A Perspective of Recent Developments in the Finite Element Simulation of Metal Forming Processes

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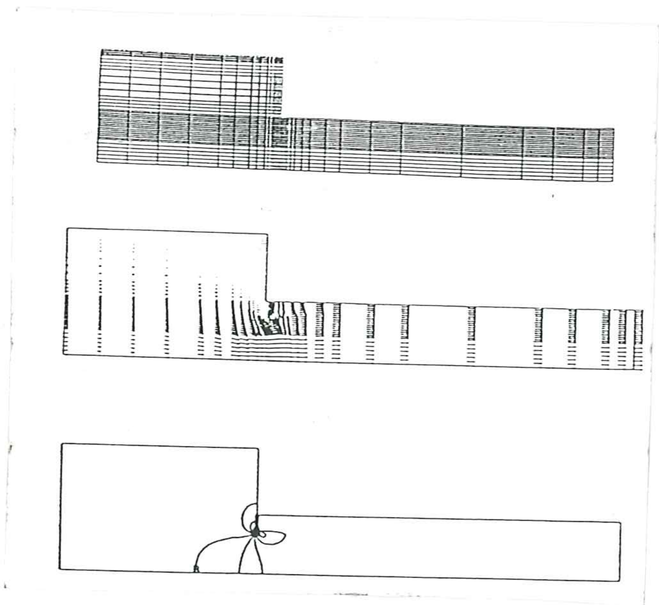

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A PERSPECTIVE OF RECENT DEVELOPMENTS IN THE FINITE ELEMENT SIMULATION OF METAL FORMING PROCESSES

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ABSTRACT

This paper presents an overview of different computational procedures for finite element analysis of metal forming problems. Both the displacement and flow aproaches are discussed together with other aspects like the treatment of temperature coupling, the techniques for geometry updating, the treatment of contact and friction, the use of quasi-static versus explicit dynamic methods and other topics of interest. Examples of applications of some of the methods proposed to extrusion, rolling, mould filling and sheet metal forming problems are presented

1. INTRODUCTION

The detailed study of metal deformation during forming processes is of great interest for many industrial applications. Examples of these kind of problems are found in the compaction of metal powers, in the filling of moulds and solidification in casting, in rolling, extrusion and forming processes and in sheet metal forming, amongst many others.

Despite of its practical interest, the development of reliable numerical procedures to predict the behaviour of metal deformation processes has encountered many serious obstacles. Together with the non linearity of the material, other important effects like the unsteady nature of the process, the large magnitude of the strains involved, the coupling with thermal effects and the importance of contact and friction at the metal-sheet interface make the study of metal forming processes so complex that its analysis justifies the use of sophisticated finite element numerical algorithms and usually leads to large scale computer requirements.

In most metal forming problems the material is subjected to a continuous deformation which induces a rapid plastification of the material. The equations of motion of this transient problem can be written in terms of the displacements of the metal points measured from an appropiate reference configuration (displacement approach) or in terms of their velocities at each deforming configuration (flow approach).

Both displacement and flow approaches can make use of elasto-plastic/viscoplastic or rigid-plastic/visco-plastic constitutive models. Also, the equations of motion can be of "quasi-static" type or else incorporate dynamic effects.

This paper presents an overview of the some of most popular alternatives for the finite element solution of metal forming problems. Both the displacement and the flow approaches are briefly presented and compared.

Other topics discussed are the treatment of incompressibility effects, the inclusion of thermal-coupling, the techniques for geometry updating, the treatment of contact and friction and the use of quasistatic versus explicit dynamic methods. Some examples of application of the methods presented to extrusion, rolling, mould filling and sheet forming problems are also included.

2. DISPLACEMENT APPROACH

This approach uses a total or updated description of the motion. The basic variables are the displacements, $\mathbf{u}$, of the deforming metal and these are related to the strains, $\boldsymbol{\varepsilon}$, by standard non linear kinematic expressions (ref. 1). On the other hand, the constitutive equations relating the appropiate stress measures, $\boldsymbol{\sigma}$, and the strains are usually written in a rate (incremental) objective form to allow for large strain computations. Here both elasto-plastic and elasto-viscoplastic constitutive models have been used with success for different sheet forming situations (refs. 2-5). Finally, the equilibrium equations can be written point wise through the adequate differential equations to be satisfied in the *reference* geometrical configuration, or (what is more usual) in a global sense through the principle of virtual work (PVW). Table 1 presents in a schematic form the basic equations of the displacement approach.

Note that in Table 1 $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}$ represent adequate conjugate stress and strain measures. The form of the PVW as written in Table 1 corresponds to the use of 2d Piola-Kirchhoff stresses and Green-Lagrange strains for $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}$, respectively. The expression of the PVW for other stress and strains definitions (i.e. Cauchy stresses and Almansi strains, etc.) can be found in many text books. (ref. 30).

3. FLOW APPROACH

This approach is typical of fluid mechanics problems where a fixed Eulerian reference frame defining a control volume through which the material flows is generally used. This method appears to be more natural for bulk forming problems like extrusion, rolling, forging, mould filling, etc. (refs. 2-7). However, it can be also applied to sheet forming problems in a straight forward manner, simply by identifying the control volume with the sheet geometry at each deforming step (refs. 8-10).

Table 1 shows also the basic equations of the flow approach. The main variables are now the *velocities* of the deforming body , $\dot{\mathbf{u}}$, and these are linearly related to the rates of deformation, $\dot{\boldsymbol{\varepsilon}}$, in a standard linear manner. The equilibrium and PVW equations are written in terms of the Cauchy stresses, $\boldsymbol{\sigma}$, showing a clear analogy with the corresponding equations for the solid approach. Note, however that V and Γ_t denote now the body volume and the traction prescribed surface in the *current* deforming configuration.

The constitutive equation for the flow approach can be written in rate form on the basis of elasto-plastic and elasto-viscoplastic constitutives models (ref. 11). However, the

DISPLACEMENT APPROACH	*FLOW APPROACH*
Displacements $\mathbf{u}$	Velocities $\dot{\mathbf{u}}$
Kinematic Equations	
$\boldsymbol{\epsilon} = (\mathbf{L} + \bar{\mathbf{L}}(\mathbf{u}))\mathbf{u}$	$\dot{\boldsymbol{\epsilon}} = \mathbf{L}\dot{\mathbf{u}}$
Equilibrium Equations	
$\mathbf{L}^T\boldsymbol{\sigma} + \mathbf{b} = \mathbf{0}$ in volume V $\mathbf{M}^T\boldsymbol{\sigma} = \mathbf{t}$ in boundary Γ_t $\mathbf{b}$: body forces, $\mathbf{t}$: surface loads $\mathbf{M}$ contains components of the unit normal to Γ_t	
Virtual Work	
$\int_V \delta\boldsymbol{\epsilon}^T\boldsymbol{\sigma}dV =$	$\int_V \delta\dot{\boldsymbol{\epsilon}}^T\boldsymbol{\sigma}dV =$
$= \int_V \delta\mathbf{u}^T\mathbf{b}dV + \int_{\Gamma_t} \delta\mathbf{u}^T\mathbf{t}d\Gamma$	$= \int_V \delta\dot{\mathbf{u}}^T\mathbf{b}dV + \int_{\Gamma_t} \delta\dot{\mathbf{u}}^T\mathbf{t}d\Gamma$
Constitutive Equations	
$\overset{\nabla}{\boldsymbol{\sigma}} = \mathbf{D}_T\dot{\boldsymbol{\epsilon}}$	$\boldsymbol{\sigma} = \mathbf{D}\dot{\boldsymbol{\epsilon}}$

Table 1. Basic equations of displacement and flow approaches.

numerical solution of the flow problem can be substantially simplified if a direct relationship between Cauchy stresses and strain rates can be obtained in the form

$$\boldsymbol{\sigma} = \mathbf{D}\,\dot{\boldsymbol{\varepsilon}} \tag{1}$$

Eq.(1) is typical of fluid mechanic problems where $\mathbf{D}$ in an appropiate constitutive matrix depending on the flow viscosity only (ref. 1).

It can be shown that eq.(1) is readily obtained for rigid-plastic and rigid-viscoplastic materials in which elastic effects have been neglected (ref. 12). In the isotropic case matrix $\mathbf{D}$ is a function of a single flow viscosity parameter, μ, given for a rigid-plastic Von-Mises material by (refs. 6,12)

$$\mu = \frac{\sigma_y}{3\dot{\bar{\varepsilon}}} \tag{2}$$

where σ_y is the Von-Mises yield stress and $\dot{\bar{\varepsilon}} = \left(\frac{2}{3}\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij}\right)^{1/2}$.

REMARK 1

Eq.(2) defines a non linear viscosity thus implying a non-Newtonian type of flow. The expression of μ for viscoplastic materials including the effect of microscopic voids can be found in (refs. 12,13). Also note that rigid zones are characterized by $\dot{\bar{\varepsilon}} = 0$ which leads to $\mu = \infty$. Therefore, a cut off value of μ should be used in these zones to prevent singularity.

REMARK 2

The form of the constitutive equation (2) for Von-Mises metals defines an *incompressible flow* problem (i.e. $\dot{\varepsilon}_{ii} = 0$). This introduces serious difficulties problems if the finite element solution is based on "solid" elements and appropiate penalty or mixed type formulations must be used as described in next section.

REMARK 3

It is interesting to note that the overall equations of the flow approach as written in Table 1, are analogous to those of standard infinitesimal (incompressible) elasticity (refs. 1.6.7). This analogy can be exploited to simplify further the computational procedure by directly using standard finite element codes written for the elasticity case simply replacing displacements and strain by velocities and strain rates, respectively, and the shear modulus by the (non linear) flow viscosity (refs. 1, 6-9).

4. TREATMENT OF INCOMPRESIBILITY EFFECTS

Several techniques have been used to deal with incompressibility effects induced by plastic deformation in metal forming processes.

In the *flow approach* techniques based on penalty methods, Lagrange multipliers, and augmented Lagrangian methods have been used with different degree of success to impose the incompressibility constraint in the metal flow (ref. 1). Perphaps the most popular approach in last 15 years is the penalty method since it allows to formulate the discretized problem in terms of velocity variables only. However, this approach must be used in conjuntion with a reduced integration of the volumetric stiffness terms to avoid numerical locking of the solution. The current tendency is to avoid the use of reduced integration based techniques since they can lead to rank-deficiency in the element matrices. This can be achieved by using mixed velocity-pressure interpolations satisfying the so called div-stability (or LBB) conditions (refs. 31-33). However, the use of a discontinuous pressure interpolation allows to eliminate the pressure variables at element level, yielding the so called B-bar method in which velocities remain as nodal variables only (refs. 1,16). Special reference should be made to the analysis of sheet metal forming problems using a flow formulations and "shell type" elements. Here the incompressibility constraint can be simply imposed by setting the Poisson's ratio equal to 0.5 in the analogous elastic problem and then updating the element thickness making use of the plane stress condition (refs. 8,9,13)

In the displacement approach the same techniques as mentioned for the flow case can be used. The main difficulty here lays in the co-existence of an incompressible (plastic) part of the deformation with the remaining elastic part, in presence of large strains. The more promising techniques nowdays are perhaps those based on mixed methods (ref. 14) and on an extension of the B-bar method to large strain computations (refs. 15,16)

5. TREATMENT OF THERMAL COUPLING EFFECTS

It is clear that temperature plays an important role in bulk metal forming problems. Most of these processes take place at high temperatures, also the internal heat generated during the deformation process can not be neglected in many cases, and in turn, temperature affects strongly the material properties of the metal. The process is, therefore, fully coupled and the equations describing the deformation of the metal must be solved jointly with the heat balance equation which can be written as

$$\rho c \left[\frac{\partial \phi}{\partial t} + \underline{\dot{\mathbf{u}}^T \nabla \phi} \right] = \nabla^T \bar{\mathbf{D}} \nabla \phi + Q \tag{3}$$

In (3) ϕ is the temperature, ρ and c the density and the specific heat, respectively, $\bar{\mathbf{D}}$ is a diagonal matrix of thermal conductivities, ∇ is the gradient operator and Q is the internal heat generated in the metal deformation process computed as a fraction ($\simeq 90\%$) of the plastic work developed. Eq.(3) is completed with the adequate boundary conditions of temperature and heat flux along the domain boundaries, and also with the known temperature values at the initial time.

REMARK 4

The underlined convective term in (3) would be equal to zero if a Lagrangian description of the movement is adopted as is the case in the displacement formulation.

The temperature field is discretized in a finite element form using generally the same interpolation as for the velocities. Following standard weighted residual procedures the following system of equations is obtained (refs, 1,7)

$$\mathbf{C}\dot{\boldsymbol{\phi}} + \mathbf{H}(\dot{\mathbf{u}})\boldsymbol{\phi} = \bar{\mathbf{f}} \tag{4}$$

In (4) $\mathbf{C}$ is the standard heat capacity matrix, $\bar{\mathbf{f}}$ is the force vector due to the heat sources and matrix $\mathbf{H}$ contains contributions from the heat conduction and convection terms. Note that the convective terms must be carefully treated using Petrov-Galerkin or similar techniques to avoid numerical in instabilities (ref. 1). These problems disappear if a displacement formulation is used.

Eq.(4) must be solved together with the equations resulting from the finite element discretization of the metal deformation problem. This process is briefly sketched in next section

6. FINITE ELEMENT DISCRETIZATION. COMPUTATIONAL ASPECTS

For both solid and flow approaches the resulting non linear equilibrium equations can be written after finite element discretization in the form (see Table 2)

$$\mathbf{r}(\mathbf{a},\mathbf{x},t)=\mathbf{p}(\mathbf{a},\mathbf{x})-\mathbf{f}(t,\mathbf{x})=0 \tag{5}$$

where $\mathbf{r}$, $\mathbf{p}$ and $\mathbf{f}$ stand for the vectors of residual forces, internal forces and external forces, respectively, $\mathbf{a}$ are displacements and velocities in the displacement and flow approaches respectively, $\mathbf{x}$ is the cartesian coordinate vector and, t represents the time in the flow approach and the load increment in the solid approach.

Eq. (5) can be iteratively solved for the values of vector $\mathbf{a}$. For the kth iteration we have

$$\Delta\mathbf{a}^k=-\left[{}^{t+\Delta t}\mathbf{S}^k\right]^{-1}\,{}^{t+\Delta t}\mathbf{r}^k \tag{6}$$

where $\mathbf{S}$ in an adequate iteration matrix. Vector $\mathbf{a}$ is subsequently updated as

$${}^{t+\Delta t}\mathbf{a}^{k+1}={}^{t+\Delta t}\mathbf{a}^k+\Delta\mathbf{a}^k \tag{7}$$

The next step is to compute the new stress field. In the rigid/plastic-viscoplastic flow approach the stresses are *directly* obtained from the updated velocity field (see Table 2). In the displacement approach (or in the elasto-plastic/viscoplastic flow case (ref. 11)) the computation of stresses implies the integration of the rate constitutive equations.

The final step is to update the metal geometry. This can be simply done from the displacement and velocity fields as shown in Table 2. However other techiques are possible as explained in next section. At this stage the mechanical properties are also updated and the contact and friction conditions are checked (see Section 7).

The updated geometrical and mechanical properties of the metal are used to compute the new increments $\Delta\mathbf{a}^k$ of the next iteration from (6). The process is continued until convergence is achieved. This is usually measured by the satisfaction of (5) using a mean quadratic norm for the residual forces.

If temperature coupling effects are present the discretized heat balance equations (4) must be solved jointly with the set (5). The simplest approach is to use a "staggered" scheme in which the computed velocity (or displacement) values at each iteration are used as inputs in eq.(4) to solve for the nodal temperatures. Thus for the quasi-static case we have

$${}^{t+\Delta t}\boldsymbol{\phi}^{k+1}=[{}^{t+\Delta t}\mathbf{H}^{k+1}]^{-1}\,{}^{t+\Delta t}\bar{\mathbf{f}}^{k+1} \tag{8}$$

where ${}^{t+\Delta t}\mathbf{H}^{k+1}$ and ${}^{t+\Delta t}\bar{\mathbf{f}}^{k+1}$ contain the contributions from the solution of the mechanical problem at the kth iteration. The temperatures provided by (8) are then used as data for the evaluation of the material properties in the next iteration of eq.(6). This process is repeated until convergence of both velocities (or displacements) and temperature fields is attained. The overall solution process for both the displacement and flow approaches is sketched in Table 2.

The detailed discussion of the computational aspects of the finite element solution falls outside the scope of this paper. Details of the different matrices and vectors appearing in Table 2 can be found in (ref. 1).

Nevertheless the following remarks should be noted at this point.

a) The iteration matrix $\mathbf{S}$ in the solid approach is usually taken as the tangent stiffness matrix computed as $\mathbf{S} = \frac{\partial \mathbf{p}}{\partial \mathbf{a}}$. The solution algorithm coincides in this case with the standard Newton-Raphson iteration scheme.

b) The selection of matrix $\mathbf{S}$ in the flow approach can be made on the basis of *secant* or *tangent* iteration procedures. The expression of $\mathbf{S}$ for the secant case coincides with that of the finite element stiffness matrix $\mathbf{K}$ for standard infinitesimal elasticity (ref. 6-10).

The form of the exact tangent matrix is complex and generally non-symmetric and approximate simpler forms are used (refs. 10,17).

c) The use of an explicit time integrations scheme ($\theta = 0$) in the flow approach results in an iterative algorithm in which the geometry at $t + \Delta t$ is known "a priori" and kept fixed during the iterations and the velocities at $t + \Delta t$ are the only possible unknows. However, in the implicit case ($\theta \neq 0$) the sheet coordinates at $t + \Delta t$ change during the iteration process and this would allow to formulate the problem in terms of the velocities at $t + \Delta t$ (as shown in Table 3), or in terms of the displacement increments between the two configurations at t and $t + \Delta t$, by noting that the geometry updating equation can be written in the form

$$\Delta \mathbf{u} = ({}^t\mathbf{a} + \Delta \mathbf{a} \theta) \Delta t \tag{9}$$

where

$$\begin{aligned} \Delta \mathbf{u} &= {}^{t+\Delta t}\mathbf{x}^{k+1} - {}^t\mathbf{x} \\ \Delta \mathbf{a} &= {}^{t+\Delta t}\mathbf{a}^{k+1} - {}^t\mathbf{a} \end{aligned} \tag{10}$$

are the displacement and velocity increment vectors, respectively. This alternative has not been fully exploited in practice and it opens new possibilities for research.

d) The terms "explicit" and "implicit" in Table 2 refer to the integration scheme chosen for updating the geometry in the flow approach. This should not be mixed up with the so called "explicit dynamic" methods based on the solution of the full second order dynamic equations with inclusion of inertia terms using an explicit backwards integration scheme. This possibility is briefly discussed later in Section 9.

Moreover note that in the quasi-static flow approach the "explicit" time integration scheme still involves necessarily the iterative solution of a system of equations for the converged velocities at $t + \Delta t$ keeping a fixed value of the geometry during the iterations.

DISPLACEMENT APPROACH	FLOW APPROACH

Basic discretization

Displacement approach	Flow approach
$\mathbf{u} = \mathbf{N}\mathbf{a}$	$\dot{\mathbf{u}} = \mathbf{N}\mathbf{a}$
$\boldsymbol{\epsilon} = \mathbf{B}_s\mathbf{a}$	$\dot{\boldsymbol{\epsilon}} = \mathbf{B}_f\mathbf{a}$
$\overset{\triangledown}{\boldsymbol{\sigma}} = \mathbf{D}_T\dot{\boldsymbol{\epsilon}} = \mathbf{D}_T\bar{\mathbf{B}}_s\mathbf{a}$	$\boldsymbol{\sigma} = \mathbf{D}\dot{\boldsymbol{\epsilon}} = \mathbf{D}\mathbf{B}_f\mathbf{a}$

Equilibrium Equation of Mechanical Problem

$$\mathbf{r} = \mathbf{p} - \mathbf{f} = \mathbf{0}$$

Displacement approach	Flow approach
$\mathbf{p} = \int_V \bar{\mathbf{B}}_s^T \boldsymbol{\sigma} dV$	$\mathbf{p} = \left(\int_V \mathbf{B}_f^T \mathbf{D}\mathbf{B}_f dV\right)\mathbf{a} = \mathbf{K}\mathbf{a}$

$$\mathbf{f} = \int_V \mathbf{N}^T\mathbf{b}dV + \int_s \mathbf{N}^T\mathbf{t}ds$$

Solution Procedure

Displacement approach	Flow approach
${}^{t+\Delta t}\mathbf{a}^o = \mathbf{0}$	${}^{t+\Delta t}\mathbf{a}^o = {}^t\mathbf{a}$
${}^{t+\Delta t}\mathbf{x}^o = {}^t\mathbf{x}$	${}^{t+\Delta t}\mathbf{x}^o = {}^t\mathbf{x} + {}^t\mathbf{a}\Delta t$

$$\boxed{\texttt{LOOP} \quad \texttt{k} = \texttt{1, NITER}}$$

$$\Delta\mathbf{a}^k = -\left[{}^{t+\Delta t}\mathbf{S}^k\right]^{-1}\left({}^{t+\Delta t}\mathbf{r}^k\right)$$

$${}^{t+\Delta t}\mathbf{a}^{k+1} = {}^{t+\Delta t}\mathbf{a}^k + \Delta\mathbf{a}^k$$

Displacement approach	Flow approach
${}^{t+\Delta t}\boldsymbol{\sigma}^k$ are computed by integrating the constitutive equations	${}^{t+\Delta t}\boldsymbol{\sigma}^{k+1} = \mathbf{D}\mathbf{B}_f\,{}^{t+\Delta t}\mathbf{a}^{k+1}$

Geometry Updating

Displacement approach	Flow approach
${}^{t+\Delta t}\mathbf{x}^{k+1} = {}^{t+\Delta t}\mathbf{x}^k + \Delta\mathbf{a}^k$	${}^{t+\Delta t}\mathbf{x}^{k+1} = {}^t\mathbf{x} + {}^t\mathbf{a}(1-\theta)\Delta t +$
	$+{}^{t+\Delta t}\mathbf{a}^{k+1}\theta\Delta t$
	$\theta = 0$ explicit solution $\theta \neq 0$ implicit solution

CHECK CONTACT

Compute ${}^{t+\Delta t}\mathbf{r}^{k+1}$, ${}^{t+\Delta t}\mathbf{f}^{k+1}$, ${}^{t+\Delta t}\mathbf{H}^{k+1}$ and ${}^{t+\Delta t}\bar{\mathbf{f}}^{k+1}$

Compute temperatures ${}^{t+\Delta t}\boldsymbol{\phi}^{k+1} = \left[{}^{t+\Delta t}\mathbf{H}^{k+1}\right]^{-1}\,{}^{t+\Delta t}\bar{\mathbf{f}}^{k+1}$

Compute error norm $E = \min\left(\frac{\|{}^{t+\Delta t}\mathbf{r}^{k+1}\|}{\|{}^{t+\Delta t}\mathbf{f}^{k+1}\|}, \|{}^{t+\Delta t}\boldsymbol{\phi}^{k+1} - {}^{t+\Delta t}\boldsymbol{\phi}^k\|\right)$

$E.GT.TOL. \xrightarrow{yes}$ continue iterations $k = k+1$

$\downarrow$ *no*

INITIATE NEW LOAD INCREMENT OR TIME STEP

Table 2. Quasi-static finite element solution algorithm for solid and flow approaches.

EXPLICIT DYNAMIC SOLUTION

DISPLACEMENT APPROACH	*FLOW APPROACH*
$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{p}(\mathbf{u}) = \mathbf{f}$	$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{p}(\dot{\mathbf{u}}) = \mathbf{f}$
$\mathbf{M}_D = \text{ diag } \mathbf{M}$	$\mathbf{p}(\dot{\mathbf{u}}) = \mathbf{K}(\dot{\mathbf{u}})\dot{\mathbf{u}}$

$(\cdot)^{n-1}$: Known values at time $n-1$

$$\mathbf{u}^n = \mathbf{u}^{n-1} + \Delta t \dot{\mathbf{u}}^{n-1} + \frac{\Delta t^2}{2}\ddot{\mathbf{u}}^{n-1}$$

Compute $\mathbf{x}^n, \mathbf{f}^n, \mathbf{p}^n$	Compute $\mathbf{x}^n, \mathbf{f}^n$
	Iterate for $\dot{\mathbf{u}}^n$, $\ddot{\mathbf{u}}^n$ (i=1, NITER)
$\ddot{\mathbf{u}}^n = \mathbf{M}_D^{-1}[\mathbf{f}^n - \mathbf{p}^n]$	$\ddot{\mathbf{u}}^n_{i+1} = [\mathbf{M} + \frac{\Delta t}{2}\mathbf{K}^n_i]^{-1}\left\{\mathbf{f}^n - \mathbf{K}^n_i(\dot{\mathbf{u}}^{n-1} + \frac{\Delta t}{2}\ddot{\mathbf{u}}^{n-1})\right\}$
$\dot{\mathbf{u}}^n = \dot{\mathbf{u}}^{n-1} + \frac{\Delta t}{2}(\ddot{\mathbf{u}}^{n-1} + \ddot{\mathbf{u}}^n)$	$\dot{\mathbf{u}}^n_{i+1} = \dot{\mathbf{u}}^{n-1} + \frac{\Delta t}{2}(\ddot{\mathbf{u}}^{n-1} + \ddot{\mathbf{u}}^n_{i+1})$

CHECK CONTACT

NEW SOLUTION AT TIME $n+1$

Table 3. Explicit dynamic solution in the displacement and flow approaches.

7. TECHNIQUES FOR GEOMETRY UPDATING

The simplest technique to update the metal geometry during the deformation process is to use a Lagrangean approach. In this, the computed velocity (or displacement) field at each time step is used to update the nodal grid points as shown in Table 2. This process may lead to distorted meshes for highly deforming stages and remeshing is then necessary.

A secong alternative is to use the so called Arbitrary Lagrangean-Eulerian (ALE) approach. Here the mesh grid nodes are decoupled from the velocity grid nodes, thus allowing for a relative mouvement between the two grids. This technique is appropiate for problems where only a small percentage of the total domain (generally the surface boundary) changes its shape during the deformation. This part of the mesh is then updated in a Lagrangean manner whereas a fixed Eulerian mesh is kept in the rest of the domain (ref. 25).

A third alternative is worth being mentioned here. This is the so called "pseudo-concentrations"method (refs. 25-29). The basic idea is to introduce a scalar function which is advected through a fixed mesh accordingly to the velocity field obtained from the solution of the flow problem. This function is defined in the whole domain and a certain isovalue is used to define the front of the deforming metal. The unfilled region is assumed to be occupied by a fictitious material (usually taken as *air*) whose physical properties are such that its motion does not effects the dynamical behaviour of the moving metal. In practice this implies to solve the following transport equation

$$\frac{\partial \psi}{\partial t} + \mathbf{u}^T \boldsymbol{\nabla} \psi = 0 \tag{11}$$

Where ψ is the scalar pseudo-concentration function defining the presence or absence of metal. For instance we may assign the value $\psi = \psi_c$ to the position of the metal front, such that the metal filled region will be identified by the values $\psi > \psi_c$ and the position of the air by $\psi < \psi_c$.

The finite element discretization of eq.(11) yields the following systems of equations

$$\bar{\mathbf{C}}\dot{\boldsymbol{\psi}} + \mathbf{R}(\dot{\mathbf{u}}) = \hat{\mathbf{f}} \tag{12}$$

where $\boldsymbol{\psi}$ denotes the nodal values of the pseudo- concentration function.

Eq.(12) is solved once the velocity field has been obtained so that the metal front can be appropiately updated for each time step. Note that eq.(11) is of hyperbolic type and therefore techniques to avoid numerical instabilities must be used (refs.). Moreover, matrix $\mathbf{R}$ in (12) is non-symmetric which introduces an aditional (although small) problem. Despite of these apparent difficulties, the "pseudo-concentration"method has attracted much attention in recent years for the solution of moving free surfaces in metal forming problems (refs. 25-29).

8. TREATMENT OF FRICTIONAL CONTACT

Contact and friction appears as a consequence of the interaction between different bodies. Such interaction is typical of metal forming problems. During the forming process the metal interacts with the tools, adding a new source of complexity to the numerical simulation due to the nonlinear nature of the boundary conditions. The numerical treatment of frictional contact problems involves two main steps. First, a *contact search* procedure must be done in order to detect the penetrations (kinematic incompatibilities) between the different bodies involved in the analysis. Second, the penetrations detected must be canceled and the *kinematic compatibility constraints* must be satisfied.

Different formulations for the numerical analysis of frictional contact problems have been proposed. In the *penalty method* a penalized functional is added to the standard functional of the unconstrained problem. The main drawback of this method is that the constraints are exactly satisfied for *infinite* values of the penalty parameter only which leads to an *infinite* ill-conditioning of the tangent operator. Otherwise, this is a very simple way to enforce the constraints and it is quite easy to implement.

Frictional contact models can be described using a plasticity theory framework where the penalty or regularization parameters may be viewed as constitutive parameters (refs. 18,19).

In the *Lagrange multipliers technique* a new field (the multipliers) is introduced by means of a contact functional. This leads to an increase of the number of the unknowns and of the system of equations to be solved. Furthermore the tangent operator is indefinite (zero diagonal block associated with the multipliers) and special care must be taken during the solution process. Its main advantage is that the constraints are satisfied *exactly*.

Using the *perturbed Lagrange multipliers* method one can bypass this drawback as the tangent operator is definite. With this approach both the penalty and Lagrange multipliers methods can be formulated in an unified manner (ref. 20).

In the *augmented Lagrangian method*, traditionally used in conjunction with *Uzawa's algorithm*, the constraints are satisfied exactly at *finite* values of the penalty parameter. This overcomes the problems associated to the choice of the penalty parameter and the ill-conditioning of the tangent operator early mentioned. However, no increase of number of the equations to be solved is produced and the multipliers are simply updated after each converged equilibrium step (nested Uzawa's algorithm) or after each equilibrium iteration (simultaneous Uzawa's algorithm) (ref. 21). In the first case an outer loop is needed but otherwise quadratic rate of convergence must be expected if consistent tangent operators have been used. In the later, no extra loops are needed but the update of the multipliers destroys the quadratic rate of asymptotic convergence of the consistent Newton-Raphson scheme (ref. 21).

Different Augmented Lagrangian formulations for frictional contact problems have been recently proposed (ref. 21,22).

In the context of frictionless contact problems a formulation based on a three-field Hu-Washizu type funcional has been proposed recently by Papadopoulus and Taylor (ref. 22). In such a formulation contact between elements rather than between node and elements are postulated, introducing an assumed gap function that is taken as an independent variable in the formulation.

A similar procedure was previously proposed by Wriggers et al. (ref. 23) using a two-field functional.

9. QUASI-STATIC VERSUS DYNAMIC METHODS

As shown in Table 2 the equations of motion must be integrated in both space and in time. In most metal forming problems this integrations needs not to include inertia contributions as they are negligible (ref. 24). The *quasi-static approach* shown in Table 2 is thus the more "natural" approach. Here the satisfaction of equilibrium conditions at time $t + \Delta t$ requires *always* the use of an iterative procedure. The use of iterative methods incorporating conjugate gradient techniques can been employed to eliminate the obstacles associated with the solution of the large system of equations typical of tridimensional problems. Unfortunately, the stiffness matrix of thin elements can be ill-conditioned which makes these iterative methods less effective for sheet metal forming problems.

Explicit dynamic methods have recently become very popular for the displacement approach as they do not require the solution of a system of equations. The basis is the solution of the dynamic equilibrium equations at time t, using an explicit integration scheme with a diagonal mass matrix The basic algorithm is shown in Table 3 for both the dispacement and flow aproaches neglecting temperture effects. The advantages of displacement-based explicit dynamic methods is that the stiffness matrix does not need to be formed and that contact conditions are accurately modelled because of the requirements of the small time steps.

Moreover, they can be easily parallelized in SIMD parallel computers. However very small time steps are required or the solution will become unstable and will grow without bound. Another drawback of explicit dynamic methods is related to the difficulties of consistently predicting the onset of local instabilities. Also the computations of spring back effects in sheet forming problems requires the additions of "ad-hoc" damping terms which are difficult to quantify in a rigourous manner.

The current debate between displacement based implicit quasi-static versus explicit dynamic methods will be clarified when improvements in the parallelization of iterative solvers for implicit codes are implemented and tested. It is envisaged that an "optimized" implicit code could provide a more suitable procedure for the effective analysis metal forming problems.

The inclusion of inertia effects in the flow approach has not received much attention although an attempt was reported by in ref. 17 to treat impact problems in automobile structures. This is most probably due to the fact that even if an explicit algorithm is used, non linear system of equations must be solved for each time step (see Table 3), thus destroying all the advantages of the direct explicit solutions also in the displacement approach.

10. EXAMPLES

The first example, shown in Figure 1, is the solution of a steady state extrusion problem. This problem has been solved with the flow approach using Q_2/P_1 quadrilateral elements with quadratic interpolation for velocities and temperature and a discontinuos linear interpolations for the pressure (refs. 1,29). Details on the geometrical and material properties can be found in ref. 7.

Figure 1a shows the finite element mesh used whereas Figures 1b, 1c and 1d show the velocity, temperature and pressure fields obtained in the computations.

The second example shown in Figure 2 is the mould filling by gravity of a rectangular cavity. The problem is solved with a flow approach and the pseudo-concentration technique described in Section 8. Figure 2a shows the mesh of Q_2/P_1 elements used in the analysis. Figures 2b and 2c show the position of the fluid front and the evolutions of pressure contours. Further information on this example can be found in ref. 29

The third example, shown in Figure 3, is the hot rolling of a rectangular slab. The solution technique has been the same as for the previous example. The geometrical and material properties can be found in ref. 7. Figure 3 shows the position of the metal front and at different times and the temperature contours for the steade state solution.

The final example is the deep drawing of a circular sheet using a cylindrical punch. The problem has been solved whith 3D membrane elements based on the flow approach. For details on the geometrical, material and frictional properties see ref. 34. Figure 4 shows the deformation pattern of the sheet at different forming stages.

11. CONCLUDING REMARKS

Its is clear that the intrinsic difficulties of the numerical analysis of metal forming problems have encouraged the developement of increasingy sophisticated computational

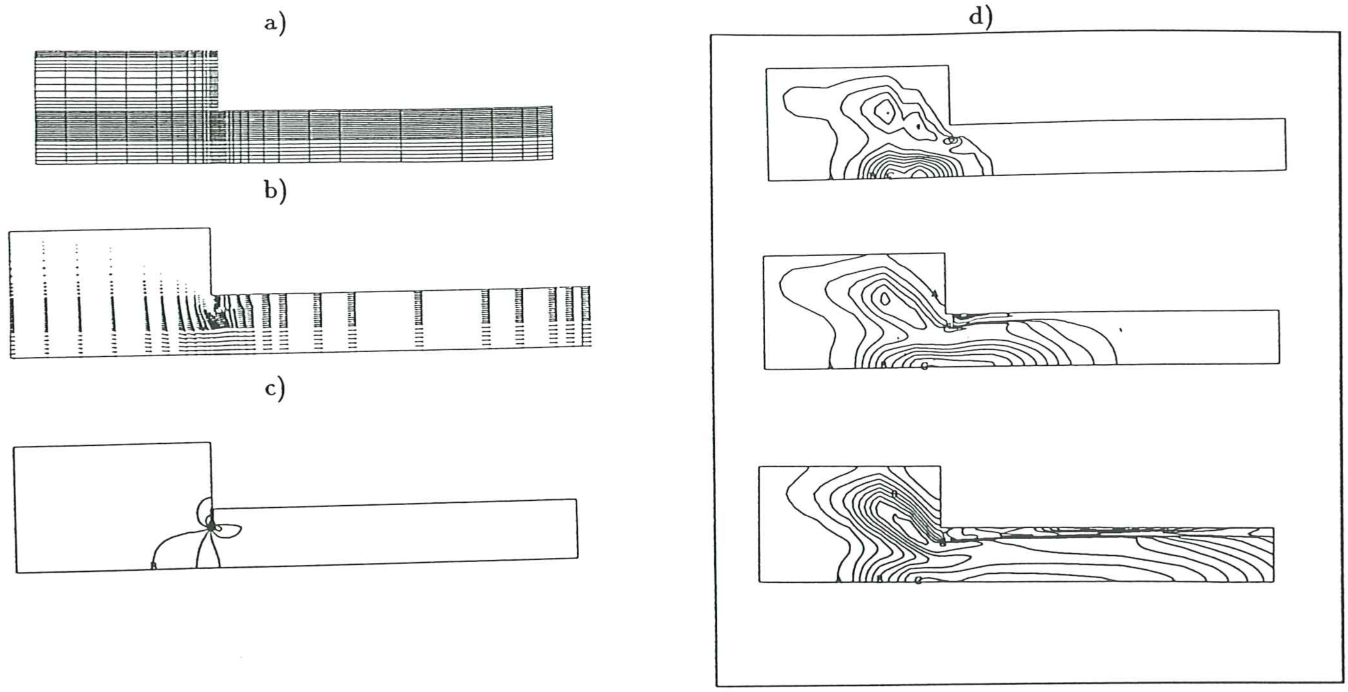


Figure 1. Plane strain extrusion. a) Geometry and mesh of 230 Q_2/P_1 mixed finite elements used. b) velocity field. c) Pressure contours and d) Evolution of temperature contours from initial to steady state stages.

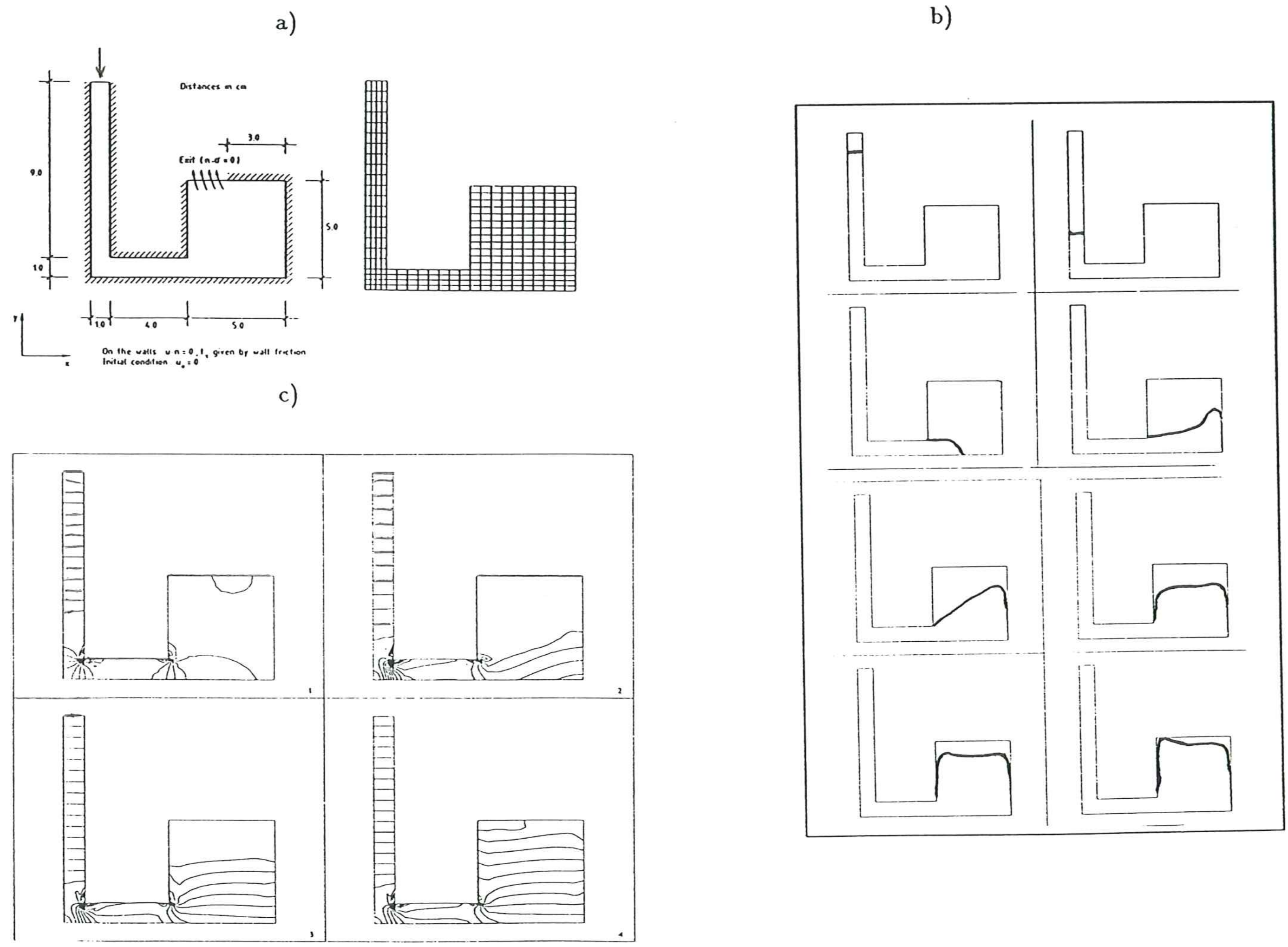
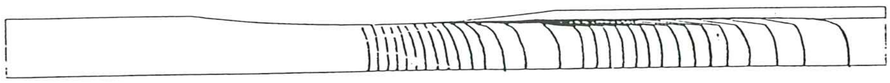


Figure 2. Mould filling by gravity. a) Geometry and mesh of 280 Q_2/P_1 mixed finite elements used. b) Evolution of metal front at different times. c) Pressure contours at different times.

a)

b)



c)

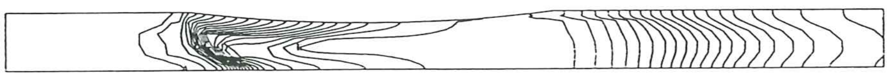


Figure 3. Hot rolling of a rectangular slab. a) Geometry and mesh of 340 Q_2/P_1 mixed finite elements used. b) Evolution of slab front with time. c) Temperature contours for steady state.

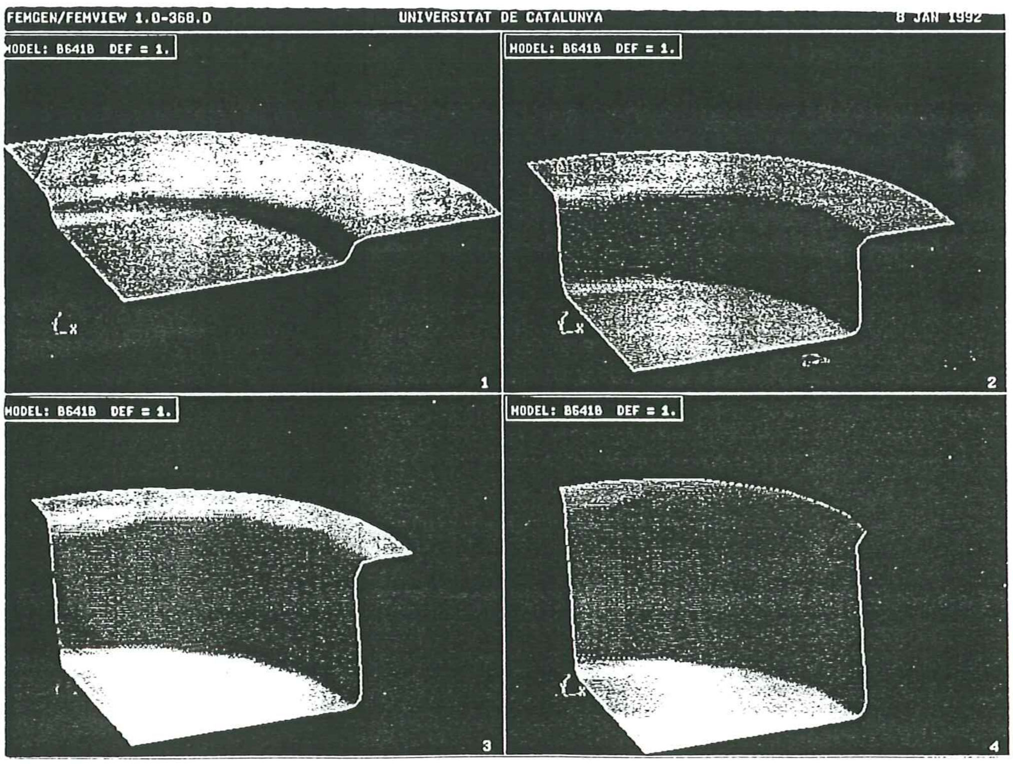


Figure 4. Deep drawing of a circular sheet with a cylindrical punch. Deformation of the sheet at different drawing stages.

procedures and a perspective of some of these has been presented in this paper. However, the solution of real problems demands more efficient numerical procedures which should be able to predict all parameters involved in the forming process in an accurate form. The need for improvements in this area is clear and topics like the development of new robust and accurate finite element methods and cost-efficient computational techniques, compatible with the new parallel machines, will certainly be the object of extensive research in next years.

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