COMMUNICATIONS IN NUMERICAL METHODS IN ENGINEERING
Commun. Numer. Meth. Engng 2002; **18**:251–258 (DOI: 10.1002/cnm.490)

A stabilized pseudo-shell approach for surface parametrization in CFD design problems

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SUMMARY

A surface representation for computational fluid dynamics (CFD) shape design problems is presented. The surface representation is based on the solution of a simplified pseudo-shell problem on the surface to be optimized. A stabilized finite element formulation is used to perform this step. The methodology has the advantage of being completely independent of the CAD representation. Moreover, the user does not have to predefine any set of shape functions to parameterize the surface. The scheme uses a reasonable discretization of the surface to automatically build the shape deformation modes, by using the pseudo-shell approach and the design parameter positions. Almost every point of the surface grid can be chosen as design parameter, which leads to a very rich design space. Most of the design variables are chosen in an automatic way, which makes the scheme easy to use. Furthermore, the surface grid is not distorted through the design cycles which avoids remeshing procedures. An example is presented to demonstrate the proposed methodology. Copyright © 2002 John Wiley & Sons, Ltd.

KEY WORDS: surface parametrization; design; shape optimization; CFD

1. INTRODUCTION

Most of the surface representation algorithms commonly used for shape design problems are based on CAD entities such as NURBS, Coon patches, Bezier and B-Splines curves and surfaces [1], or some kind of analytical curves or surfaces [2]. These are constructed before, or after, a surface grid is obtained to define the shape deformation modes. In general, such shape functions are defined by patches, and the deformation of the surface is controlled by some control points or knots. Another common approach is to use global shape functions defined over the complete surface, such as Hicks–Henne functions, standard NACA functions or mappings [3]. The deformation modes are then controlled by parameters associated with these functions.

In these approaches, the design space is restricted to the control or knot points that define the deformation modes (CAD approach), or to the function parameters defining the global

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Received 22 March 2001
Accepted 26 July 2001

shape functions. The key idea of the present work is a scheme in which almost every point on the surface can be chosen as design parameter. In this way, the design space will be only constrained by the surface grid density. In addition, the scheme generates surfaces free of singularities which can degrade or inhibit the convergence of the selected optimization methodology.

2. PSEUDO-SHELL SCHEME

The main goal of any surface parametrization scheme for shape design problems is to generate sufficiently smooth geometries during the whole optimization process. In general, a CAD or global shape function representation assures that not only the normal movements of the points on the surface are continuous, but also their first (rotations) and second (curvatures) derivatives with respect to the surface tangential co-ordinates (C^2 continuity).

Let Ω_s be the surface to parameterize and $\mathscr{B}(\mathbf{x}) := \{\mathbf{t}^1, \mathbf{t}^2, \mathbf{n}\}$ an orthonormal basis defined on each point $\mathbf{x} \in \Omega_s$, with $\mathbf{t}^1$ and $\mathbf{t}^2$ two unit vectors tangent to Ω_s and $\mathbf{n}$ the normal one. If some normal deflections are imposed to a set of points $\mathbf{x} \in \Omega_s$, a C^2 continuous displacement field may be computed by solving the following PDE problem (the usual summation convention is implied in the whole paper):

$$\theta_l - \frac{\partial u_n}{\partial \mathbf{t}^l} = 0 \quad \text{for } l = 1, 2, \quad \theta_n = 0, \quad \frac{\partial}{\partial \mathbf{t}^l}\left(\theta_l - \frac{\partial u_n}{\partial \mathbf{t}^l}\right) = 0 \quad \text{for } l = 1, 2 \tag{1}$$

In (1), $\boldsymbol{\theta}$ is the rotation field on Ω_s, $\theta_l = \boldsymbol{\theta} \cdot \mathbf{t}^l$ its l tangential component, $\theta_n = \boldsymbol{\theta} \cdot \mathbf{n}$ its normal one, $\mathbf{u}$ the displacement field on Ω_s and $u_n = \mathbf{u} \cdot \mathbf{n}$ its normal component. The boundary conditions of (1) are given by the imposed normal deflections and by geometrical restrictions.

Problem (1) assures the required continuity of the normal surface deflections. However, it cannot support tangential displacement fields, which are required in most design cases. To take into account such on-plane modes, problem (1) is enriched with the following elasticity problem on Ω_s:

$$\frac{\partial}{\partial \mathbf{t}^k}\left[\mu\left(\frac{\partial u_l}{\partial \mathbf{t}^k} + \frac{\partial u_k}{\partial \mathbf{t}^l}\right)\right] + \frac{\partial}{\partial \mathbf{t}^l}\left[\left(\frac{\mu v}{1 - 2v}\right)\frac{\partial u_k}{\partial \mathbf{t}^k}\right] = 0 \quad \text{for } l, k = 1, 2 \tag{2}$$

where $u_l = \mathbf{u} \cdot \mathbf{t}^l$ is the l tangential component of the displacement field, and μ and v are the material shear modulus and Poisson ratio, respectively. In this work the values of these properties were fixed to $\mu = 1$ and $v = 0.499$, to obtain a quasi-incompressible on-plane deformation. This reduces the grid distortion across the design cycles and, therefore, the need for remeshing.

Problems (1) and (2), together with compatible boundary conditions given by the imposed displacements and by the geometrical constraints, have to be solved in a coupled manner to obtain smooth surface movements.

3. STABILIZED FINITE ELEMENT SOLUTION

Equation (2) can be approximated using a standard finite element approximation without major obstacles. However, the standard Galerkin approximation of problem (1) may yield undesirable instabilities due to the divergence constraint (third equation in (1)). This pathology is very well known by authors working in the finite element solution of Reissner–Mindlin bending plates and incompressible flow problems [4]. From a mathematical point of view, the finite element interpolations used to approximate the rotations ($\boldsymbol{\theta}$) and the deflections (u_n) in (1) must satisfy a compatibility condition (or Babuška–Brezzi [5] condition), which translates in the use of mixed interpolations. Such shortcoming can be circumvented by using a stabilized formulation that allows the same interpolation for the two fields, and the use of low-order linear elements, two highly desirable numerical features from a computational point of view. The stability of problem (1) may be fulfilled by employing an algebraic subgrid scale (ASGS) type formulation [4] as follows: Find $(\theta_l, u_n) \in V_l \times W_n$ such that,

$$\int_{\Omega_s} \left(v_l - \frac{\partial w_n}{\partial \mathbf{t}^l} \right) \left(\theta_l - \frac{\partial u_n}{\partial \mathbf{t}^l} \right) \mathrm{d}\Omega + \int_{\Omega} \alpha \left(v_l - \frac{\partial w_n}{\partial \mathbf{t}^l} \right) \left(\theta_l - \frac{\partial u_n}{\partial \mathbf{t}^l} \right) \mathrm{d}\Omega = \int_{\Gamma_s} w_n \left(\theta_l - \frac{\partial u_n}{\partial \mathbf{t}^l} \right) t_l^n \, \mathrm{d}\Gamma \tag{3}$$

$\forall (v_l, w_n) \in V_l \times W_n$ and for $l = 1, 2$. Here V_l and W_n are the standard finite element spaces where the l rotation component and the normal deflection are interpolated, respectively, α the stabilizing parameter that depends on the element size h, and t_l^n the l component of the unit vector tangent to Ω_s and normal to its boundary Γ_s. The first left-hand side and the right-hand side terms are the standard Galerkin contributions obtained after integrating by parts the Laplacian terms, and the last left-hand side term is the ASGS stabilizing contribution. Note that the formulation is consistent in the sense that the solution of (1) is solution of (3). The right-hand side term is equal to zero, because at Γ_s or u_n is prescribed, which implies $w_n = 0$, or the rotation is equal to the deflection gradient $\theta_l = \partial u_n / \partial \mathbf{t}^l$ (natural boundary condition).

At this point, the only value to define in (3) is α. It is easy to see that (1) is totally analog to the problem of a thin plate in which the shear terms has been neglected [4]. The ASGS stability analysis dictates that for an infinitely thin plate α turns out to be:

$$\alpha = \frac{k}{h^2} - 1 \quad \text{with } k = \frac{c_1 \mu}{12(1 - \nu)} \tag{4}$$

and c_1 a positive constant with an optimal value of $c_1 = 4$. The values used for the material properties μ and ν are the same used for the on-plane deformation equation (2).

In summary, the final discrete form of the pseudo-shell problem may be written as: Find $(\boldsymbol{\theta}, \mathbf{u}) \in \mathbf{V} \times \mathbf{W}$ such that,

$$\int_{\Omega_s} v_n \theta_n \, \mathrm{d}\Omega + \int_{\Omega_s} \frac{k}{h^2} \left(v_l - \frac{\partial w_n}{\partial \mathbf{t}^l} \right) \left(\theta_l - \frac{\partial u_n}{\partial \mathbf{t}^l} \right) \mathrm{d}\Omega + \int_{\Omega_s} \mu \frac{\partial w_l}{\partial \mathbf{t}^k} \left(\frac{\partial u_l}{\partial \mathbf{t}^k} + \frac{\partial u_k}{\partial \mathbf{t}^l} \right) \mathrm{d}\Omega + \int_{\Omega_s} \frac{\mu \nu}{1 - 2\nu} \left(\frac{\partial w_l}{\partial \mathbf{t}^l} \right) \left(\frac{\partial u_k}{\partial \mathbf{t}^k} \right) \mathrm{d}\Omega = 0, \quad \forall (\mathbf{v}, \mathbf{w}) \in \mathbf{V} \times \mathbf{W} \quad \text{and for } l = 1, 2 \tag{5}$$

Again, $\mathbf{v} = (v_1, v_2, v_n)$ and $\mathbf{w} = (w_1, w_2, w_n)$ are the vectorial test functions for the rotation and displacement fields, respectively. The last two terms of (5) are the standard Galerkin discrete

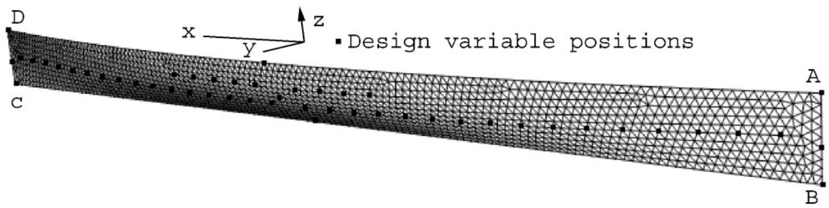

Figure 1. Surface mesh of the Wigley hull.

form associated to problem (2), after integrating by parts and assuming only Dirichlet and natural boundary conditions.

As a final remark, it can be observed that the second term of (5) has the form of a penalty contribution. The value of k/h^2 is several orders of magnitude larger than the other formulation constants for usual mesh sizes. Such term enforces in a weak form a smooth variation of the rotation field, and in this sense, the required C^2 continuity for the normal displacements.

3.1. Boundary conditions and design parameters selection

The boundary conditions for (5) in a shape optimization problem are dictated by the design parameter displacement and the geometrical restrictions. Even though for most cases such a set of constraints is enough to solve the pseudo-shell problem (5), numerical experience has shown that a better control over the whole surface movement is obtained if the boundary of Ω_s, i.e. Γ_s, is moved following a set of cubic B-spline curves. This movement is totally compatible with the allowable deformation given by the pseudo-shell approach. Given a finite element partition of Ω_s, the general procedure to solve (5) is:

Algorithm 1

(i) Select a set of design parameters.
(ii) Compute the boundary Γ_s.
(iii) Adjust Γ_s to a set of cubic B-spline curves.
(iv) Compute the displacements of the design parameters with the optimization algorithm.
(v) Move the design parameters on Γ_s, and all Γ_s following the B-spline curves.
(vi) Solve (5) using the displacements of Γ_s, the displacements of the interior design parameters (not in Γ_s) and the geometric restrictions, as boundary conditions.

All the steps of Algorithm 1 may be done in a quasi-automatic way. This is better explained in the example described below.

Figure 1 shows the surface mesh of a Wigley hull. Due to symmetry only half of it is analysed. The unique design parameters that have been chosen by the user, and therefore their deformation directions, are the four vertices (A–D) of the design area. The boundary Γ_s can be computed automatically from the discretization of the hull surface (mesh edges that belong to just one element). After obtaining Γ_s, the B-spline curves were defined by joining the segments of Γ_s between corners. One design parameter on the middle of each B-spline curve was added, and the deformation modes were automatically defined in such a way that they fulfill the following geometric restrictions: Points on the plane $z=0$ (hull's top) can be moved only on the plane, and points on the plane $y=0$ can also have only on-plane displacements. Finally, interior design points were automatically selected allowing minimum three edges of separation among them, and among them and any B-spline point. The deformation direction of such interior design parameters is normal to Ω_s. These points have been highlighted in Figure 1.

4. DESIGN ALGORITHM

This section describes how the proposed pseudo-shell surface representation may be used in an optimization environment. The design algorithm utilized for this purpose is shown below.

Algorithm 2

- Evaluate the objective function I in the original geometry Ω_s.
- FOR $k=1, n_d$
  * Perturb the co-ordinates of the k design variable in its deformation direction by a small ε. The rest of design parameters are not moved.
  * Move Γ_s using the B-spline curves if it is necessary.
  * Solve the pseudo-shell problem (5) using the boundary conditions described in step (vi) of Algorithm 1. Then, a perturbed geometry Ω'_s is obtained.
  * Evaluate the objective function I' in the perturbed geometry Ω'_s.
  * Obtain the gradient of the objective function respect to the k design variable by finite differences as $(I-I')/\varepsilon$.
- END FOR
- Make a line search in the negative gradient direction to find a minimum.
- Perform a new design cycle beginning with the final geometry of the line search procedure.

In Algorithm 2, the pseudo-shell problem has to be solved n_d times, where n_d is the number of design variables, to compute the gradients, and another several times during the line search procedure. This may performed in a very fast manner if the linear system of equations associated to (5) is solved by using a direct LU descomposition, and the LU matrices are computed and saved once at the beginning. The solution of the pseudo-shell problem is then performed by very fast backward–forward substitutions. However, the right-hand side of the system has to be rebuilt each time due to the change in the boundary condition values. Again, this can be performed in a very fast manner if the original matrix and right-hand side, without imposed boundary conditions, are stored as well. The amount of memory needed to save the LU matrices and the original matrix and RHS is very small (the problem is practically 2D) compared with the capacity of modern computers.

5. NUMERICAL EXAMPLE

In this section the wave drag minimization of the Wigley hull presented in Figure 1 is performed. The problem restrictions were: the volume enclosed by the hull and the plane

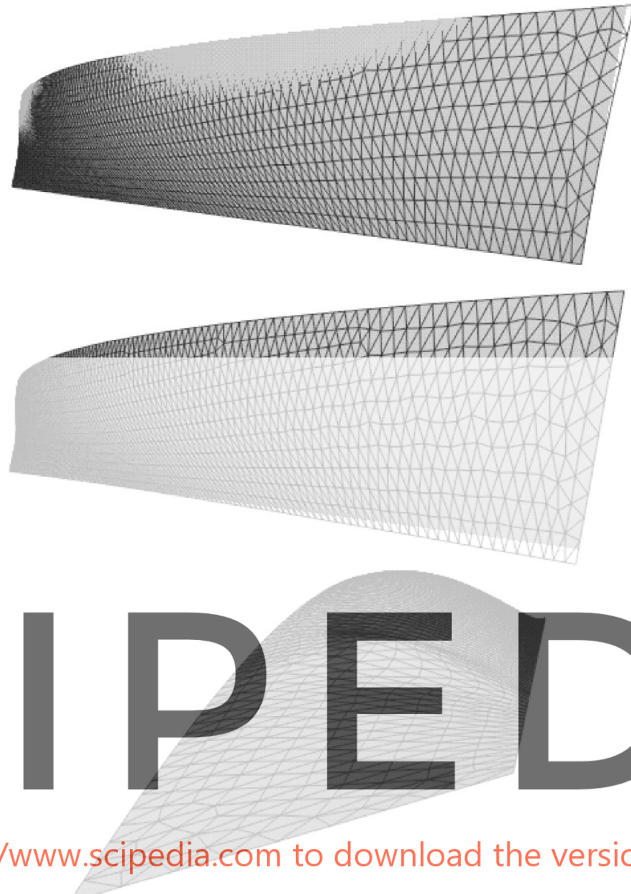

Figure 2. Examples of deformation modes: top corner, bottom corner and interior design variable.

$y=0$ must remain constant, and the initial waterplane ($z=0$) moment of inertia should be conserved too. The problem may be written as the minimization of the following objective function:

$$I = w_1 \frac{C_D}{C_D^*} + w_2 \frac{|V^* - V|}{V^*} + w_3 \frac{|J^* - J|}{J^*} \tag{6}$$

where C_D and C_D^* are the wave drag and its initial value, V and V^* the enclosed volume and its initial value, J and J^* the waterplane moment of inertia and its respective initial value, and w_1, w_2 and w_3 are relative weights. The zeroth-order slender-ship approximation [6] was used to evaluate the wave drag. The exceptional simplicity of this calculation method renders it ideally suited for routine hull form design and optimization.

The procedure described in the Section 3.1 generated 52 design variables (see Figure 1). In Figure 2 the deformation modes of some of them are shown. After eight design cycles, a 90 per cent drag reduction was obtained while the volume and moment of inertia constraints were

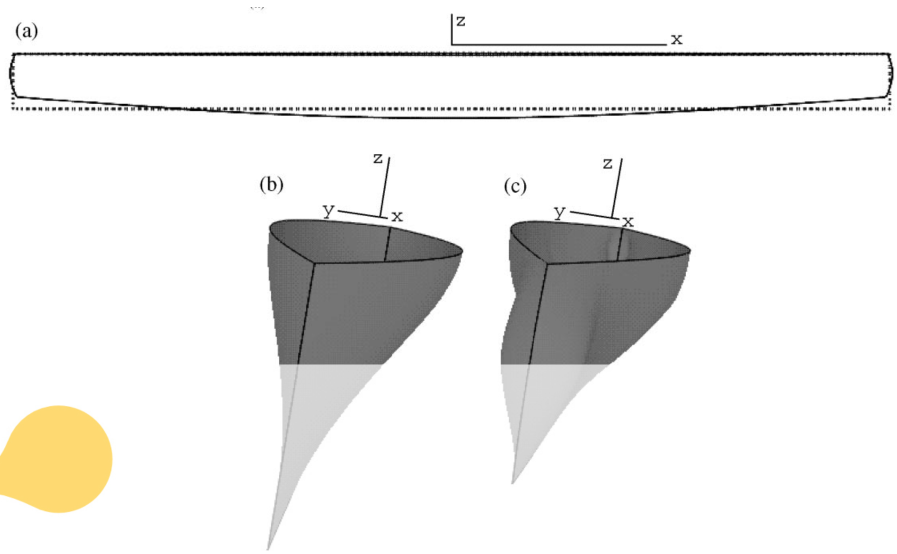

Figure 3. (a) Initial hull (dashed line) and final hull (continuous line); (b) initial hull; and (c) final hull.

fulfilled. The final hull is shown in Figure 3. The CPU time for the entire design process was of 385 seconds using a single R1000 Silicon Graphic processor. The initial LU descomposition for the pseudo-shell approach spent 1.13 s of CPU, while each subsequent backward–forward substitution took only 0.05 s.

6. CONCLUSIONS

A very fast pseudo-shell approach that produces smooth singularity-free shapes was presented to parameterize surfaces in CFD optimization problems. The user has to generate only the original surface mesh and a few design variables. The rest of the design parameters and their respective deformation modes can be generated automatically by the method. This makes the scheme easy to use. Given that almost every point on the surface is chosen as design variable, the design space is very rich, leading to fast convergence to a good optimal solution. Due to the way the scheme is constructed, mesh distortion is minimized, avoiding remeshing procedures. The proposed pseudo-shell representation is currently being used in more elaborated optimization algorithms based on an adjoint approach to compute the gradients and the Navier–Stokes equations for the flow solution [7].

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