Space-time goal oriented error estimation and adaptivity for discretization and reduced order modeling errors

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ABSTRACT

In this presentation, we present a uniform framework in which the dual-weighted residual (DWR) method is used for spatial and temporal discretization error control [1], as well as the control of the reduced order modeling error for the proper orthogonal decomposition (POD).

In the first part of this presentation, the DWR method is applied to a space-time formulation of nonstationary Navier-Stokes flow. Tensor-product space-time finite elements are being used to discretize the variational formulation with discontinuous Galerkin finite elements in time and inf-sup stable Taylor-Hood finite element pairs in space. To estimate the error in a quantity of interest and drive adaptive refinement in time and space, we demonstrate how the DWR method for incompressible flow [2] can be extended to a partition of unity based error localization [3, 4]. Our methodology is being substantiated on the two dimensional flow around a cylinder benchmark problem.

In the second part of this presentation, we apply the DWR method to obtain a certified incremental proper orthogonal decomposition based reduced order model [5]. For the full order model, we utilize a tensor-product space-time discretization and reduce the spatial modes by POD. We then introduce a novel approach that marries the space-time reduced order model and an incremental proper orthogonal decomposition [6, 7] based basis generation with a goal-oriented error control based on DWR estimates. We aim to solve the reduced system without any prior knowledge or exploration of the solution manifold such that no offline phase is required. Instead, we solve from the beginning on the reduced order model and –if necessary– update the reduced basis on-the-fly during the simulation with high fidelity finite element solutions by means of the incremental POD. For this purpose, we estimate the error in the goal functional and update the reduced basis if the estimate exceeds a given threshold. This allows an adaptive enrichment of the reduced basis in case of unforeseen changes in the solution behavior. We demonstrate our methodology on the heat equation and elastodynamics.

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