

MULTI-BODY ROPE APPROACH FOR THE FORM-FINDING OF SHAPE OPTIMIZED GRID SHELL STRUCTURES

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Abstract. *Over the past decades, different approaches, physical and geometrical, were implemented to identify the optimal shape, reducing the internal stresses, of grid shells and vaults. As far as their original organic shape is concerned, the design of grid shell structures inspired architects and structural engineers worldwide and in any time. The method, here presented, is developed and extended, from its original formulation, employing a self-made code based on the dynamic equilibrium, ensured by the d'Alembert principle, of masses interconnected by rope elements in the space-time domain. The equilibrium corresponding the optimized shape to be defined, is obtained through an iterative process in the falling masses connected by a net for the definition of the "catenary surface" coinciding with the best shape of the shell (form minimizing the bending moment) according to the conditions of zero velocities and accelerations of the nodes. The implementation of the method is realized in MATLAB and set up for Python in an interpreted high-level general-purpose programming language. By the use of this code as well as its object-oriented architecture the MRA Python code will be linked to the Grasshopper environment for the direct visualization of the shapes and their fast-parametrization phase.*

1 INTRODUCTION

In the last times, new architectural requirements like internal distribution flexibility and the practise of free-form for large span roofing structures, encouraged the use of ground-breaking double curved shells and domes as a valid result for column-free buildings [1]. To this purpose the reticulated, lattice or grid shells is giving a valid option able to propose advanced

solution joining aesthetic purposes and structural necessities in a sole product. As mentioned, by many authors, a grid shell is essentially a structure with a single thin layer with a thickness very small in comparison to the main span of the roof. The grids are frequently optimized in order to reduce the bending moment inside the structural elements [1,2]. As in the case of very famous examples of the recent past this kind of structures are even today progressively used and characterized by double curvature geometrical domains and parts of the roof with very high shallowness ratio. The captivating constructions of the roof of the Yas Viceroy Hotel in Abu Dhabi built in 2009 and the Chadstone grid shell (Chadstone, Australia), recently finished, are just some of the most recent examples of these kind of structures (see Figs 1a and b). Certainly, these architectures and particularly their shapes were designed considering the aesthetic influence as one of the most important and underlying idea.

Different approaches, physical and mathematical, were used to discover the shape diminishing the internal stresses [1-5]. This kind of structures are characterized by high technology levels in the construction and by the necessity of stability analysis during the design phase. The optimized shape very often is altered by partial or global collapse due to buckling, snap-through and coupled instability. [6-10]. The form-finding usually accepted (optimization techniques, genetic algorithms etc.) leads to the description of a particular shape in which the stresses are minimized for a certain loading arrangement. The importance of finding a funicular shape for 3D shells lies in the fact that the evenly distributed gravity load underwrites largely to the load to be resisted. For over 40 years Heinz Isler used physical suspended models as the most suitable way to describe three dimensional systems [11]. Similarly, Frei Otto, during his research activity in Stuttgart in the '70s, developed accurate physical models for the form finding methodology definition (e.g. the models for the Multihalle of Mannheim construction). In the early 20th Century, Antoni Gaudí employed hanging models in the form-finding process for the chapel of the Colonia Guell [11] and the arches of the Casa Mila. Robert Hooke recognized in the eighteenth century that tension forms could be inverted to find the shape of structural forms acting in pure compression under the same loading conditions. In a unique sentence: "As hangs the flexible line, so but inverted will stand the rigid arch." [11]. However, today, most commercially available structural analysis software are suited for analysing grid shell structures. Very often, large displacements are not supported and the form finding based on the suspended shape results to be hardly appropriate [11,12].

Shape optimization of grid shells has been carried out using different techniques including among them linear software design [13] and gradient optimization [14]. At the same time, discrete truss topology method [15], graphed based design [16], simulated annealing [17], and cut-and-branch methods [18] have been used. Moreover, genetic algorithms have been recently employed for the optimization of three-dimensional discrete system, such as spatial structures planar structures and geodetic domes [19]. Multi-objectives optimization scheme have been developed by Winslow for free form grid shell constituted by elements with variable orientation [20]. At the same time, a coupled form-finding and grid optimization has been anticipated by Richardson et al. [2]. Form-finding approaches such as the force density method [21] and the dynamic relaxation (DR) [22] have been introduced to weightless configurations. Among these last kinds of systems Kilian and Ochsendorf [11] proposed a shape-finding tool for statically determinate systems based on particle-spring model. At the same time, Block and Ochsendorf proposed the thrust network analysis to establish the shape of pure compression systems in particular for masonry structures [23]. Recently, in addition to the overall grid shell

form also the selection of the grid type is considered as an important key point. As reported by Richardson et al. [1] the grid configuration generated by computer aided design software is transposed to static layout. Triangulated grids are the most basic and intuitive means of configuring the grid on a curved surface. However, this grid is not essentially the most efficient choice for a given form: triangulated grids tend to be higher-priced [2], since not all elements are essential for stability. Quadrangular grid configurations with planar faces are a good substitute of triangulated grids. Adriaenssens et al. [2] used a strain energy origami approach to enforce planar face constraints in the form-finding of an irregular configured grid shell to achieve ideal planarity of the faces. In this context, it is also necessary to discriminate the solutions in which the selected mesh is shaped by triangular units founded by elements of dissimilar length but with the same section and configurations in which the mesh is instead quadrangular and the elements that are forming the diagonal layer are absent (pure quadrangular mesh) as in the case of the Manneheim Multihalle (1975), or belong to another hierarchy, constituting the bracing effect, as in the case of the courtyard roof of the Museum of Hamburg History (1989) [1].

In the present paper, different shapes were obtained by the dynamic study of a hanging grid formed by free masses connected by flexible ropes with a certain slack coefficient (sc). In this case, any kind of loads can be assumed as the input for the step-by-step analysis and both 2D and 3D systems can be taken on. With this approach, named multi-bodies-rope approach (MRA), solving the mathematical model describing the model of the whole system, it is possible to achieve the equilibrium configuration of the net for the masses [24,25]. Originally, due to the large numbers of variable of the model the author adopted a numerical approach to solve the scheme by a multi-bodies numerical code using Runge-Kutta solution method. By this way, it is possible to define the configuration of the structure as the upturned model consistent to the last step (equilibrium step). In addition, in the case of roofs with a very large number of nodes, a calculation procedure is presented here. It is based, obviously, on the dynamic model proposed, providing, in the preliminary phase, a geometric model built using NURBS surfaces based on Bézier curves [26,27]. This combined approach allowed to obtain good results with a lower cost in term of time consuming if compared to simulations for complex form in which the form-finding is obtained for grid where the initial conditions very far from the optimal shape. The MRA approach is used to define the shape of three circular grid shell varying the sc . The implementation of the method is realized in Matlab and Python in an interpreted high-level general-purpose programming language. The adopted design philosophy emphasizes the code readability by other languages with respect to the traditional model realized in Visual Nastran 4D. By the use of this code as well as its object-oriented architecture the MRA Python code will be linked to the Grasshopper environment for the direct visualization of the shapes and their fast-parametrization phase.

2 MRA APPROACH FOR THE FORM-FINDING

As mentioned before different approaches have been examined in the last periods in order to range the target shapes for grid shells structures. Among these, very interesting outcomes have been obtained by particle-spring models, consisted of particles linked by rotational and translational springs elongating during the forming phases [11,12]. In these models the self-

weight of the nodes and the load of the rods are focused in the nodes (elements). In the present paper, the proposed model considers real ropes in order to simulate the part of the hanging net creating the suspended shape. The ropes are characterized by different *s.c.* permitting shapes more or less curvilinear for the final shapes [24,25]. The main difference between particle spring model (PSM), the dynamic relaxation model (DRM) and the MRA (multi-body rope approach) consisted into the system of forces acting on the nodes. In the first cases the forces due to the linear (k_u) or the non-linear translational spring stiffens (k_u^n) and the bending due to the rotational contribution ($k_r\theta$) are considered together with the external load to describe the resultant for each node, see Eq. (1) and (2). In the method, here offered, the connection (constraints) between two nodes is realized by a proper rope. From this point of view, the rope does not put on reactions at all when the distance between the endpoints (x), starting from initial positions corresponding to an initial distance (l_i), are less than the prefixed rope length (l_f). When the distance between the nodes is equal to the rope length, forces are applied at the endpoints equal in magnitude and opposite in direction, while no bending is applied excluding the limitation of any degrees of freedom, see Eq. (3) and (4).

$$F = ku, \quad F = ku + a ku^n, \quad (1)$$

$$M = k_r\theta, \quad (2)$$

$$F = 0, \quad l_i \leq x \leq l_f, \quad (3)$$

$$F = F_{\max} \quad x \geq l_f, \quad (4)$$

From this point of view the proposed method demonstrated to be consistent to the experimental models and to the form finding. The static configuration of the hanging net can be got by an iterative technique applied to the grid using the equations of the equilibrium of the nodes in a three-dimensional in the time domain [24,25]. Between one stage and the next, the node coordinates matched to a time step, their difference is characterized by the velocity of the falling masses (nodes) and their accelerations. In order to guarantee the convergence of the iterative process, using an actual step time, it is possible to simulate numerically the falling of the net by the dynamic equilibrium with inertial actions.

In Figure 1c a generic node "i" of a grid with quadrilateral mesh is reported. The node is a generic internal node adjacent to four other nodes. At this element four rope elements (a, b, c, d) are converging. The node is identified by the coordinates x_i, y_i, z_i , expressed respect to the Cartesian space. The elements connected the node i to the four adjacent nodes j, k, l, m . In the node i a generic load p_i represented the external load and the proper load (p_{ix}, p_{iy}, p_{iz}) due to the mass node (m_i). In the equation system of equilibrium, the inertial and dissipative actions are taken into account as proportional to the velocity and the acceleration of each nodes of the suspended model. The equilibrium of the node i , as referring in Figure 1, is the following:

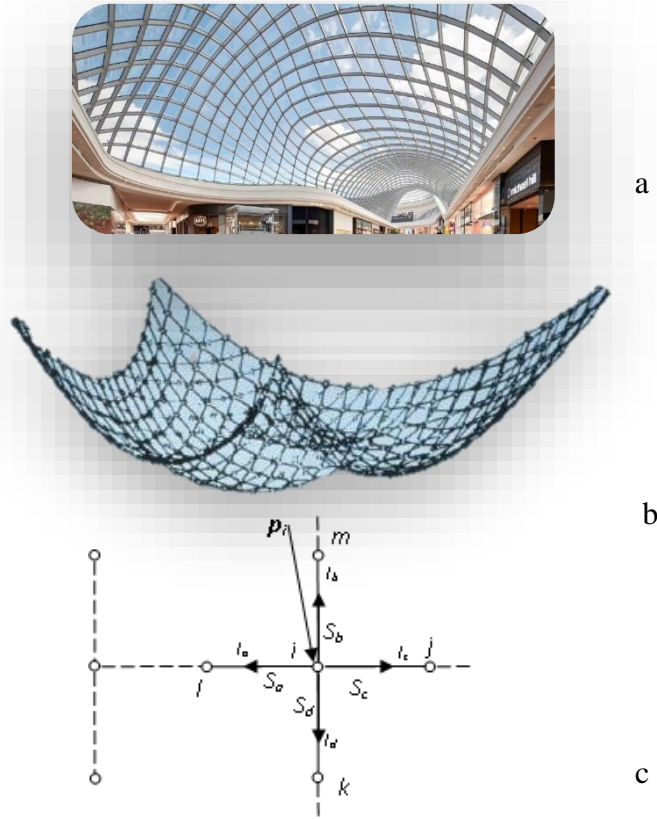


Figure 1: Example of free-form grid shell (Chadstone shopping centre Grid shell, Melbourne 2016) (a). Suspended model for the 3D definition of the catenary surface (b). Elementary portion of the grid: the node to which the rope elements converge (node i) connected four adjacent nodes (c) [25].

$$\sum_{i=1}^N \mathbf{F}_{e_i} + \mathbf{F}^I = \mathbf{0} = \mathbf{R}_i \quad (5)$$

where the summation is equal to \mathbf{R}_i representing the resultant in the node i (generic node in the net). \mathbf{F}^I is representing the effects of the inertial force (F^I) with a module equal to the product between the mass of the node and the amplitude of the acceleration vector with a direction equal to the opposite direction of the acceleration and the dissipative force (F'') assumed equal to the product of a constant times the velocity vector with a direction equal to the opposite of the velocity. The contribution of \mathbf{F}_{e_i} is constituted by S_a , S_b , S_c and S_d represented the resultants along the ropes a , b , c and d (see Fig. 1 c) and by the external loads. In this way, it is possible to take as the initial position of the grid nodes a configuration also very far from the final balance (final suspended shape). The convergence of the system, indeed, was guaranteed by the convergence of the iterative process as a physical process of the three-dimensional suspended grid. In this case the non-linear system of equations (6), (7) and (8) was a system of non-linear differential equations to be solved by numerical methods in the time domain. According to this system the solutions are found according to the dynamic balance equations by a step-by-step analysis [25]:

$$\frac{x_i - x_j}{l_a} S_a + \frac{x_i - x_k}{l_b} S_b + \frac{x_i - x_l}{l_c} S_c + \frac{x_i - x_m}{l_d} S_d + p_{ix} - m_i \ddot{x}_i - c_i \dot{x}_i = 0, \quad (6)$$

$$\frac{y_i - y_j}{l_a} S_a + \frac{y_i - y_k}{l_b} S_b + \frac{y_i - y_l}{l_c} S_c + \frac{y_i - y_m}{l_d} S_d + p_{iy} - m_i \ddot{y}_i - c_i \dot{y}_i = 0, \quad (7)$$

$$\frac{z_i - z_j}{l_a} S_a + \frac{z_i - z_k}{l_b} S_b + \frac{z_i - z_l}{l_c} S_c + \frac{z_i - z_m}{l_d} S_d + p_{iz} - m_i \ddot{z}_i - c_i \dot{z}_i = 0. \quad (8)$$

The contribution of velocity and acceleration for each node can be expressed as:

$$\begin{aligned} \dot{x}_i &= \frac{\partial x_i}{\partial t}; \dot{y}_i = \frac{\partial y_i}{\partial t}; \dot{z}_i = \frac{\partial z_i}{\partial t} \\ \ddot{x}_i &= \frac{\partial^2 x_i}{\partial t^2}; \ddot{y}_i = \frac{\partial^2 y_i}{\partial t^2}; \ddot{z}_i = \frac{\partial^2 z_i}{\partial t^2} \end{aligned}, \quad (9a,b)$$

and the length of le ropes (such as rope a) were obtained as the following respect to the Cartesian coordinates:

$$l_a = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2}. \quad (10)$$

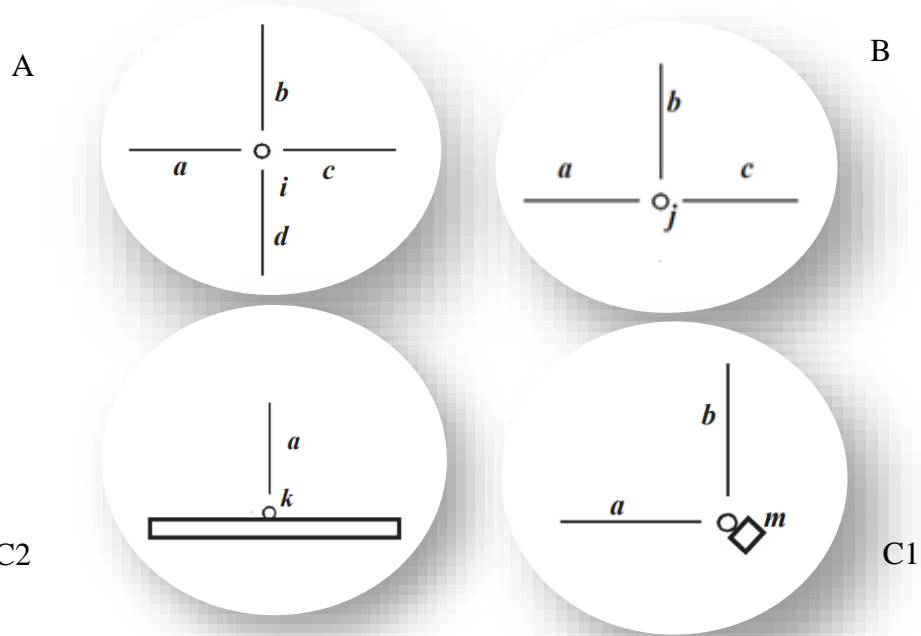


Figure 2: The node typologies are reported. Considering different types of nodes composing the net it is possible to consider the correspondent hypostatic mechanism.

The main conditions to create the model through the MRA approach are represented by an appropriate level of hypostatic condition of the suspended configuration at the initial step, and by the constraint typology defined for the rope elements. For the realization of the first condition it will be sufficient that the number of degrees of freedom (D.o.F) of the three-dimensional system is $>$ to the number of degrees of tridimensional constraint of the whole system (D.o.C). In particular it is possible to define the number of D.o.C as:

$$D.o.C = n_A(9) + n_B(6) + n_{C1}(6) + n_{C2}(3) + n_t(1) \quad (11)$$

where n_A , n_B , n_{C1} , n_{C2} are the numbers of nodes in the mesh characterized by constraint level as reported in Fig. 2 (n_t is the number of the node at all). At the same time, the number of degrees on freedom can be defined as:

$$D.o.F = n_t(6) \quad (12)$$

The final condition of true equilibrium is considered fulfilled when for each component of the system it will be possible to consider the velocity and acceleration tending to zero or otherwise $\leq e$ with e as a negligible value. The form-finding found leads to the definition of a particular shape in which the stresses are minimized for a certain loading configuration. In this way, the same structural scheme of shells and arches may be affected by instability problems, even up to the collapse, changing the boundary conditions (loading and constraints) or with the emerging of defects [28-41].

3 APPLICATION OF MRA BY THE LENGTH CONTROL

Within the framework of the code that was developed, a specific effort was made in order to explore the problems related to the creation of a shape through a form-finding process able to ensure the use of rods (ropes) characterized by the same length. The search for shapes that are optimized for force distribution (bending moment minimization) and that are, on the other hand, consisting a system, a layer (mesh), that allows to have the biggest number as possible of rods characterized by the same length. This condition is assumed to be a key concept in the design and construction of shells that are marked by extremely free and complex shapes. The use of free forms, in fact, for roofs and shells with increasingly large span is widely spread. Some examples may be the roof of the shopping center at Chadstone in Australia and the project for the roof of the shopping center Pompeii Maximall in Italy. Considering the process of finding the adopted form, however, the final configuration is the result of several parameters.

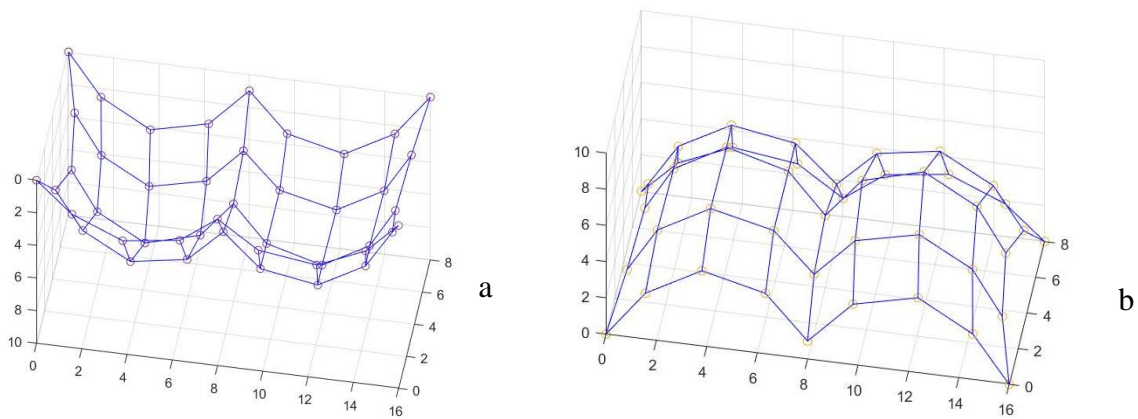


Figure 3: Mesh composed of 9×4 elements with 6 suspension points has been simulated. At the end of the steps according to the equilibrium condition all ropes were checked to ensure that they were stretched. Suspended model (a). Reversed shape: dome (b).

The initial slack coefficient, the shape of the edges (edge beams or suspension points), the number of nodes in the initial mesh and their initial distance, the last but not the least the constraint pattern assumed for the definition of the characteristics of the rope. As mentioned before, the developed code aims to solve problems related to the use of elements of equal length. In the figure 3 the case of a mesh composed of 9×4 elements with 6 suspension points has been simulated. Later it was possible to perform patterns with a much larger number of elements and characterized by many frames of the edges. Searching for a very complex configurations of the final shape (see Fig. 4).

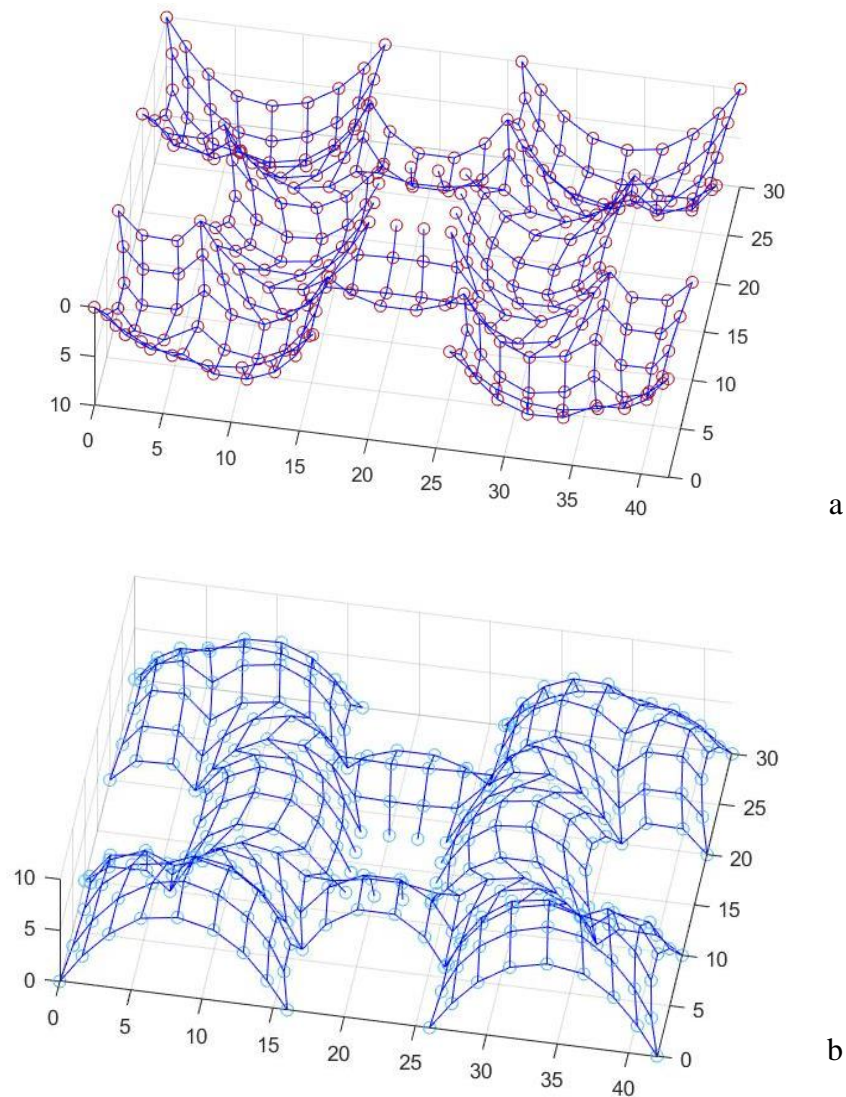


Figure 4: Patterns with a large number of elements and characterized by many frames of the edges. Suspended model (a). Reversed shape (b).

CONCLUSIONS

The code developed offer the solution for a structural form-finding of shells where the equilibrium corresponding to the optimized shape to be defined, is obtained through an iterative process of falling masses connected by a net in order to define the "catenary surface" coinciding with the best shape of the shell (form minimizing the bending moment). The implementation of the method is realized in MATLAB and predisposed to be implemented in Python in an interpreted high-level general-purpose programming language. The adopted design philosophy emphasizes the code readability by other languages with respect to the traditional model realized in Visual Nastran 4D. By the use of this code as well as its object-oriented architecture the MRA Python code will be linked to the Grasshopper environment for the direct visualization of the shapes and their fast-parametrization phase. Moreover, the code was developed in order to explore the problems related to the creation of a shape through a form-finding process able to ensure the use of rods (ropes) characterized by the same length. The search for shapes that are optimized for force distribution (bending moment minimization) and that are, at the same time, consisting of a system allowing the biggest number as possible of rods with the same length, seems to be a key concept in the design and shells characterized by large span, extremely free and complex shapes.

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