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ARTICLE

Efficient Mean Estimation Using Repeated Measurements in PPS Sampling with Scientific Applications

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ABSTRACT: Research studies in many scientific disciplines need efficient estimation methods for estimating the parameters of quantitative variables, including, but not limited to, tensile strength, wind speeds, air quality, and temperature, etc. Statisticians employ a sampling design to get a random sample and estimate the population means based on the observed sample. If the units of a finite population have different selection probabilities, unequal probability methods of sample selection are used. The existing unequal probability sampling methods allocate probabilities proportional to size (PPS) to the population units by using a one-time measurement approach. Memory–type estimation methods, on the other hand, use multiple measurements and provide more precise estimates than the traditional estimators. This research study introduces a novel memory-type estimator using a PPS sampling design. Various properties of the suggested mean estimator are analyzed, and the efficiency conditions are derived. A few real-world data sets have been used from previous studies to evaluate the performance of the suggested and competing mean estimators. Our comparative study suggests that the suggested memory-based estimator performs much better than the competing estimators, which means that the suggested memory-type estimator is an appropriate estimator for application in real-life sample surveys.

KEYWORDS: Efficient estimator; PPS sampling; real data; repeated measurements; sample survey

1 Introduction

Survey statisticians often face challenges in finding efficient estimates of the average of quantitative variables, including, but not limited to, distance, temperature, height, and weight, etc. For the estimation of the population mean, researchers select a subset of units from the population of interest and use the selected data to compute the numerical value of the estimator under investigation. This section provides a short history of the main developments in previous research works.

In sampling theory, Cochran [1] pioneered the development of the ratio method of estimation based on auxiliary information. Robson [2] evaluated survey situations in which the auxiliary variable has a negative relationship with the variable of interest. A research study by Das and Tripathi [3] analyzed variance estimation for studying the variations in the values of variables. Srivenkataramana [4] analyzed the ratio and product estimators of the population mean and evaluated their efficiency. The research study of Sisodia and Dwivedi [5] developed a new estimator utilizing auxiliary information and assessed its properties. Prasad [6] also developed optimal mean estimators using the existence of supplementary information. The research work of Bahl and Tuteja [7] developed an exponential estimator and showed improvement in performance over the competitor estimators.

Rao [8] assessed various types of mean estimators using auxiliary information in sample surveys. The research study of Upadhyaya and Singh [9] evaluated the mathematical transformation and its impact on the estimation of population parameters. Singh and Espejo [10] analyzed numerous properties of mean estimators under auxiliary data. Using a two-phase sampling approach, Choudhury and Singh [11] suggested improved variants of the ratio estimators for the population mean. Azeem and Hanif [12,13] assessed the effects of non-sampling error on the estimates of a finite population mean. Gupta and Yadav [14] utilized the information related to sample size in the development of novel optimal estimators of population parameters. Zahid and Shabbir [15] analyzed the impact of response error on mean estimation in sample surveys.

In the available literature, many research studies have developed and analyzed mean estimators based on different sampling schemes. The study of Muhammad et al. [16] introduced optimal ratio estimators using random sampling. Koç and Koç [17] also developed some precise forms of mean estimator based on stratified sampling from finite populations. Yousuf et al. [18] and Koçyiğit and Kadilar [19] evaluated the performance of ratio estimators using rank-based sample selection. Ajayi et al. [20] introduced mean estimators under a ranked set sampling design and analyzed their performance. Khan et al. [21,22] developed optimal estimators of the mean under different sampling methods. Using systematic sampling, Azeem et al. [23] evaluated the performance of the mean estimator in sample surveys where a linear tendency exists among population units. Khan et al. [24] introduced optimal mean estimators and evaluated their performance under PPS sampling. Alomani et al. [25] also presented enhanced estimators using the PPS sampling approach.

In recent years, survey statisticians have taken a keen interest in the development of memory-based estimation methods. Such memory-based approaches utilize the measurements of the subjects recorded at multiple points in time. These estimators allocate weightage to the units in such a way that the newest observed measurement receives the largest weight, whereas the oldest observed measurement gets the smallest weight. A weight-based average of measurements gives the numerical value of a memory-based estimator, commonly referred to as an Exponentially Weighted Moving Average (EWMA) estimator.

In survey sampling, Noor-ul-Amin [26,27] suggested the idea of memory-type methods of estimation by taking inspiration from control charts. Alomair and Iftikhar [28] and Qureshi et al. [29], by taking motivations from Haq [30], have also recently developed improved EWMA estimators for optimal estimation. Kumar and Kumar [31,32] analyzed neutrosophic imputation methods using ranked set sampling. Azeem et al. [33] introduced a technique that uses repeated measurements for mean estimation. Kumar and Verma [34,35] studied mean estimation for imprecise data sets using neutrosophic methods.

Some recent research studies have attempted to develop new techniques for dealing with data streams and sequential query answering. Various approaches have been used by researchers to deal with incomplete data in different fields. Li et al. [36] suggested a novel load profile inpainting technique for restoring missing load data segments. The suggested method can be used to estimate the baseline for a demand response event. The study of Li et al. [37] dealt with incomplete data by presenting a functional-coefficient quantile regression model for panel data for situations where the response variables are subject to censoring. Recently, a study on the life prediction of sliding bearings in nuclear power plant shielded pumps was conducted by Wang et al. [38]. Zheng et al. [39] developed a novel sketching solution that efficiently deals with subset queries over data streams, utilizing various statistical attributes.

In sampling theory, the existing EWMA estimators are based on the conventional equal probability—based sample selection methods. In many practical sample surveys, researchers face real-life cases in which the selection probabilities vary between units, hence making the PPS sampling preferable over traditional designs for sample selection. Using information on auxiliary variables under the PPS sampling design, a novel memory-tape mean estimator has been presented. For performance comparison, some real-life data sets have been considered, and the improvement over the competing estimators has been shown.

2 Available Estimators

Consider a population consisting of N subjects from which a sample of n subjects is chosen by utilizing PPS sampling scheme with replacement. Let us denote the variable of interest by Y with parameter $\overline{Y}$ and S_Y^2, and let X denote a supplementary variable with parameter $\overline{X}$ and S_X^2. Assuming some degree of relationships between variable *Y* and *X*, let us use the notation ($\bar{y}$, $\bar{x}$) to denote sample mean of variable *Y* and *X*. We transform *Y* and *X* to *U* and *V* using the transformation:

$$u_i = \frac{y_i}{Np_i},$$

$$v_i = \frac{x_i}{Np_i},$$

where p_i is the selection probability of the *i*th population unit. The probabilities p_i may be assigned to units based on an auxiliary variable, prior to selection of the sample. For example, let *Z* be an auxiliary variable, the probability of selection of unit '*i*' may be determined from:

$$p_i = \frac{z_i}{\sum_{i=1}^{N} z_i}.$$

The above equation shows that the population units with larger values of *Z* get higher probabilities of selection. Let us symbolize the population and sample means of variable *U* and *V* as $\left(\bar{U}, \bar{V}\right)$ and $\left(\bar{u}, \bar{v}\right)$. Further, we use the notations:

$$S_u^2 = \sum_{i=1}^{N} p_i \left(u_i - \bar{Y}\right)^2,$$

$$S_v^2 = \sum_{i=1}^{N} p_i \left(v_i - \bar{X}\right)^2,$$

$$\rho_{uv} = \frac{\sum_{i=1}^{N} p_i \left(u_i - \bar{Y}\right)\left(v_i - \bar{X}\right)}{S_u S_v}, \quad C_u = \frac{S_u}{\bar{Y}},$$

and

$$C_v = \frac{S_v}{\bar{X}}.$$

In conventional sampling methods, measurements of units are observed only at one time-point for both the main and auxiliary variables. Conversely, time-based sampling methods involve measurements observed at many points. Let ' λ ' represent a predefined smoothing constant and let us define a memory-type (or EWMA) estimator for the population mean of *Y* as under:

$$\hat{U}_t = \lambda \bar{u} + \left(1-\lambda\right) U_{t-1}, \tag{1}$$

where $0 \le \lambda \le 1$ and *t* denotes the specific time at which the measurement is recorded, with $t = 1, 2, 3, \ldots$. The use of smoothing constant is a motivation from control charts which has been used by some recent researchers in ratio estimation. For further details about the use of smoothing constants, readers may refer to the studies [27–29]. For $t = 1$, we can use the initial estimator value $\hat{U}_0$ which may be determined either from a pilot survey or from past knowledge. To explain the framework, let us take an example. Suppose $\lambda = 0.1$, $\bar{u} = 45$, and $\hat{U}_0 = 50$, then the observed value at first time-point (t = 1) will be $\hat{U}_1 = 0.1(45) + 0.9(50) = 49.5$. Now at time t = 2, the observed response at second time-point will be $\hat{U}_2 = \lambda\bar{u} + (1-\lambda)U_1 = 0.1(45) + 0.9(49.5) = 49.05$, and so on. This means as the observations at different time-points are recorded, the value of the estimator approaches the initially chosen value. This shows the importance of the initial value. It should ideally be chosen in such a manner that it gives an initial guess of the true value (population mean). The tendency to the initial value may be faster or slower, depending on the value of the smoothing constant λ. In Eq (1), $\bar{u}$ denotes the estimator of the population mean of the main variable. Likewise, in Eq (2), $\bar{v}$ denotes the estimator of the population mean of the auxiliary variable.

It is worth noting that under repeated measurements framework, the repeated measurements are taken on the same units at different time points which may be days, months or years.

An EWMA estimator for X may be formulated as follows:

$$\hat{V}_t = \lambda\bar{v} + (1-\lambda)V_{t-1}. \tag{2}$$

Using the notations discussed above, we now revisit some of the competitor EWMA estimators in PPS sampling. The simple estimator may be formulated in the form:

$$T_{0(t)} = \hat{U}_t = \lambda\bar{u} + (1-\lambda)U_{t-1}. \tag{3}$$

The simple estimator $T_{0(t)}$ has sampling variance as follows:

$$Var\left(T_{0(t)}\right) = \frac{\bar{Y}^2}{n}\left(\frac{\lambda}{2-\lambda}\right)C_u^2. \tag{4}$$

The exponential ratio estimator in PPS sampling design is written as:

$$T_{1(t)} = \hat{U}_t \exp\left(\frac{\hat{V}_t - \bar{X}}{\hat{V}_t + \bar{X}}\right). \tag{5}$$

The estimator $T_{1(t)}$ is biased with bias and mean squared error (MSE):

$$Bias\left(T_{1(t)}\right) \approx \frac{\bar{Y}}{n}\left(\frac{\lambda}{2-\lambda}\right)\left[C_v^2 + \frac{1}{2}\rho_{uv}C_uC_v\right], \tag{6}$$

and

$$MSE\left(T_{1(t)}\right) \approx \frac{\bar{Y}^2}{n}\left(\frac{\lambda}{2-\lambda}\right)\left[C_u^2 + \frac{C_v^2}{4} + \rho_{uv}C_uC_v\right]. \tag{7}$$

The Choudhury and Singh [40] estimator, using multiple measurements, may be described as follows:

$$T_{2(t)} = \hat{U}_t\left[\alpha\frac{\bar{X}}{\hat{V}_t} + (1-\alpha)\frac{\hat{V}_t}{\bar{X}}\right], \tag{8}$$

where α represents an optimizing constant. The estimator given in Eq. (8) has bias and MSE in the form:

$$Bias\left(T_{2(t)}\right) \approx \frac{\bar{Y}}{n}\left(\frac{\lambda}{2-\lambda}\right)\left[\alpha C_v^2 + (1-2\alpha)\rho_{uv}C_uC_v\right], \tag{9}$$

and

$$MSE\left(T_{2(t)}\right) \approx \frac{\bar{Y}^2}{n}\left(\frac{\lambda}{2-\lambda}\right)\left[C_u^2 + (1-2\alpha)^2 C_v^2 + 2(1-2\alpha)\rho_{uv}C_uC_v\right]. \tag{10}$$

The Kanwai et al. [41] estimator is expressed as:

$$T_{3(t)} = \frac{1}{2}\hat{U}_t\left(\frac{\bar{X}}{\hat{V}_t^*} + \frac{\hat{V}_t^*}{\bar{X}}\right)\exp\left(\frac{\bar{X}-V_t}{\bar{X}+V_t}\right). \tag{11}$$

The estimator given in Eq. (11) has bias and MSE in the form:

$$Bias\left(T_{3(t)}\right) \approx \left(\frac{\lambda}{2-\lambda}\right)\frac{\bar{Y}}{n}\left[\frac{7}{8}C_v^2 - \frac{1}{2}\rho_{uv}C_uC_v\right], \tag{12}$$

and

$$MSE\left(T_{3(t)}\right) \approx \left(\frac{\lambda}{2-\lambda}\right)\frac{\bar{Y}^2}{n}\left[C_u^2 + \frac{C_v^2}{4} - \rho_{uv}C_uC_v\right]. \tag{13}$$

3 Proposed Estimator

Taking motivations from Kanwai et al. [41], we suggest the following EWMA estimator:

$$T_{p(t)} = \frac{1}{2}\hat{U}_t\left(\frac{\hat{V}_t^*}{\bar{X}}\right)^{\alpha}\left(\frac{\bar{X}}{\hat{V}_t^*} + \frac{V_t}{\bar{X}}\right)\exp\left(\frac{\bar{X}-\hat{V}_t^*}{\bar{X}+V_t}\right), \tag{14}$$

where α is a predetermined constant $(0<\alpha<1)$, and

$$\hat{V}_t^* = \frac{N\bar{X} - n\hat{V}_t}{N-n}. \tag{15}$$

Eq. (15) is basically a transformation of the auxiliary variable which uses the sample and population mean of the auxiliary variable. This transformation represents the mean of the non-sampled observations.

Theorem 1: *The proposed estimator is biased with the bias expression given as:*

$$Bias\left(T_{p(t)}\right) \approx \frac{\bar{Y}}{n}\left(\frac{\lambda}{2-\lambda}\right)\left[GC_v^2 - B\rho_{uv}C_uC_v\right], \tag{16}$$

where

$$B = A\alpha + \frac{1}{2}, \tag{17}$$

and

$$G = \frac{1}{2}\left\{\alpha\left(\alpha - 1\right)A^2 + A\alpha + \frac{7}{4}\right\}, \tag{18}$$

where

$$A = \frac{n}{N-n}. \tag{19}$$

Proof: We introduce the notations:

$$e_0 = \frac{\hat{U}_t - \bar{Y}}{\bar{Y}}, \tag{20}$$

and

$$e_1 = \frac{\hat{V}_t - \bar{X}}{\bar{X}}. \tag{21}$$

From Eqs. (20) and (21), we get:

$$\hat{U}_t = \bar{Y}\left(1 + e_0\right), \tag{22}$$

and

$$\hat{V}_t = \bar{X}\left(1 + e_1\right). \tag{23}$$

The error terms have expected values:

$$E\left(e_0\right) = E\left(e_1\right) = 0, \tag{24}$$

and

$$\left.\begin{aligned} E\left(e_0^2\right) &= \left(\frac{\lambda}{2-\lambda}\right)\left[1-\left(1-\lambda\right)^{2t}\right]\frac{C_u^2}{n}, \\ E\left(e_1^2\right) &= \left(\frac{\lambda}{2-\lambda}\right)\left[1-\left(1-\lambda\right)^{2t}\right]\frac{C_v^2}{n}, \\ E\left(e_0 e_1\right) &= \left(\frac{\lambda}{2-\lambda}\right)\left[1-\left(1-\lambda\right)^{2t}\right]\frac{1}{n}\rho_{uv}C_u C_v. \end{aligned}\right\} \tag{25}$$

As $t \to \infty$, the term $\left(1-\lambda\right)^{2t} \to 0$, thus Eq. (25) becomes:

$$\left.\begin{aligned}
E\left(e_0^2\right) &= \left(\frac{\lambda}{2-\lambda}\right)\frac{C_u^2}{n},\\
E\left(e_1^2\right) &= \left(\frac{\lambda}{2-\lambda}\right)\frac{C_v^2}{n},\\
E\left(e_0 e_1\right) &= \left(\frac{\lambda}{2-\lambda}\right)\frac{1}{n}\rho_{uv}C_u C_v.
\end{aligned}\right\} \tag{26}$$

In Eq. (26), we can see that the time t vanishes, which means that the bias of the proposed estimator is independent of t.

Using Eq. (15) in Eq. (14) gives:

$$T_{p(t)} = \frac{1}{2}\hat{U}_t\left[\frac{N\bar{X} - n\hat{V}_t}{\left(N-n\right)\bar{X}}\right]^{\alpha}\left(\frac{\bar{X}}{\hat{V}_t} + \frac{V_t}{\bar{X}}\right)\exp\left(\frac{\bar{X} - \hat{V}_t}{\bar{X} + V_t}\right). \tag{27}$$

Putting Eqs. (22) and (23) in Eq. (27) yields:

$$T_{p(t)} = \frac{1}{2}\bar{Y}\left(1+e_0\right)\left[\frac{N\bar{X} - n\bar{X}\left(1+e_1\right)}{\bar{X}\left(N-n\right)}\right]^{\alpha}\left[\frac{\bar{X}}{\left(1+e_1\right)\bar{X}} + \frac{\left(1+e_1\right)\bar{X}}{\bar{X}}\right]\exp\left[\frac{\bar{X} - \bar{X}\left(1+e_1\right)}{\bar{X} + \left(1+e_1\right)\bar{X}}\right],$$

or

$$T_{p(t)} = \frac{1}{2}\bar{Y}\left(1+e_0\right)\left(1-Ae_1\right)^{\alpha}\left[\left(1+e_1\right)^{-1} + \left(1+e_1\right)\right]\exp\left[\frac{-\bar{X}e_1}{2\bar{X}\left(1+\frac{1}{2}e_1\right)}\right],$$

where

$$A = \frac{n}{N-n}.$$

On expansion of the series, ignoring higher order terms, and simplification, we get the equation:

$$T_{p(t)} \approx \frac{1}{2}\bar{Y}\left(1+e_0\right)\left(1-Ae_1\right)^{\alpha}\left[1-e_1+e_1^2+1+e_1\right]\exp\left[-\frac{e_1}{2}\left(1+\frac{1}{2}e_1\right)^{-1}\right],$$

or

$$T_{p(t)} \approx \frac{1}{2}\bar{Y}\left(1+e_0\right)\left[1-\alpha Ae_1 + \frac{\alpha\left(\alpha-1\right)}{2!}A^2e_1^2\right]\left(2+e_1^2\right)\left(1-\frac{e_1}{2}+\frac{3}{8}e_1^2\right),$$

or

$$T_{p(t)} \approx \frac{1}{2}\bar{Y}\left(1+e_0\right)\left[1-\alpha Ae_1 + \frac{\alpha\left(\alpha-1\right)}{2!}A^2e_1^2\right]\left(2-e_1+\frac{7}{4}e_1^2\right),$$

or

$$T_{p(t)} - \bar{Y} \approx \bar{Y}\left[e_0 - Be_1 + Ge_1^2 - Be_0e_1\right], \tag{28}$$

where

$$B = A\alpha + \frac{1}{2},$$

and

$$G = \frac{1}{2}\left\{\alpha\left(\alpha-1\right)A^2 + A\alpha + \frac{7}{4}\right\}.$$

Taking expectation on Eq. (28) yields:

$$Bias\left(T_{p(t)}\right) \approx \bar{Y}\left[E\left(e_0\right) - B\cdot E\left(e_1\right) - B\cdot E\left(e_0e_1\right) + G\cdot E\left(e_1^2\right)\right]. \tag{29}$$

Putting Eqs. (24) and (26) in Eq. (29), we get the required result as:

$$Bias\left(T_{p(t)}\right) \approx \frac{\bar{Y}}{n}\left(\frac{\lambda}{2-\lambda}\right)\left[GC_v^2 - B\rho_{uv}C_uC_v\right].$$

□

Theorem 2: *The MSE of* $T_{p(t)}$ *can be expressed in the form:*

$$MSE\left(T_{p(t)}\right) \approx \frac{\bar{Y}^2}{n}\left(\frac{\lambda}{2-\lambda}\right)\left[C_u^2 + \left\{A\alpha + \frac{1}{2}\right\}^2 C_v^2 - 2\left\{A\alpha + \frac{1}{2}\right\}\rho_{uv}C_uC_v\right].$$

Proof: Applying squares on Eq. (28) and using expected values, we get:

$$E\left(T_{p(t)} - \bar{Y}\right)^2 \approx \bar{Y}^2 E\left(e_0 - Be_1\right)^2,$$

or

$$MSE\left(T_{p(t)}\right) \approx \bar{Y}^2 E\left[e_0^2 + B^2e_1^2 - 2Be_0e_1\right],$$

or

$$MSE\left(T_{p(t)}\right) \approx \bar{Y}^2\left[E\left(e_0^2\right) + B^2E\left(e_1^2\right) - 2BE\left(e_0e_1\right)\right]. \tag{30}$$

Using Eq. (23) in Eq. (30) yields:

$$MSE\left(T_{p(t)}\right) \approx \frac{\bar{Y}^2}{n}\left(\frac{\lambda}{2-\lambda}\right)\left[C_u^2 + B^2C_v^2 - 2B\rho_{uv}C_uC_v\right]. \tag{31}$$

Using $B = A\alpha + \frac{1}{2}$ in Eq. (31) yields the required MSE as follows:

$$MSE\left(T_{p(t)}\right) \approx \frac{\bar{Y}^2}{n}\left(\frac{\lambda}{2-\lambda}\right)\left[C_u^2 + \left\{A\alpha + \frac{1}{2}\right\}^2 C_v^2 - 2\left\{A\alpha + \frac{1}{2}\right\}\rho_{uv} C_u C_v\right]. \tag{32}$$

Remark: *The minimum value of* α *can be derived by differentiating Eq. (32) with respect to* α*:*

$$\frac{\partial MSE\left(T_{p(t)}\right)}{\partial \alpha} = 0,$$

or

$$\frac{\bar{Y}^2}{n}\left(\frac{\lambda}{2-\lambda}\right)\frac{\partial}{\partial \alpha}\left[C_u^2 + \left\{A\alpha + \frac{1}{2}\right\}^2 C_v^2 - 2\left\{A\alpha + \frac{1}{2}\right\}\rho_{uv} C_u C_v\right] = 0,$$

or

$$A\alpha + \frac{1}{2} = \rho_{uv}\frac{C_u}{C_v}, \tag{33}$$

or

$$\alpha_{opt} = \frac{1}{A}\left[\rho_{uv}\frac{C_u}{C_v} - \frac{1}{2}\right]. \tag{34}$$

Putting Eq. (34) in (32) gives the optimum MSE of the estimator as follows:

$$MSE_{opt}\left(T_{p(t)}\right) \approx \frac{\bar{Y}^2}{n}\left(\frac{\lambda}{2-\lambda}\right)C_u^2\left(1-\rho_{uv}^2\right). \tag{35}$$

4 Derivation of Analytical Conditions

We compare the suggested estimators with its different existing competitors to get the efficiency conditions using PPS sampling design.

4.1 Comparison with Exponential Estimator

The efficiency condition in PPS sampling scheme can be obtained as:

$$MSE\left(T_{p(t)}\right) < MSE\left(T_{1(t)}\right),$$

or

$$\left[\left\{A\alpha + \frac{1}{2}\right\} - \frac{1}{2}\right]C_v < 2\rho_{uv} C_u,$$

or

$$\rho_{uv} > \frac{A\alpha C_v}{2C_u}.$$

4.2 Comparison with Choudhury and Singh [40] Estimator

The efficiency condition in PPS sampling scheme can be obtained as:

$$MSE\left(T_{p(t)}\right) < MSE\left(T_{2(t)}\right),$$

or

$$\frac{\bar{Y}^2}{n}\left(\frac{\lambda}{2-\lambda}\right)\left[C_u^2 + \left\{A\alpha + \frac{1}{2}\right\}^2 C_v^2 - 2\left\{A\alpha + \frac{1}{2}\right\}\rho_{uv}C_uC_v\right]$$

$$< \frac{\bar{Y}^2}{n}\left(\frac{\lambda}{2-\lambda}\right)\left[C_u^2 + \left(1-2\alpha\right)^2 C_v^2 + 2\left(1-2\alpha\right)\rho_{uv}C_uC_v\right],$$

or

$$\rho_{uv} > \frac{C_v}{2C_u}\left[\left(A+2\right)\alpha - \frac{1}{2}\right].$$

4.3 Comparison with Kanwai et al. [41] Estimator

In PPS sampling design, the condition can be derived using the inequality:

$$MSE\left(T_{p(t)}\right) < MSE\left(T_{3(t)}\right),$$

or

$$\frac{\bar{Y}^2}{n}\left(\frac{\lambda}{2-\lambda}\right)\left[C_u^2 + \left\{A\alpha + \frac{1}{2}\right\}^2 C_v^2 - 2\left\{A\alpha + \frac{1}{2}\right\}\rho_{uv}C_uC_v\right] < \frac{\bar{Y}^2}{n}\left(\frac{\lambda}{2-\lambda}\right)\left[C_u^2 + \frac{C_v^2}{4} - \rho_{uv}C_uC_v\right],$$

or

$$\rho_{uv} > \frac{C_v}{2C_u}\left(A\alpha + 1\right).$$

4.4 Comparison between EWMA and Simple Estimators

In PPS sampling, our suggested mean estimator, in the case of one-time measurements, may be formulated as follows:

$$T_p = \frac{1}{2}\bar{u}\left(\frac{\bar{v}^*}{\bar{X}}\right)^{\alpha}\left(\frac{\bar{v}}{\bar{X}} + \frac{\bar{X}}{\bar{v}}\right)\exp\left(\frac{\bar{X}-\bar{v}}{\bar{X}+\bar{v}}\right), \tag{36}$$

where

$$\bar{v}^* = \frac{N\bar{X} - n\bar{v}}{N-n}. \tag{37}$$

In order to obtain the MSE of T_p, one may simply replace $\frac{\lambda}{2-\lambda} = 1$ in Eq. (32) to get:

$$MSE\left(T_p\right) \approx \frac{\bar{Y}^2}{n}\left[C_u^2 + \left\{A\alpha + \frac{1}{2}\right\}^2 C_v^2 - 2\left\{A\alpha + \frac{1}{2}\right\}\rho_{uv} C_u C_v\right]. \tag{38}$$

We can derive the efficiency condition as follows:

$$MSE\left(T_{p(t)}\right) < MSE\left(T_p\right).$$

Using Eqs. (32) and (38), the above condition reduces to the form:

$$\lambda < 1.$$

5 Real-World Applications

This section provides some real-world applications of the suggested memory-type estimator. Four real-life populations were taken from previous research papers. The following are the details of each data set:

5.1 Food Irradiation Data

The data on food irradiation was taken from Abdelfattah and Sayed [42] who gathered data about the foods using microwave radiation and gamma radiation techniques. The data quantified microwave radiation as 1, 2, and 3 min whereas quantification of the gamma radiation was 6, 8, and 10 kilos. We recommend readers to refer to the research works of Alomani et al. [25] and Abdelfattah and Sayed [42]. A summary of the data is displayed in Table 1. Using the data, the MSE of various estimators is presented in Table 2.

5.2 Data Related to Air Quality

We considered the air quality data given in the study of Chambers [43]. The data presented ozone levels recorded against corresponding temperatures on surface of Earth, where the ozone levels were regarded as the values of the study variable, while treating the temperatures as values of the auxiliary variable. The two variables are correlated since higher temperatures tend to be associated with higher ozone levels. A summary of the data is provided in Table 1, whereas the MSEs of different estimators are presented in Table 3 for various α and λ values.

5.3 Demographic Data

The data related to vital statistics was taken from the paper of Singh [44] where the total fertility rate was recorded against crude birth rate. We treated the total fertility rates as our main variable and the crude birth rates were regarded as auxiliary variable. Summary statistics of the data are provided in Table 4.

5.4 Fish-Trapping Data

The data on fish trapping was taken from the research study of Singh [44]. The number of fishes trapped in 1995 was considered as main variable whereas the number of fishes trapped in 1994 was taken as an auxiliary variable. The MSE of the competing estimators are given in Table 5 for different sets of values of α and λ.

Table 1: Summaries of different data sets.

Parameters/Statistics	Food Radiation Data	Air Quality Data	Demographic Data	Fish-Trapping Data
N	20	4071	96	69
n	5	120	20	12
$\bar{Y}$	94.55	1012.588	3.5017	4514.90
$\bar{X}$	113	46,535.20	26.0115	4954.435
C_u	1.4551	1.84	0.7488	0.4061
C_v	0.6939	1.54	0.697	0.3829
ρ_{uv}	0.9745	0.7124	0.9927	0.7546

The improvement in efficiency in the proposed memory-type estimator over the competitor mean estimators may clearly be observed in Tables 2–5.

Table 2: MSEs based on radiation data.

α	λ	$Var\left(T_{0(t)}\right)$	$MSE\left(T_{1(t)}\right)$	$MSE\left(T_{2(t)}\right)$	$MSE\left(T_{3(t)}\right)$	$MSE\left(T_{p(t)}\right)$
0.1	0.3	668.05	1016.49	1262.01	395.58	380.12
	0.5	1261.88	1920.03	2383.79	747.21	718.00
	0.7	2038.42	3101.59	3850.75	1207.03	1159.84
	0.9	3097.34	4712.81	5851.13	1834.05	1762.35
0.3	0.1	199.24	303.16	280.57	117.98	104.45
	0.3	668.05	1016.49	940.72	395.58	350.20
	0.5	1261.88	1920.03	1776.92	747.21	661.49
	0.9	3097.34	4712.81	4361.54	1834.05	1623.66
0.5	0.1	199.24	303.16	199.24	117.98	95.93
	0.3	668.05	1016.49	668.05	395.58	321.64
	0.5	1261.88	1920.03	1261.88	747.21	607.53
	0.9	3097.34	4712.81	3097.34	1834.05	1491.22
0.7	0.1	199.24	303.16	132.42	117.98	87.81
	0.3	668.05	1016.49	444.00	395.58	294.42
	0.5	1261.88	1920.03	838.66	747.21	556.13
	0.9	3097.34	4712.81	2058.53	1834.05	1365.04

Table 3: MSEs based on air quality data.

α	λ	$Var\left(T_{0(t)}\right)$	$MSE\left(T_{1(t)}\right)$	$MSE\left(T_{2(t)}\right)$	$MSE\left(T_{3(t)}\right)$	$MSE\left(T_{p(t)}\right)$
0.1	0.30	5104.95	9042.77	12,263.72	2955.15	2947.54
	0.50	9642.71	17,080.80	23,164.80	5581.94	5567.58
	0.70	15,576.68	27,592.09	37,420.06	9016.95	8993.79
	0.90	23,668.44	41,925.61	56,859.05	13,701.11	13,665.89
0.3	0.10	1522.54	2696.98	2419.42	881.37	874.62
	0.30	5104.95	9042.77	8112.18	2955.15	2932.55
	0.50	9642.71	17,080.80	15,323.00	5581.94	5539.26
	0.90	23,668.44	41,925.61	37,611.00	13,701.11	13,596.37
0.5	0.10	1522.54	2696.98	1522.53	881.37	870.23
	0.30	5104.95	9042.77	5104.96	2955.15	2917.82
	0.50	9642.71	17,080.80	9642.70	5581.94	5511.44
	0.90	23,668.44	41,925.61	23,668.45	13,701.11	13,528.09
0.7	0.10	1522.54	2696.98	966.93	881.37	865.91
	0.30	5104.95	9042.77	3242.06	2955.15	2903.36

	0.50	9642.71	17,080.80	6123.90	5581.94	5484.12
	0.90	23,668.44	41,925.61	15,031.38	13,701.11	13,461.02

Table 4: MSEs based on demographic data.

α	λ	$Var\left(T_{0(t)}\right)$	$MSE\left(T_{1(t)}\right)$	$MSE\left(T_{2(t)}\right)$	$MSE\left(T_{3(t)}\right)$	$MSE\left(T_{p(t)}\right)$
0.1	0.3	0.06	0.13	0.18	0.02	0.02
	0.5	0.11	0.25	0.35	0.03	0.03
	0.7	0.19	0.40	0.56	0.05	0.05
	0.9	0.28	0.60	0.85	0.08	0.08
0.3	0.1	0.02	0.04	0.03	0.01	0.00
	0.3	0.06	0.13	0.11	0.02	0.01
	0.5	0.11	0.25	0.22	0.03	0.03
	0.9	0.28	0.60	0.53	0.08	0.06
0.5	0.1	0.02	0.04	0.02	0.01	0.00
	0.3	0.06	0.13	0.06	0.02	0.01
	0.5	0.11	0.25	0.11	0.03	0.02
	0.7	0.19	0.40	0.19	0.05	0.03
	0.9	0.28	0.60	0.28	0.08	0.05
0.7	0.1	0.02	0.04	0.01	0.01	0.00
	0.3	0.06	0.13	0.02	0.02	0.01
	0.5	0.11	0.25	0.05	0.03	0.02
	0.9	0.28	0.60	0.11	0.08	0.04

Table 5: MSEs based on fish-trapping data.

α	λ	$Var\left(T_{0(t)}\right)$	$MSE\left(T_{1(t)}\right)$	$MSE\left(T_{2(t)}\right)$	$MSE\left(T_{3(t)}\right)$	$MSE\left(T_{p(t)}\right)$
0.1	0.3	49,437.14	95,598.68	133,843.59	25,250.56	24,714.29
	0.5	93,381.26	180,575.29	252,815.66	47,695.50	46,682.54
	0.7	150,846.66	291,698.54	408,394.53	77,046.58	75,410.26
	0.9	229,208.56	443,230.25	620,547.54	117,070.77	114,584.42
0.3	0.1	14,744.41	28,511.89	25,234.08	7530.87	7085.90
	0.3	49,437.14	95,598.68	84,608.38	25,250.56	23,758.62
	0.5	93,381.26	180,575.29	159,815.82	47,695.50	44,877.39
	0.7	150,846.66	291,698.54	258,164.02	77,046.58	72,494.24
	0.9	229,208.56	443,230.25	392,275.20	117,070.77	110,153.59
0.5	0.1	14,744.41	28,511.89	14,744.41	7530.87	6847.36
	0.3	49,437.14	95,598.68	49,437.14	25,250.56	22,958.78
	0.5	93,381.26	180,575.29	93,381.26	47,695.50	43,366.59
	0.7	150,846.66	291,698.54	150,846.66	77,046.58	70,053.71
	0.9	229,208.56	443,230.25	229,208.56	117,070.77	106,445.25
0.7	0.1	14,744.41	28,511.89	8449.26	7530.87	6655.28
	0.3	49,437.14	95,598.68	28,329.88	25,250.56	22,314.78
	0.5	93,381.26	180,575.29	53,511.99	47,695.50	42,150.14
	0.7	150,846.66	291,698.54	86,442.45	77,046.58	68,088.68
	0.9	229,208.56	443,230.25	131,347.62	117,070.77	103,459.42

6 Stability Analysis

The proposed estimator is a function of sample statistics $\hat{U}_t$ and $\hat{V}_t$ which in turn are functions of the sample means $\bar{u}$ and $\bar{v}$. In Eqs. (1) and (2), we see that the values of the statistics $\hat{U}_t$ and $\hat{V}_t$ are based on repeated measurements. In this section, we analyze the stability of the proposed estimator by calculating its value for various values of α, $\hat{U}_t$ and $\hat{V}_t$. The results have been presented in Tables 6 and 7 where observe that our proposed estimator remains stable across sample sizes and α values. The stability of the proposed estimator across sample sizes is practically beneficial as small sample sizes would provide reliable estimates. This helps the researchers avoid high costs of data collection often associated with large samples. Moreover, the results also show that as the value of $\hat{U}_t$ increases, the value of the proposed estimator also increases.

Table 6: Values of the Proposed Estimator for $\bar{X} = 30$, $N = 1000$.

$\hat{U}_t$	$\hat{V}_t$	$\alpha = 0.2$			$\alpha = 0.4$		
		$n = 50$	$n = 100$	$n = 200$	$n = 50$	$n = 100$	$n = 200$
30	20	39.83	39.99	40.34	39.97	40.28	40.99
	40	26.99	26.89	26.62	26.90	26.68	26.16
	60	26.58	26.24	25.37	26.30	25.63	23.95
50	20	66.39	66.64	67.23	66.62	67.13	68.31
	40	44.99	44.81	44.37	44.83	44.47	43.61
	60	44.30	43.74	42.28	43.83	42.72	39.92
70	20	92.95	93.30	94.12	93.27	93.98	95.64
	40	62.99	62.73	62.12	62.76	62.26	61.05
	60	62.02	61.24	59.19	61.36	59.81	55.88

Table 7: Values of the Proposed Estimator for $\bar{X} = 80$, $N = 1000$.

$\hat{U}_t$	$\hat{V}_t$	$\alpha = 0.2$			$\alpha = 0.4$		
		$n = 50$	$n = 100$	$n = 200$	$n = 50$	$n = 100$	$n = 200$
30	20	117.06	118.03	120.22	117.97	119.94	124.43
	40	52.61	52.90	53.58	52.88	53.48	54.86
	60	36.14	36.25	36.49	36.24	36.45	36.93
50	20	195.10	196.72	200.37	196.62	199.90	207.38
	40	87.68	88.17	89.30	88.14	89.13	91.43
	60	60.24	60.41	60.81	60.40	60.74	61.56
70	20	273.15	275.41	280.52	275.27	279.86	290.33
	40	122.75	123.44	125.03	123.39	124.79	128.01
	60	84.33	84.58	85.14	84.56	85.04	86.18

7 Simulation Study

To compare the performance of various competitor estimators in PPS sampling, we generated an artificial population of $N = 5000$ units. To generate artificial data, we considered a normal distribution with mean 30 and variance 9 and chose samples of different sizes from the population. In order to assign probabilities to population units, we generated an additional supplementary variable Z correlated with Y. For each sample size, we averaged the results across 1000 iterations. The results of simulation are presented in Table 8 where the superiority of our proposed estimator can be seen. We can see that as the sample size increases, the MSEs of all estimators decline.

Table 8: Comparison of Simulated MSEs.

n	α	**Estimators**				
		$Var\left(T_{0(t)}\right)$	$MSE\left(T_{1(t)}\right)$	$MSE\left(T_{2(t)}\right)$	$MSE\left(T_{3(t)}\right)$	$MSE\left(T_{p(t)}\right)$
100	0.25	0.07218	0.02002	0.01507	0.01703	0.01505

	0.50	0.07218	0.01508	0.01507	0.02699	0.01501
	0.75	0.07218	0.01639	0.01507	0.04496	0.01498
	1	0.07218	0.02396	0.01507	0.07094	0.01496
200	0.25	0.03578	0.00961	0.00730	0.00852	0.00726
	0.50	0.03578	0.00729	0.00730	0.01384	0.00724
	0.75	0.03578	0.00816	0.00730	0.02326	0.00722
	1	0.03578	0.01221	0.00730	0.03678	0.00721
300	0.25	0.02172	0.00601	0.00470	0.00557	0.00468
	0.50	0.02172	0.00470	0.00470	0.00896	0.00467
	0.75	0.02172	0.00534	0.00470	0.01487	0.00467
	1	0.02172	0.00793	0.00470	0.02328	0.00468
400	0.25	0.01601	0.00443	0.00342	0.00399	0.00340
	0.50	0.01601	0.00342	0.00342	0.00639	0.00339
	0.75	0.01601	0.00383	0.00342	0.01062	0.00339
	1	0.01601	0.00572	0.00342	0.00343	0.00341
500	0.25	0.01219	0.00357	0.00276	0.00308	0.00273
	0.50	0.01219	0.00276	0.00276	0.00472	0.00272
	0.75	0.01219	0.00297	0.00276	0.00768	0.00271
	1	0.01219	0.00421	0.00276	0.01196	0.00273

8 Discussion of Results

In survey sampling, memory-based estimators are formulated for precise estimation of various parameters, viz., mean, variance, and proportion, etc. Unlike the conventional estimators, where single measurements of subjects are used, memory-based estimators utilize the measurements of variables observed at more than one time point. Relative weights can be allocated to various observations in such a way that the newest observed measurement gets the largest weight, and the oldest observed measurement gets the smallest weight. The existing memory-based estimators utilize traditional sampling designs, which may lead to misleading results in many real-life problems where the units have unequal probabilities of selection in the sample. The rarity of memory-type EWMA estimation methods in PPS sampling in the available literature is the main motivation of this study. We introduced a novel EWMA estimator for use with the real-life challenges of efficient mean estimation.

Different properties of the proposed PPS sampling—based EWMA estimator were studied in previous sections. For efficiency comparison under identical situations, some of the available competitor estimators were also extended to the case of multi-time surveys under PPS sampling design. Four different real data sets were taken from previous research studies for performance comparison under the PPS sampling design. The findings of the comparison have been presented in tables.

Tables 2–5 suggest that the suggested memory-based EWMA estimator achieves significant improvement over its competitor estimators of the population mean. From these tables, we see that the MSE of each estimator increases as the value of the smoothing constant λ approaches 1. Further, in each of the four data sets, we can clearly observe a smaller MSE for the suggested EWMA estimator than other competitor mean estimators. The tables also suggest that, after the proposed memory-type estimator, the Kumar et al. [32] estimator is the second-best of all competitors for population mean estimation, under each of the four data sets.

The proposed estimator has some strengths and weaknesses. It uses a single auxiliary variable, which makes it mathematically simpler than most of its competitors, which use two or more auxiliary variables. Besides simplicity, improvement in efficiency also makes the proposed estimator superior to its competitors. A limitation of the proposed work is that the real-world populations studied in this paper lack cases of low correlations between variables. We leave this as future research work where the researchers can demonstrate the efficiency of ratio estimators under low correlations.

Future researchers can conduct studies on analyzing different properties of our proposed EWMA estimator in the presence of different kinds of non-sampling errors. Moreover, future researchers can also work on evaluating our proposed EWMA estimator using various complex sampling schemes.

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