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ADVANCED STRUCTURE MODELLING USING THE BONDED-PARTICLE METHOD: ENABLING COMPLEX CAPABILITIES FOR STRUCTURES IN DEM SIMULATIONS

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Abstract: Bonded-particle modelling is a powerful approach for simulating particle-made structures with enhanced abilities, i.e., structures that are able to deform, brake/fracture, and thus interact with other particles of a DEM simulation in an enhanced complex manner. This modelling approach is based on connecting particles with beam elements (commonly termed bondings, with each bonding connecting two particles) to create a bonded-particle model (BPM), typically consisting of a large number of particles and bondings. The behaviour of such a BPM can be adjusted in two categories: via properties related to the particles, e.g. their mass, shape, or frictional characteristics, and via properties related to the bondings, e.g. their Young's moduli or breakage criteria.

This contribution presents developments in complex bonded-particle modelling for achieving flexible/deformable and breakable/fracturable structures and highlights the enhanced capabilities made possible by using this approach. For this purpose, various applications for bonded-particle modelling in DEM simulations are presented, with selected case studies each demonstrating the effectiveness of bonded-particle modelling in solving practical engineering problems, including:

- 1D BPMs, e.g., deformable beams or ropes and chains,
- 2D BPMs, e.g., membranes, textiles, nets, bags, shell-like parts (e.g. silos), or conveyor belts (with particular reference to dynamic belt simulation), and
- 3D BPMs, e.g., complex-breakable structures (e.g. with considering crack formation or dynamic impact handling; e.g. illustrated on filter cake material).

Furthermore, possible interaction scenarios, including interactions of BPMs to particles, rigid parts, other BPMs, or even in terms of SPH (Smoothed-Particle Hydrodynamic) or MBD (Multibody Dynamics), are addressed.

Finally, current developments and potential future directions in this field are outlined, and potentials for extending this modelling approach to further application areas are discussed.

Keywords: Discrete Element Method, DEM, Bonded-Particle Model, BPM, Flexible Structures, Breakable Structures

1 INTRODUCTION

The field of Discrete Element Method (DEM; cf. [1, 2]) simulation has become a cornerstone in the study and analysis of bulk solids. As a particle-based numerical method, DEM allows for the detailed simulation of granular materials, offering insights into their behaviour under various conditions, such as required for virtual prototyping of particle-handling and -processing equipment. Within this realm, bonded-particle modelling (using a bonded-particle model, BPM; cf. [3, 4]) has established itself as a pivotal technique in state-of-the-art DEM simulations. It provides a robust framework for constructing structures with advanced capabilities, which is the focus of this paper.

Bonded-particle models (BPMs) serve as a versatile tool for simulating structures that are not rigid objects but dynamically interactive components in DEM simulation setups. These structures can exhibit a range of behaviours, from deformation and flexibility to fracture and breakage. Thus, the utilisation of BPMs extends beyond mere structural representation of components, as they enable complex (bi-directional) interactions with other elements of the simulation.

This paper's main part is structured to offer a categorising overview in terms of various applications of BPMs, which is followed by selected visualising examples that are associated with those categories and which intend to underline the potentials enabled for the different categories stated. This categorisation is introduced as based on a BPM's dimension, more specifically defining 1D, 2D, and 3D BPMs. Additionally, the BPM's interaction partner is used for these purposes as to consider BPMs interacting with particles, rigid parts, other BPMs, or even fluids modelled via Smoothed-Particle Hydrodynamics (SPH; cf. [5]), elements belonging to a Multibody Dynamics (MBD; cf. [6, 7]) model (joints). The selected visual examples, specifically focusing on the first sort of classification (dimensions), will further illustrate the application of BPMs in an engineering context, moreover providing a practical perspective on the capabilities of bonded-particle modelling.

In summary, this paper aims to highlight the practicality and versatility of using bonded-particle modelling in DEM simulations, focusing on the creation of dynamically interactive structures that can deform/flex or even fracture and break under various conditions.

2 BACKGROUNDS REGARDING APPLIED METHODS

2.1 Bondings

In DEM simulation setups, bondings (also referred to as bonding elements, bonds, or joints) serve as a specific type of virtual connection between two particles. Technically, they can be categorised as a specialised form of contact model, defining the interactional behaviour between exactly two (bonded) particles. In ThreeParticle/CAE [8], for instance, bondings are implemented as slave contact models, applied in superposition to a master contact model that defines general particle interaction (e.g. a common Hertz-Mindlin contact model [9–11]). [12]

The bonding, in mechanical sense a beam-like connection between two particles, is present as a virtual connection model with no contact surface, volume, or mass, thus only to transfer loads between the two bonded particles, as visualised in Figure 1.

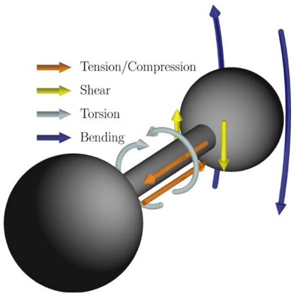


Figure 1: Different load types transmittable via the bonding between two bonded particles [13]

To handle and transmit such types of loads, including tension/compression, shear, torsion, and bending (as shown in Figure 1), the mechanics of bondings are fundamentally related to the approach of one-dimensional beam elements, for which the Timoshenko beam theory (cf. [14–16]) is commonly applied.

A bonding with its two bonded particles – on the left shown in relaxed, undeformed state and on the right in a loaded, thus deformed state – is further shown in Figure 2.

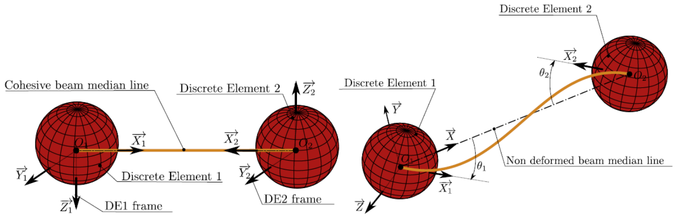


Figure 2: Bonding as a virtual beam element between two particles in the relaxed state (left) and in a loaded thus deformed state (right)] [17]

2.2 Bonded-particle models (BPMs)

Bonded-particle models (BPMs) are complex structures formed by connecting more than two (commonly a relatively large multitude) of particles through bondings. These structures not only maintain a distinctive form but also exhibit complex behaviour, particularly in terms of deformability and, optionally, fracture.

Bonded-particle models (BPMs) are complex structures formed by connecting more than two (and commonly even a relatively large multitude) of particles through bondings. These structures not only maintain a distinctive form but also exhibit complex behaviour, particularly in terms of deformability and, optionally, fracture of the resulting total BPM.

BPMs are versatile and find applications in various fields, such as material science, geomechanics, mining, mineral and material processing, mechanical engineering, etc. Due to the aforementioned capabilities of bondings regarding the inclusion of deformability and breakage, BPMs are particularly useful for simulating advanced material characteristics that require such characteristics to be included.

In the context of DEM simulation, BPMs can be utilised for the modelling of complex particles, but furthermore also the modelling of complex parts (in contrast to conventionally rigid parts) – with this paper focusing on the latter.

Figure 3 ([18]) shows a representative example of a BPM application: a cylindrical pillar made from a multitude of bonded particles – with the particle model shown on the left and the underlying particle network visualised in an isolated view on the right.

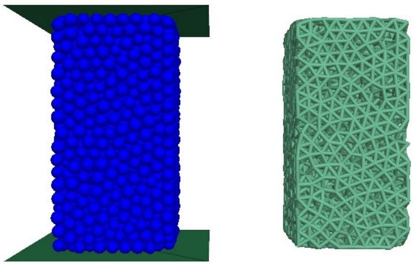


Figure 3: A bonded-particle model (BPM) made of a multitude of spherical particles (left) with the underlying bondings visualised separately (right) [18]

2.3 Adjusting the behaviour of BPMs

The behaviour of BPMs can be fine-tuned through properties related to either the particles or the bondings that connect them. For particles, typical adjustable characteristics include the particles' densities (or masses) and geometries. For bondings, properties like the bondings' Young's moduli, cross-sectional areas, and optional breakage criteria can be adjusted. It is noteworthy that a BPM can be set up heterogeneously, as made from different particles and different bondings.

Furthermore, advanced bonding models, such as the one introduced by Fimbinger [19] (as discussed later in Chapter 4.3 in terms of dynamic belt modelling) that incorporates a reduction factor to allow for easy-to-apply adaptation of the bonding's bending behaviour, even extend the commonly available capabilities of bondings and consequently the resulting BPMs. Other such bonding model adaptations may include accumulated damage or weakening due to repeated loading. Also, the implementation of enhanced models regarding the particles may be applied, such as to include wear or temperature-related aspects to BPM simulations.

3 CATEGORISING OVERVIEW OF BPM APPLICATION TYPES

BPMs are versatile, finding applications across a range of disciplines and scenarios. Their general utility can be broadly classified based on two key criteria: the dimensionality of the BPM and the type of interactions involved. It is worth noting that these classifications are not mutually exclusive. For example, a 1D wire BPM can be part of a 2D grid to form a net. Similarly, a conveyor belt modelled as a BPM can interact with both the particles it transports and the rigid idlers that keep it on track.

In this paper, specifically in the next Chapter 4, presenting selected examples of BPM applications, the focus is primarily on categorising BPMs based on their dimensionality, as interaction handling is generally consistent with those in standard DEM simulations.

Each subsection will feature a simple sketch to illustrate the category in question.

In addition to the aspects covered in these two general categories, more detailed and typically application-specific modelling methods and approaches can be applied that further define BPMs. For this purpose, a large pool of BPM-based expanding methods/approaches exist, e.g. concerning the use of complex particle geometries (e.g. cuboidal particles; see Chapter 4.3), surface reconstruction (e.g. with the PFacet approach; see Chapter 4.2), advanced load handling (e.g. regarding impact loads; see Chapter 4.5), etc.

3.1 In terms of the BPM's dimension

One-dimensional (1D) BPMs

A 1D BPM forms as a single line of bonded particles, as sketched in Figure 4, with each (regular) particle connected to only two adjacent ones.

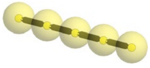


Figure 4: Simple sketch of a 1D BPM

Two-dimensional (2D) BPMs

A 2D BPM forms as a single layer of bonded particles (thus with a thickness of only one particle), as representatively sketched in Figure 5, with each (regular) particle connected to several adjacent ones within this layer. Common grid variants are quadrilateral (connection to four adj. particles; see Figure 5) or triangular (connection to six adj. particles; see Figure 9). Some further variants are also double-split triangular (connection to eight adj. particles; see Figure 11) or hexagonal (connection to three adj. particles; see Figure 10).

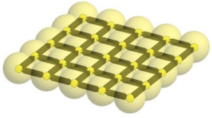


Figure 5: Simple sketch of a 2D BPM with a quadrilateral grid structure

Three-dimensional (3D) BPMs

A 3D BPM forms as a three-dimensional structure consisting of multiple bonded particles, either with multiple layers as sketched in Figure 6, but also in a random arrangement, as shown previously in Figure 3, with each (regular) particle connected to adjacent ones in all three spatial directions. A common variant – as a pendant to the 2D quadrilateral grid – is when each (regular) particle is connected to six adjacent ones (i.e., two in each spatial direction), which is specifically shown in Figure 6. Other variants correspondingly exist.

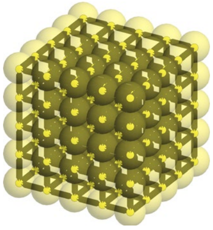


Figure 6: Simple sketch of a 3D BPM with a quadrilateral grid structure

Hybrid-dimensional BPMs (combinations of 1D/2D/3D)

A hybrid-dimensional BPM forms as a combination of 1D/2D/3D sections that are bonded to form one single model. An exemplary variant forming a 1D-2D-hybrid BPM is the model sketched in Figure 7, representing a hammock-like model made of a mat (2D quadrilateral section) hung via four ropes (1D sections; e.g. fixed on each outermost particle).

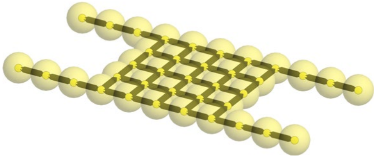


Figure 7: Simple sketch of a 1D-2D-hybrid BPM, made of four 1D sections bonded to a 2D (quadrilateral grid) section

3.2 In terms of the BPM’s interaction partner

BPM to particle interaction: trivial particle-to-particle contact, where one particle belongs to a BPM.

BPM to rigid part interaction: trivial particle-to-part contact, where the particle belongs to a BPM.

BPM to BPM interaction: trivial particle-to-particle contact, where both particles belong to separate BPMs.

BPM self interaction: (non-trivial) particle-to-particle contact within a BPM, e.g. with no contact reactions further considered, except those from the bondings (explained in detail in [19], termed particle-overlapping), but furthermore concerning the handling of fractured but not fully-broken BPM when initially bonded-particles are coming in contact again (i.e. closing a fractured gap), or when a BPM requires to get in surface-contact with itself (explained in detail in [19], termed self-contact).

BPM to SPH interaction: particle-to-sph-liquid – which corresponds to common interaction handling between DEM particles and the SPH spherical elements representing the (meshless) liquid. (e.g. cf. [12]).

BPM to MBD interaction: particle-to-multibody-dynamics-system – which corresponds to the ability to attach MBD markers (for implementing MBD joints) to particles that further belong to a BPM (which then allows the inclusion of a BPM into an MBD system, such as for chains or ropes, for which this ability was initially introduced as available in ThreeParticle/CAE [8] since late 2022).

4 EXAMPLES

The selected case studies shown in the following demonstrate the applicability and effectiveness of using BPMs in solving practical engineering problems in a wide range of industrial fields. These examples refer to and, moreover, extend the respective types of applications described in the previous Chapter 3.

4.1 1D: Rope- and beam-like elements

For showcasing 1D BPMs, the example in Figure 8 shows a ring net barrier modelled with three different 1D BPMs, including BPMs that represent a steel wire rope, several shackles, and several ring net elements. The reaction of this total model consisting of multiple BPMs interacting with one another and further with an impacting particle that represents a boulder is shown on the right.

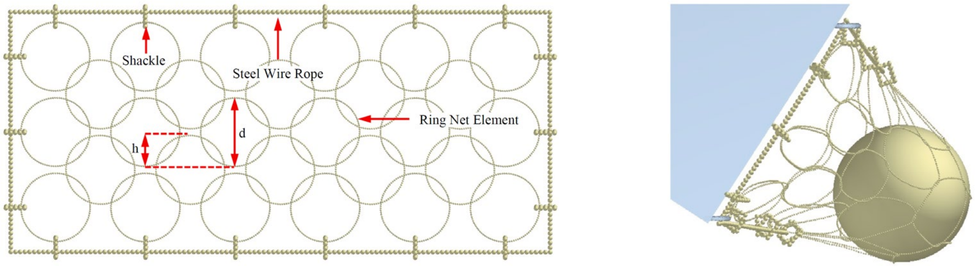


Figure 8: Ring net barrier, modelled with several 1D BPMs (rings, shackles, and a surrounding wire rope; left); also shown reacting deformable during the impact of a boulder (right) [20]

4.2 2D: Membranes, nets, bags, bins etc.

For simple-in-principle 2D BPMs, as for representing simple surface-geometries, such as rectangular or cylindrical shells with common spherical particles, a large number of examples exist, with the two BPMs shown in Figure 9 and Figure 10 as representative selected examples.

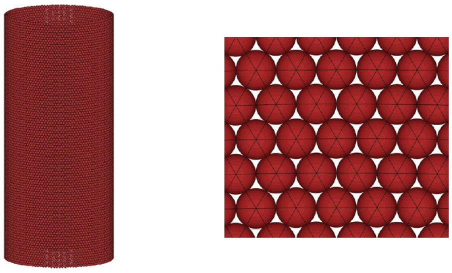


Figure 9: Cylindrical membrane modelled as a 2D triangular BPM [21]

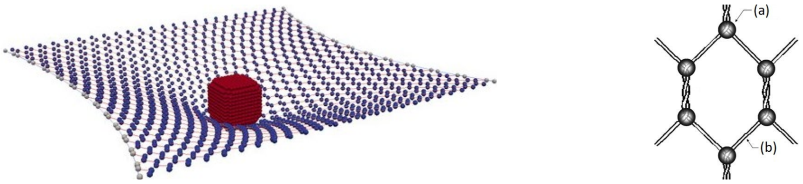


Figure 10: Hexagonal wire net as a 2D hexagonal BPM deformed by a mass block [22], with detail of such a grid made of particles (a) and wire-representing bondings (b) [23]

A noteworthy adaptation in this context is the approach of using PFacet modelling by Effeindzourou (see [24, 25]) for surface smoothing, as shown and explained in Figure 11.

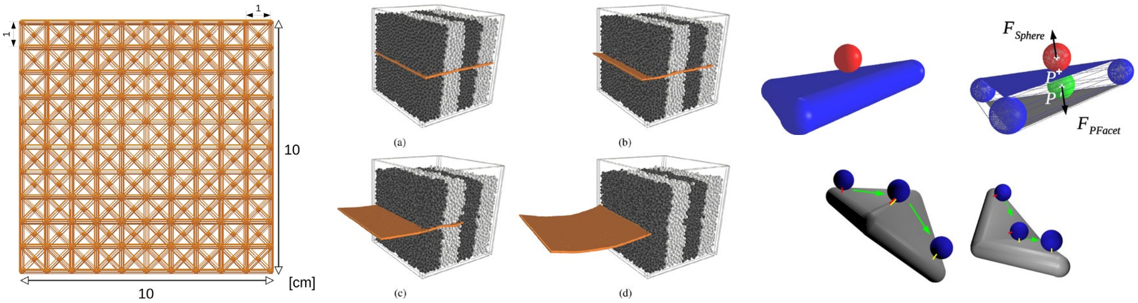


Figure 11: Membrane as a 2D double-split triangular BPM (left) pulled out of a particle-filled box (middle; a-d) [24], with the applied PFacet approached visualised (contact handling concept; right) [25]

A practical extension regarding 2D BPM modelling is the creation of geometries made of multiple (and/or further also complex-shaped) surfaces. Figure 12 shows three illustrative examples from practice: a bulk-solid-filled big bag made of five rectangular 2D BPMs that are connected along their edges to form a cuboidal bag, a filled chips bag made of two 2D BPMs, and a washing machine that contains several 2D BPMs representing garment items, which in this case are not set up as distinct forms, but such objects would in principle be well suited for this purpose (this specific example may also regard BPM to SPH interaction). All three of these examples further concern softening in terms of the bending resistance of bondings, as explained in the next Chapter (4.3, regarding the implementation of a bonding reduction factor; cf. [19]).

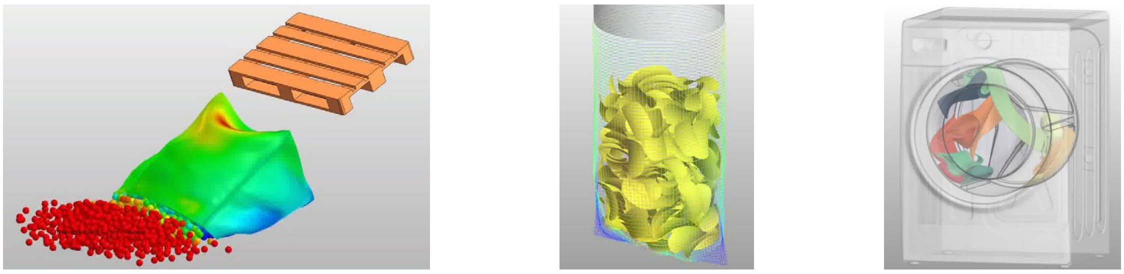


Figure 12: Examples of 2D BPMs: A big bag, a chips bag, and clothes in a washing machine [26]

Other objects that are depictable with surfaces can also be created from 2D BPMs, for example, containers made of sheet metal, such as silos or troughs.

4.3 2D: Conveyor belts

Specifically for enabling dynamically deformable belts in DEM simulations, Fimbinger [19] introduced a respective methodology that comprises, amongst other details, the following major aspects:

- Defining belt-typical behaviour to the BPM via a bonding reduction factor that allows softening in terms of bending whilst remaining a higher tensile behaviour, as further illustrated/explained in Figure 14.
- Defining anisotropic belt behaviour via fibre-modelling, which allows different bonding definitions in transverse and longitudinal belt direction.
- Computation-efficient surface smoothing by using cuboidal particles and enabling particle overlapping within the BPM by disabling inner-BPM-contacts other than regarding the bondings (with exception for enabling specific belt-to-belt-contacts).
- Initialisation of any belt geometry as a BPM supported by a CAD-to-DEM conversion algorithm embedded in a free-to-use software tool that reads (meshed) CAD data and returns DEM data of pre-deformed (pre-tensioned) and optionally also running BPM belts. The process in this regard is shown in Figure 15.
- Using smooth-surfaced cylinders via a CAD-to-DEM converter that enables the implementation of cylindrical objects instead of triangulated ones for idlers and pulleys (with similar usability as the belt conversion software).

An exemplary application of a relatively complex belt conveyor system, a sandwich conveyor containing two belts that clamp bulk material between them for vertical transportation, is shown in Figure 13.

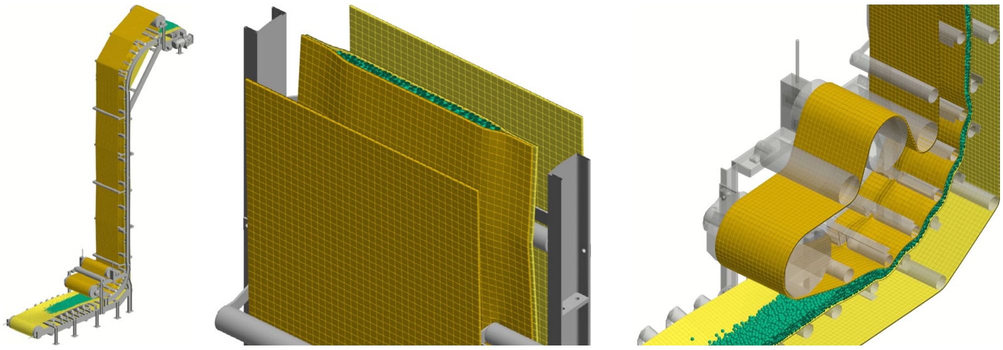


Figure 13: A sandwich conveyor applied for dynamic belt simulation, with bulk material conveyed between the two belts; cross-sectional views showing the deformation of the belts as 2D BPMs [19]

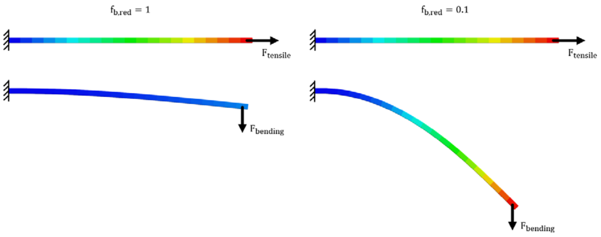
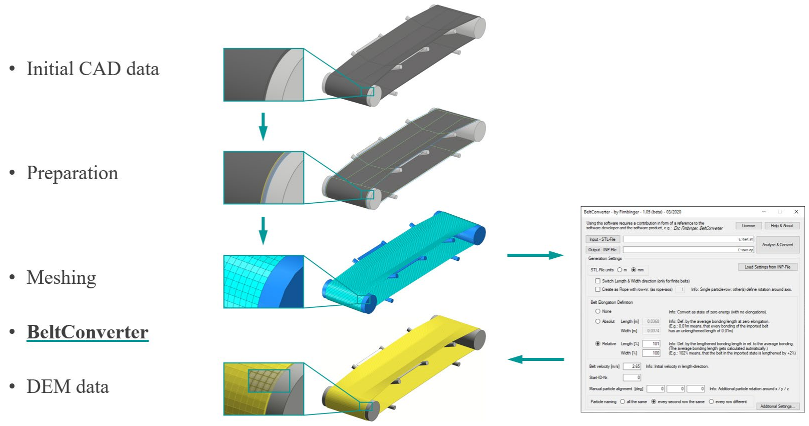


Figure 14: Effects of using the bonding reduction factor: A 2D BPM seen from the side under tensile load (top) and bending load (bottom), with a reduction factor of 1 (left) and 0.1 (right) [19]



BeltConverter - by Fimbinger - 1.05 (beta) - 03/2020

Using this software requires a contribution in form of a reference to the software developer and the software product, e.g.: *Eric Fimbinger, BeltConverter*

License | Help & About

Input - STL-File: E:\belt.stl

Output - INP-File: E:\belt.inp

Analyze & Convert

Generation Settings

Load Settings from INP-File

STL-File units ○ m ● mm

☐ Switch Length & Width direction (only for finite belts)

☐ Create as Rope with row-nr. (as rope-axis) 1 Info: Single particle-row; other(s) define rotation around axis.

Belt Elongation Definition

○ None — Info: Convert as state of zero energy (with no elongations).

○ Absolut — Length [m] 0.0368, Width [m] 0.0374 — Info: Def. by the average bonding length at zero elongation. (E.g.: 0.01m means, that every bonding of the imported belt has an unlengthened length of 0.01m)

● Relative — Length [%] 101, Width [%] 100 — Info: Def. by the lengthened bonding length in rel. to the average bonding. (The average bonding length gets calculated autmatically.) (E.g.: 102% means, that the belt in the imported state is lengthened by +2%)

Belt velocity [m/s] 2.65 Info: Initial velocity in length-direction.

Start-ID-Nr. 0

Manual particle alignment [deg] 0 0 0 Info: Additional particle rotation around x / y / z

Particle naming ○ all the same ● every second row the same ○ every row different

Additional Settings...

Figure 15: Visualisation of the CAD-to-DEM belt conversion process [27]

The option to consider belt breakage is also given, as showcased in Figure 16.

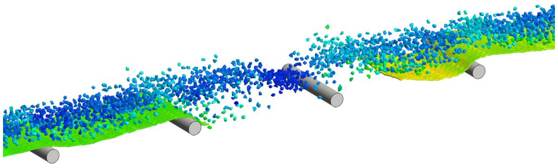


Figure 16: Dynamic effects shortly after belt breakage [19]

4.4 3D/Hybrid: Heterogenous structures

For showcasing a heterogenous 3D BPM, Figure 17 shows a reinforced concrete beam with the reinforcement structure within the beam clearly visible in the analysis.

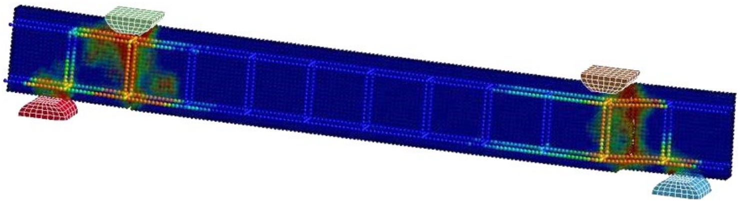


Figure 17: Heterogenous 3D BPM of a reinforced beam under 4-point bending [28]

Figure 18 shows a maize stubble structure made as a BPM.

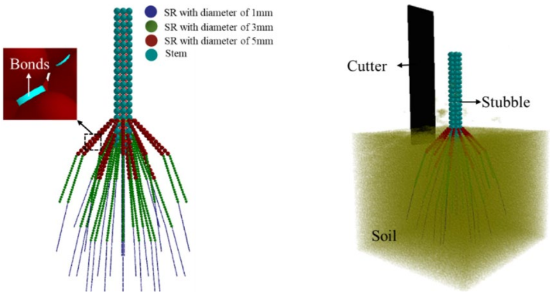


Figure 18: Hybrid/heterogeneous BPM of a maize stubble structure [29]

4.5 3D: Fracturable structures

Applications of rock cutting where the rock is made as a brittle structure are representable examples for fracturable BPMs, as shown in Figure 19.

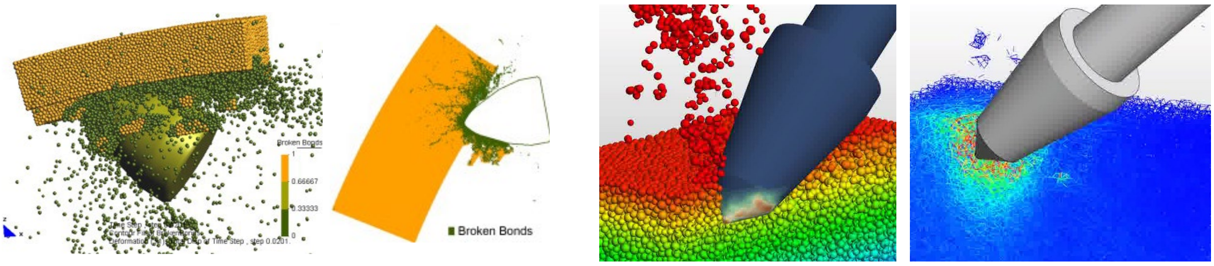


Figure 19: Rock as a fracturable BPM cut with a pick [30], with wear analysis on the tool (right) [26]

Extensive developments and analyses in terms of bonded-particle modelling, specifically regarding enhanced fracture capabilities of BPMs, are presented in the Master's thesis by Platzer [31] (cf. [32]). In addition to defining the fractural behaviour of BPMs in (quasi-)static state via four-point-bending calibration, Platzer introduced a Dynamic Increase Factor (DIF) for BPMs that suppresses breakage of bondings on dynamic impacts. This feature is visualised in Figure 20, showing a BPM without DIF on the left (correspondingly fracturing on impact due to high dynamic load peaks leading to certain bonds to break), the same BPM adapted with a DIF in the middle, thus leading to the BPM to survive this impact without fracture, as it should be the case according to a lab test as shown on the right (which was furthermore reproducible, as shown).

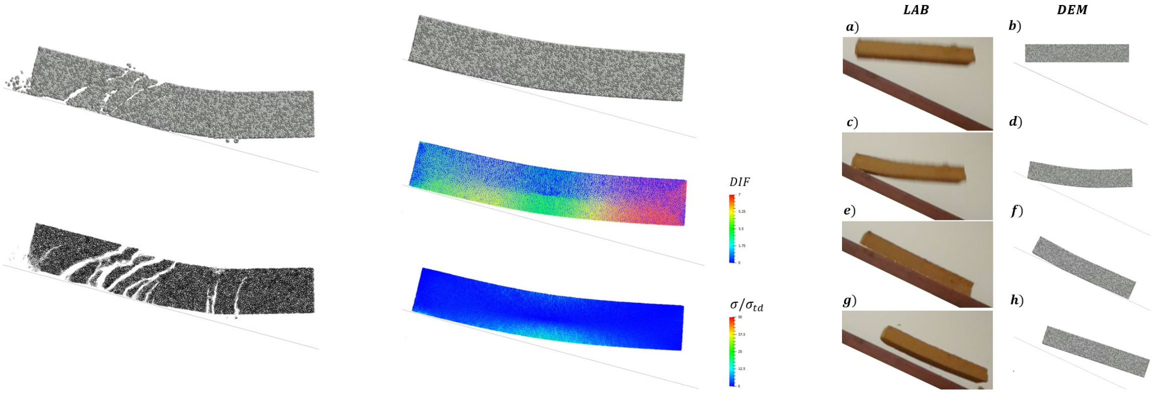


Figure 20: Implementation/effects of the DIF, suppressing BPM breakage due to dynamic loads [31]

In addition, Platzer [31] presented noteworthy advancements:

- An algorithm for transforming complex 3D geometries given in CAD data to BPMs by filling it with particles and adding bondings (as shown in Figure 21).
- Considering mass-volume-related inconsistencies when BPMs break down via allowing initial particle overlaps.
- A cluster detection method for identifying BPMs as individual objects, e.g. for fragment analysis (as shown in Figure 22 on the right).

- The Bonded-Particle Replacement Method (BPRM) for replacing complex-shaped rigid particles with BPMs when certain criteria are met, e.g. when an (initially rigid) particle enters an area where it is required to deform/break, thus replacing it with a corresponding BPM (as shown in Figure 22).

Figure 21: Visualisation of the filling algorithm by Platzer [31]

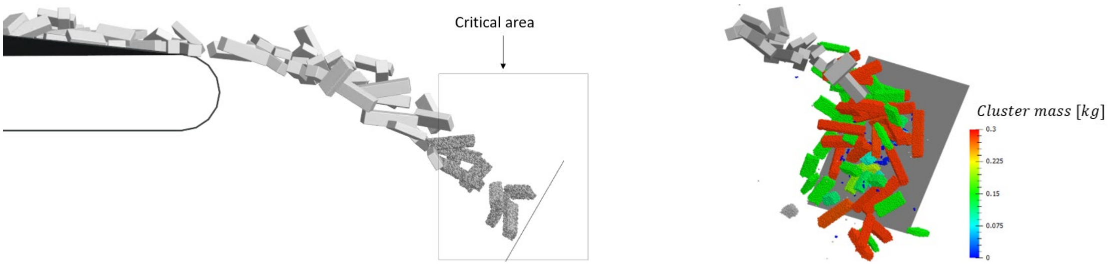


Figure 22: Bonded-Particle Replacement Method (BPRM) left, with rigid particles replaced with BPMs when entering the critical area where breakage is expected to occur (left); and different sizes of BPMs after breakage detected as individual clusters (right) [31]

5 CURRENT DEVELOPMENTS AND POTENTIAL FUTURE TRENDS

Using BPMs shows potential application in several particle simulations where structures need to react to loads, either by deforming or breaking.

Future research could also extend this to liquid-to-solid-structure simulations involving computational fluid dynamics (CFD), in this context particularly smoothed-particle hydrodynamics (SPH).

Further specific bonding adaptations, like the reduction factor discussed in the context of belt conveyor modelling in Chapter 4.3, allow for more nuanced control of the bondings' behaviour within a BPM, which can, for example, consider changing behaviour as depending on aspects such as ageing, weakening, or changing temperature.

Further areas where BPMs can potentially be utilised are for the simulation of sintering, freezing, crystallisation, or similar effects, as bondings between particles can be set at a certain point of the simulation, thus depicting the forming of a larger structure built from smaller particles (a BPM).

6 CONCLUSIONS

This paper highlights the versatility and practicality of using bonded-particle modelling to set up non-common structures in DEM simulations. More specifically, this regards the extension of said structures to react with enhanced complex capabilities in terms of deformation and even fracture behaviour.

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