2. Statistical distribution of residual USSR

If considering only the 30th percentile value, it is clear there are large differences between methods. Robertson (2022) returned a value of 0.145. Olsen and Stark (2002) returned a value of 0.056 which is significantly lower than Robertson (2022). The Been and Jefferies method returned a value of 0.215, which is closer to what would be expected for a peak USSR.

Without knowing the details of the methods and how their formulae are applied, one could assume that Robertson (2022) provides a good middle value, with Olsen and Stark (2002) and Been and Jefferies (2016) resulting in over- and un-conservatives estimates.

For this specific study a large portion of data points had been omitted from the distribution due to the limits of the Robertson (2022) method, as most of the data points were obtained at effective overburden stresses greater than 300 kPa. Thus, it does not consider a representative range of data points.

In the case of Been and Jefferies (2016), the large amount of data points at higher effective over burden stresses skews the 30th percentile value towards higher residual USSR values.

The issue here does not lie in the choice of interpretation method, but rather in the way the methods are being applied to the model. By attempting to determine an average value for residual USSR, the results are unconsciously biased toward the majority of the data points.

As will be discussed in the following section, a better approach to applying the methods would be to discretize the profile into different ranges from which a more applicable specific method can be selected for interpretation.

6. Hybrid interpretation

To distinguish from conventional interpretation and to eliminate the limitations of using only a single method of interpretation, a hybrid method was used to apply strength parameters to the LE stability model. For the hybrid interpretation method, a combination of the three methods was used.

Robertson (2022) was used as the main method of interpretation where the effective overburden stresses were less than 300 kPa. Using this method at larger overburden stresses included only material considered clay-like based on a soil behaviour type index, Ic. Considering that the tailings exhibited sand-like behaviour and was determined to be non-plastic, interpretation of material classified as being clay-like was avoided.

At effective overburden stresses greater than 300 kPa, the Been and Jefferies (2016) method was used as there is no effective overburden stress limit applied to the method. Thus, providing a greater number of data points.

As mentioned in previous sections, there are instances where both the Robertson (2022) and the Been and Jefferies (2016) interpretations methods may result in conservatively low residual USSRs. In such instances the Olsen and Stark (2002) value was applied as a lower bound cut-off.

The tailings profile was then delineated into “weaker” layers and “stronger” layers. The weaker layers were identified as layers which were liquefiable, saturated and for which a low residual USSR was determined. Figure 3 demonstrates how the layers (contained in the yellow highlighted zones) were identified using typical CPTu data along with the estimated values for residual USSR and state parameter. The 30th percentile residual USSR

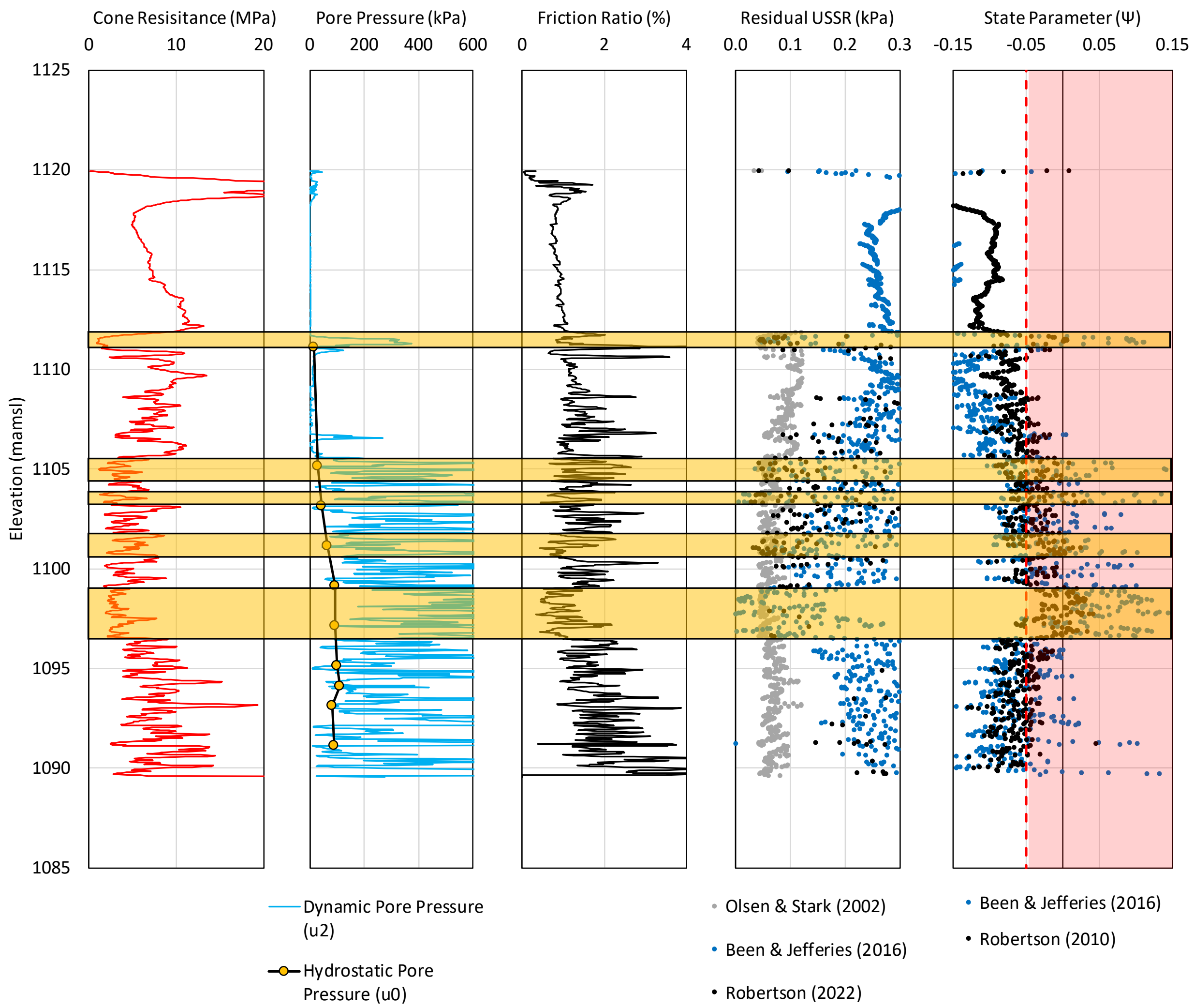

Figure 3. Typical state parameter profile plot

value was determined and applied for each of the weak layers.

For the surrounding materials, a similar approach was used as in the conventional interpretation where the 30th percentile value was determined from a statistical distribution. The only difference being that the weak layers were excluded from the distribution. An interesting finding from separating the "weaker" and the "stronger" layers from the distribution was that the results obtained from Robertson (2022) and Been and Jefferies (2016) seemed to converge. This highlights the unconscious bias the methods may have towards "weaker" or "stronger" zones.

Based on the distribution of state parameter over the testing profile liquefiable zones could be identified and applied in developing cross sections for stability analyses.

6.1. Delineation of weaker layers

The weaker potentially liquefiable layers were selected for inclusion in the stability models based on a set of criteria which included layer thickness, continuity throughout profile, pore pressure response and residual USSR value. To assess the continuity throughout the section the residual USSR plots for each probing location in the section (TL1 to TL7) were placed next to each other. Figure 3 shows the residual USSR plot for one of the probing locations.

If all the criteria were met the weaker layer was included in the LEM model. The residual USSR values for the stronger layers above and below the weaker layer was calculated and included in the model. The residual USSR values ranged between 0.05 to 0.11 for the weaker layers and 0.12 to 0.33 for the stronger layers. This clearly indicates how weak the weak layers can be and how much stronger the stronger layers are, especially when considering that these are the residual USSRs of the tailings material.

It should also be noted that the pore water response from the CPTu probing for the line indicated as u_0 exceeded 0 kPa in localized layers in the profile. These localized zones could be linked to the weaker layers that were identified. Thus, the assumed water level could be lowered, while still accounting for the saturated layers above this level. Figure 4 shows the difference in phreatic surface location should a conventional vs hybrid interpretation method be adopted. Figures 5 show what effect it has on the placement of the phreatic surface in TL1 for the stability model if you consider all $u_0 > 0$ kPa to be part of the capillary zone. Whereas Fig.6 includes the weaker layers and thus the phreatic surface could be included where u_0 consistently exceeds 0 kPa. This is critical for the stability of the slope. It should also be mentioned that the hybrid interpretation was better aligned to vibrating wire piezometer data at the same location than the conventional interpretation.

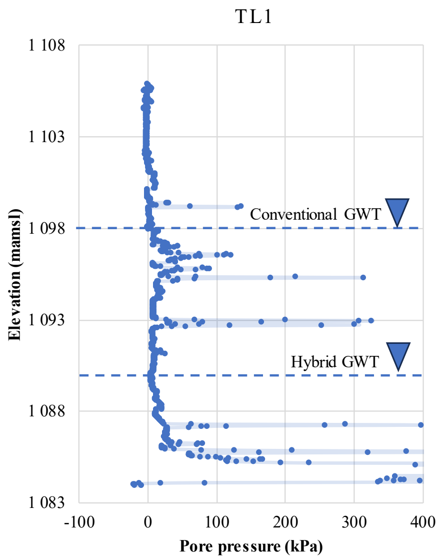

Figure 4. GWT location based on interpretation method

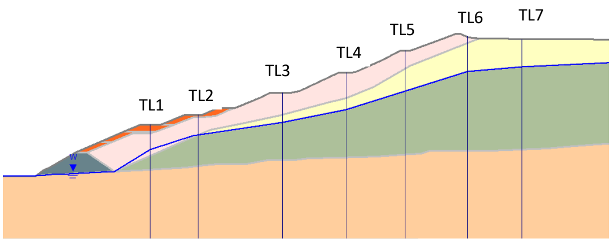

Figure 5. Limit equilibrium model with typical zoning (no delineation)

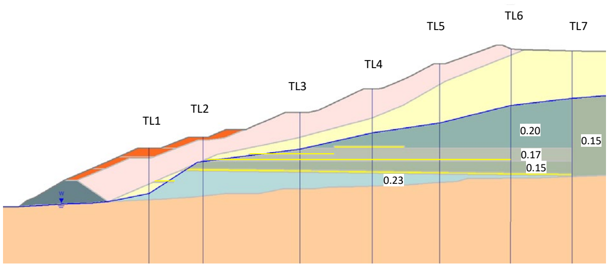

Figure 6. Limit equilibrium model with delineated layering

7. Impact of conventional and hybrid interpretation on slope stability assessment

7.1. Conventional stability analyses

The conventional stability analysis uses the average residual USSR determined per line applied to all overflow tailings below the ground water table (green zone shown in Fig.7). The stability assessment shows that the global failure plane runs through the interface of the foundation and tailings overflow material. The factor of safety in this case was equal to 0.71, which indicates that the slope will not be stable if liquefaction is triggered.

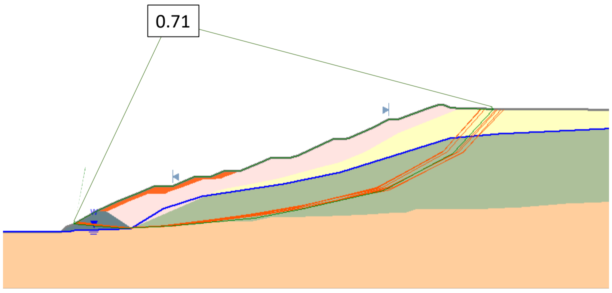

Figure 7. Slope stability analysis results (FoS = 0.71)

7.2. Hybrid stability analyses

The hybrid stability analyses consider the inclusion of the weaker and prominent stronger layers that are moderately to completely continuous throughout the assessed profile. Drained material strength parameters are applied to the stronger layers above the phreatic surface. The weaker layers' residual USSR is applied both above and below the phreatic surface in the overflow tailings. Figure 8 shows the analysis results. The failure plane is guided by a weaker layer instead of failing through the foundation as seen in the conventional approach's stability assessment. The factor of safety was equal to 0.84. This is marginally higher than for the conventional stability analysis but similarly indicates that the slope will not be stable in the event that the material is strained to its residual USSR value under some trigger. Figure 9 shows the comparison should the GWT be adjusted but no prominent weaker layers are included.

In the event that a deformation trigger assessment indicates that it is plausible that the liquefaction could be triggered, the slope would have to be stabilized. This is generally done by buttressing the slope. The design of the buttress will need to take into account the weaker layers as the failure plane's path is clearly dictated by it in the slope that was assessed. This will hence have a profound effect on the size of the buttress.

It should also be noted that it is only in this specific case that a higher factor of safety was obtained using the hybrid method. The use of the hybrid method could result in a lower post-liquefied factor of safety.

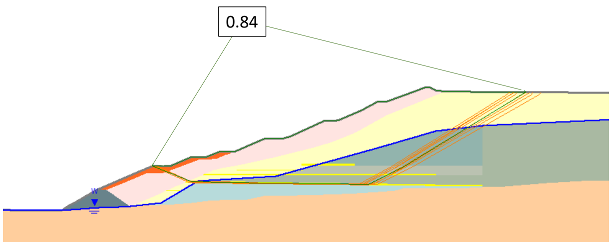

Figure 8. Slope stability analysis results for inclusion of delineated layers (FoS = 0.84)

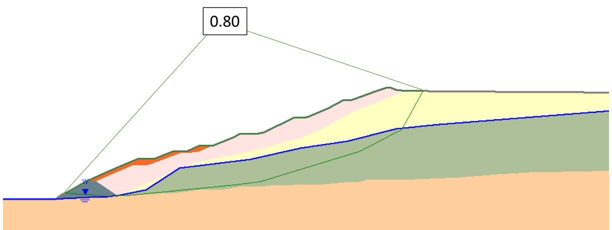

Figure 9. Slope stability analysis results without delineated layers and GWT adjustment (FoS = 0.80)

8. Conclusions and recommendations

The aim of the CPTu testing campaign, conducted on a decommissioned platinum tailings storage facility in South Africa, was to confirm and supplement the existing knowledge database on the tailings material properties, such as the capillary zone, the phreatic surface, the liquefaction potential, and the residual USSR.

A comparison was made between the different interpretation methods to derive the liquefaction potential and residual undrained shear strength ratio (USSR) from CPTu data.

The methodologies and key differences behind each of the methods were assessed as part of the interpretation. It was clear that each of the methods possessed limitations. These limitations could lead to over- or un-conservative estimates of the tailings material strength. These limitations highlight the drawbacks of using a single method or a statistical distribution to determine a representative value for the residual USSR for tailings material.

To mitigate the potential impact of these limitations when utilising a conventional interpretation, the authors propose a hybrid interpretation method which uses a combination of interpretation methods to account for the heterogeneity throughout the tailings profile.

The hybrid method can provide a more calibrated and realistic outcome than the conventional interpretation method, by using the appropriate method for different ranges of effective overburden stress, and by delineating the weaker layers based on state parameter, residual USSR and dynamic pore pressure response.

The post-liquefied stability analyses conducted demonstrated how discretising the tailings profile to include weaker layers could alter the location of the phreatic surface and failure plane which has a profound impact on the factor of safety determined.

It is recommended that a hybrid method, such as that discussed, be applied for post-liquefied limit equilibrium slope stability analyses to reduce uncertainty pertaining to the limitations of using a single method of interpretation for assessing the strength of a material. This should be done regardless of the method selected. It is further recommended that a better understanding of the method of interpretation be obtained to ensure that all limitations have been determined and mitigated.

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