15th World Congress on Computational Mechanics (WCCM-XV)
8th Asian Pacific Congress on Computational Mechanics (APCOM-VIII)
Virtual Congress: 31 July – 5 August 2022
S. Koshizuka (Ed.)

2D ANALYTICAL SOLUTION FOR MULTI-SEGMENTED AUMINIUM-STEEL COMPOSITE PANEL - AN AEROSPACE APPLICATION

Poonam Kumari[1] and Viwek Kumar[2]

[1] Indian Institute of Technology Guwahati
Department of Mechanical Engineering, Indian Institute of Technology Guwahati,
Guwahati 781039, India
kpmech@iitg.ac.in

[2] Indian Institute of Technology Guwahati
Department of Mechanical Engineering, Indian Institute of Technology Guwahati,
Guwahati 781039, India
viwek176103111@iitg.ac.in

Key words: Composites, 2D exact, EKM, Panel, Dissimilar material

Abstract. *E-vehicles and light weight structural parts in automotive and aerospace industry has led to design and development of new structures where traditional materials aluminium/steel are joined with composite laminated materials. This approach has led the engineers/researchers to reduce weight and ultimately save fuel consumption and reduce carbon footprints. Moreover, prosthetic limbs are also designed to have varying material along the length for better suitability. The above problems cannot be analysed using functionally graded theories/concepts. In theory, material properties vary linearly, exponentially, or power-law-like along x-values but for the above cases, material property does not follow a particular variation. Further even, it is not always practical to produce or manufacture components having very smooth variations along the length. Number of research articles are reported on the development of joining techniques for the dissimilar materials.[1,2] Bending, free vibration and buckling analysis is also needed for theses case.[3] At the joining or interface point, local inplane and transverse stresses may rise sharply and may cause debonding and failure of structures. One dimensional analysis based on classical assumption or higher order can not accurately predict the stress behaviour at theses places. In this paper, an attempt is made to develop the 2D analytical solution for multi-segmented Al/ steel-composite panel under transverse loading. Extended Kantorovich method is used for developing governing equations. Continuity of displacement and stresses are satisfied at interface of each segment. Two segmented panels having aluminium/steel equal segment are considered. The deflection and stresses are compared with the finite element solution and found in good agreement.*

1 INTRODUCTION

The demand of motor vehicles nowadays are increased due to various reasons such as population rise, transportation, luxary, etc., which causes a huge consumption of fuel as well. This increase in fuel consumption causes more carbon emissions which affects the human lives. So, it is the biggest challenge for scientists, researchers, and designers to improve the fuel efficiency of the motor vehicles. In this aspect, some new steps are taken every day to improve the fuel efficiency, such as vehicle design modifications, materials advancement, etc. One of the step that is found more effective in reducing fuel consumption is reduction in weight of the vehicle body. Simultaneously, the vehicle body should also have high strength for the safety of passengers. To meet both the demands with cost-effectiveness, Al and its alloy is for the weight reduction, and Steel for the high strength is found more suitable.[4] So, Al-Steel lap and butt joined parts are widely used for the manufacturing of different body parts in automobiles, aerospace, sheet joining industries, etc. However, some parts of composite materials are also used in the car body which needs to be joined with metallic frames. These manufactured parts after assembling may behave as a structural member such as a beam or panel or plate within the system. Where these parts are subjected under different working conditions. So, it must be needed to check the joint strength of the parts under different loading and boundary conditions.

The EKM based solution for bending of edge bonded composite plates with dissimilar dimensions, materials, foundation, and loading conditions have been presented.[1] The mechanical behaviour of similar and dissimilar materials adhesively bonded single lap joints using the finite element method approach is anlyzed.[2] The strengths of adhesive joints consisting of metal and composites using the modified damage zone theory have been predicted.[5] The failure process, mode and strength of unidirectional composite single lap bonded joint with three different bonding methods are investigated.[6] Very recently, a multi-segmental beam model has been developed to analyze the effect of thickness and support conditions on the stresses & deformation at the interfaces for different metal-composite bonded models.[7] In this work, a two segmented panel made of Al and Steel having equal length under different support condition is analyzed.

2 THEORETICAL FORMULATION

A single layered panel with span length a along x-direction and total thickness h along z-direction as shown in figure 1 is considered for the study. It can have any arbitrary boundary conditions at $x = 0$ and a, and is subjected to uniformly distributed pressure loads of q_1 and q_2 applied on the bottom and top surfaces, respectively. The panel has two segments with equal length along x axis as shown in figure 1. The following assumption are made

- Each segment can have different isotropic or orthotropic material.
- The segments are perfectly bonded at the interface along x-axis. Continuity conditions at the interface is satisfied exactly.
- Segment length may be or may not be same and for a typical segment.

The panels having materials with the principal material axis x_1 oriented along the x-direction. The junction between two segment is called as interface. The segment superscript is omitted unless needed for clarity. Here for the analysis only two segments having equal length are considered. Since the width is very long along the y-direction so the displacement along this direction is ignored i.e. plane strain condition.[8] The displacement u and w considered along x-axis and z-axis respectively which are independent of y coordinate.

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{zx} \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \bar{s}_{11} & \bar{s}_{12} & \bar{s}_{13} & 0 & 0 & \bar{s}_{16} \\ \bar{s}_{12} & \bar{s}_{22} & \bar{s}_{23} & 0 & 0 & \bar{s}_{26} \\ \bar{s}_{13} & \bar{s}_{23} & \bar{s}_{33} & 0 & 0 & \bar{s}_{36} \\ 0 & 0 & 0 & \bar{s}_{44} & \bar{s}_{45} & 0 \\ 0 & 0 & 0 & \bar{s}_{45} & \bar{s}_{55} & 0 \\ \bar{s}_{16} & \bar{s}_{26} & \bar{s}_{36} & 0 & 0 & \bar{s}_{66} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{zx} \\ \tau_{xy} \end{bmatrix} \tag{1}$$

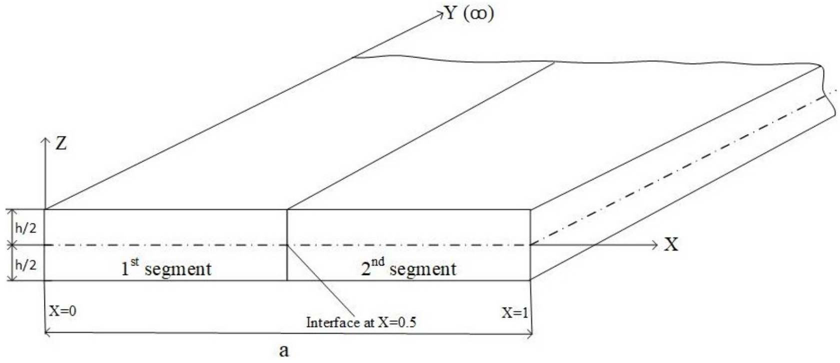

Figure 1: Mathematical geometry of the panel

For such plane strain condition, 3D constitutive equations of an orthotropic lamina, when transformed from the material coordinate system (x_1, x_2, x_3) to the rectangular coordinate system (x, y, z) can be written for panel as in equation 1.[8]

The $\bar{s}_{ij}$ are transformed elastic compliances, whose expressions in terms of the engineering properties, namely, Young's moduli Y_i, shear moduli G_{ij} can be given.[9] To ensure numerical stability in the solution process, all entities are expressed in non-dimensional forms in such a way that, on substitution of dimensionless entities, the form of all governing equations remains unchanged.

The strain-displacement relations are,

$$\varepsilon_x = u_{,x}; \quad \varepsilon_y = 0; \quad \varepsilon_z = w_{,z}; \quad \gamma_{xy} = v_{,x}; \quad \gamma_{yz} = v_{,z}; \quad \gamma_{zx} = w_{,x} + u_{,z} \tag{2}$$

where a subscript comma denotes differentiation.

The mixed principle of virtual work for bending analysis of the panel, without any body force source can be expressed as,[8]

$$\begin{aligned}\int_V [\delta u(\sigma_{x,x} + \tau_{xz,z}) + \delta v(\tau_{xy,x} + \tau_{yz,z}) + \delta w(\tau_{zx,x} + \sigma_{z,z}) + \delta\sigma_x(\varepsilon_x - u_{,x}) + \delta\sigma_z(\varepsilon_z - w_{,z}) + \\ \delta\tau_{yz}(\gamma_{yz} - v_{,z}) + \delta\tau_{xy}(\gamma_{xy} - v_{,x}) + \delta\tau_{zx}(\gamma_{zx} - u_{,z} - w_{,x})]\, dV = 0, \quad \forall\, \delta u, \delta v, \delta w,\ \delta\sigma_i,\ \delta\tau_{ij}\end{aligned} \tag{3}$$

Where V denotes the volume of panel for per unit width along the y-direction. The more details about the method can be found in Ref. [7] and the field variables are solved by using similar approach. The multi-term EKM can be written for the field variables as in equation 4.

$$X_l = \sum_{i=1}^{n} f_l^i(\xi) g_l^i(\zeta) + \delta_{l5}[q_a + z q_d] \quad for \quad l = 1, 2, \ldots 8 \tag{4}$$

Where, the dimensionless in-plane coordinates ξ for the segment along the length and a local thickness coordinate ζ are introduced, which varies from 0 to 1. The function f_l^i are known functions, integration along x-direction are evaluated. After solving, it generates a set of ODEs and linear algebric equation for each layer as obtained below.

$$\mathbf{M}\bar{\mathbf{G}}_{,\zeta} = \bar{\mathbf{A}}\bar{\mathbf{G}} + \hat{\mathbf{A}}\hat{\mathbf{G}} + \bar{\mathbf{Q}}_{\mathbf{p}}^{\mathbf{m}} \tag{5}$$

$$\mathbf{K}\hat{\mathbf{G}} = \tilde{\mathbf{A}}\bar{\mathbf{G}} + \tilde{\mathbf{Q}}_{\mathbf{p}}^{\mathbf{m}} \tag{6}$$

Now, $\hat{\mathbf{G}}$ is obtained from equation 6 and put into equation 5 which yields a set of first-order homogeneous ODEs as:

$$\bar{\mathbf{G}}_{,\zeta} = \mathbf{A}\bar{\mathbf{G}} + \mathbf{Q_p} \tag{7}$$

Now $g_l^i(\zeta)$ is known from the first step, whereas $f_l^i(\xi)$ are considered as unknown. So in this step, arbitrary variation is considered in f_l^i functions. Similarly, algebraic equation for f_l^i is obtained below.

$$\mathbf{N}\bar{\mathbf{F}}_{,\xi} = \bar{\mathbf{B}}\bar{\mathbf{F}} + \hat{\mathbf{B}}\hat{\mathbf{F}} + \bar{\mathbf{P}} \tag{8}$$

$$\mathbf{L}\hat{\mathbf{F}} = \tilde{\mathbf{B}}\bar{\mathbf{F}} + \tilde{\mathbf{P}} \tag{9}$$

Now, $\bar{\mathbf{F}}$ is obtained from equation 9 and put into equation 8 which yields a set of first-order ODEs with constant coefficients.

$$\bar{\mathbf{F}}_{,\xi} = \mathbf{B_0}\bar{\mathbf{F}} + \mathbf{P_0} \tag{10}$$

Above equation 10 represent a system of $4n$ non-homogeneous first order ODEs with constant coefficient. This can be solved using the approach discussed in Ref. [7].

3 NUMERICAL RESULTS AND DISCUSSIONS

The numerical results are presented and discussed for Steel-Al type of panel as shown in Fig. 2. The panel has two equal segments, first and second segments are made of Steel and Aluminium (Al) respectively. Thickness of the panel is taken as h. If a panel is subjected to clamped support at x=0 and free at edge at x=a, then it is designated as C-F. The material properties considered here are presented in the table 1.

Table 1: Material Properties

Sl no.	Material	Young's Modulus (GPa)	Shear Modulus (GPa)	Poisson's Ratio (μ)
1	Steel	200	76.923	0.30
2	Aluminium (Al)	70	26.923	0.30

The present EKM results are compared with 2D FE results for different combination of boundary conditions. Since it is a panel, plane strain element of ABAQUS can be used.[10] Therefore, the 2D plane panel with length a along x-direction and thickness h along z-direction is modeled in ABAQUS using the element type CPE8R with a mesh size of 100 (length)$\times$20 (thickness). Whereas, for the non-dimensionalization Y_0=200 GPa, $S = a/h$ and q_0=1 is considered. The converged FE results are presented below in the figures along with EKM results.

Longitudinal variation of u, w, σ_{xx} and τ_{zx} are presented for thick (S=5) and thin (S=10) panel for all ξ locations along the length. The results are presented for simply supported (S-S), clamped-simply support (C-S), clamped-clamped (C-C) and clamped - free (C-F) support conditions. Present results (EKM) with single term n=1 with at least two iterations are presented to show the convergence, accuracy and efficacy of the method. From the figures, it can be seen that the present results are in good agreement with 2D FE except at the edges. This disaggreement at the very clamped support is well reported.[11] There is no much significant effect of S is observed for the S-S case, while for C-S, C-C and C-F cases the effect is clearly visible on the variables.

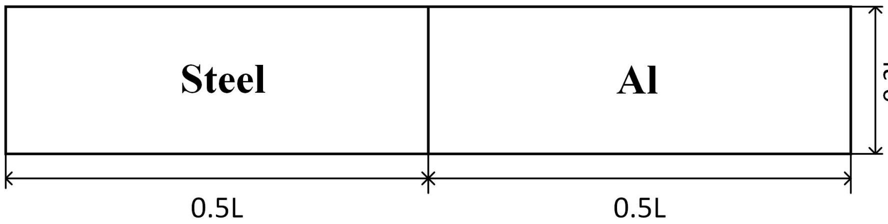

Figure 2: Geometry of the Steel-Al panel for S=5

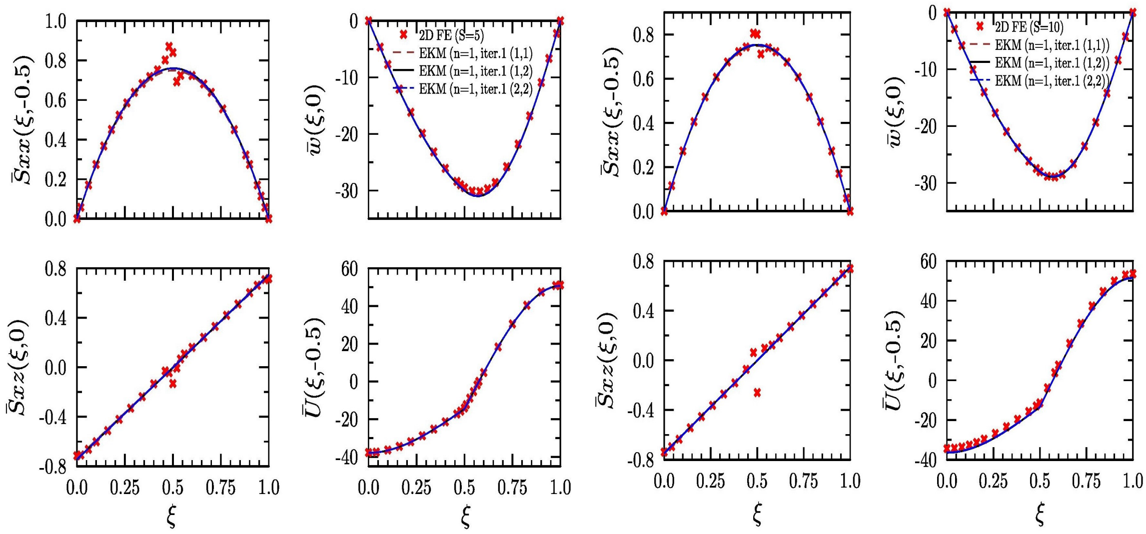

Figure 3: Variation of deflections and stresses for panel with S=5 & 10 under S–S case

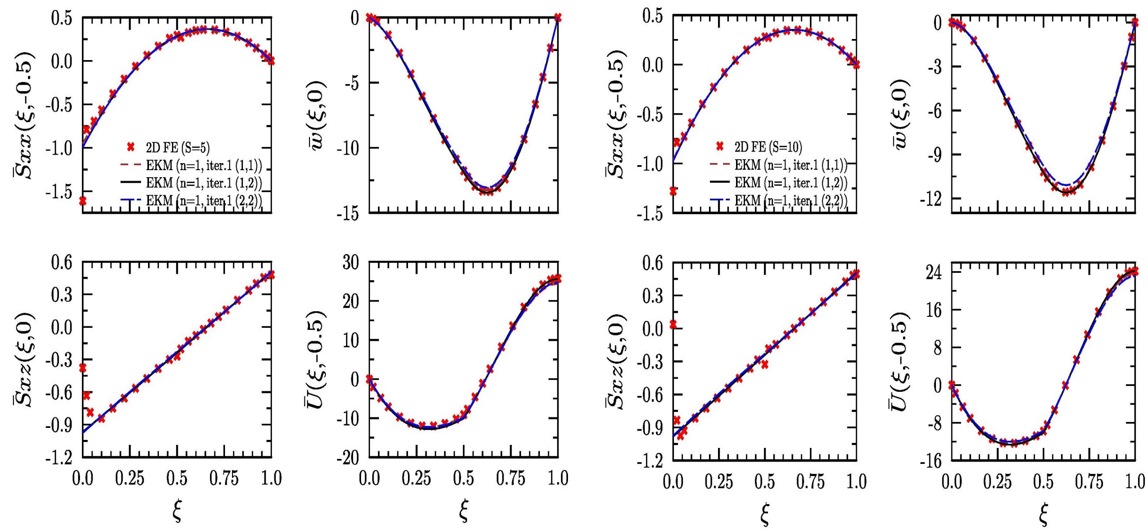

Figure 4: Variation of deflections and stresses for panel with S=5 & 10 under C–S case

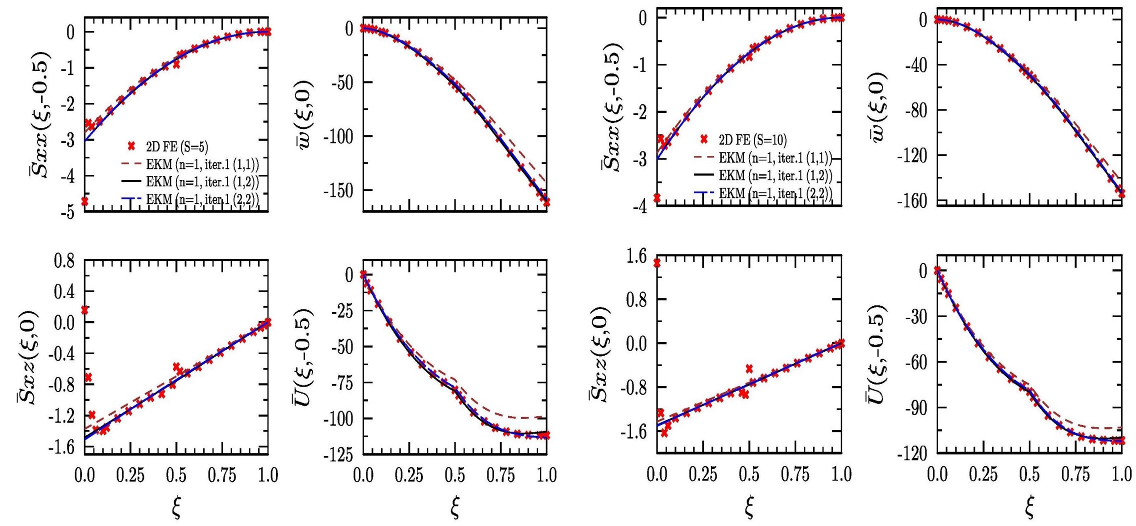

Figure 5: Variation of deflections and stresses for panel with S=5 & 10 under C–F case

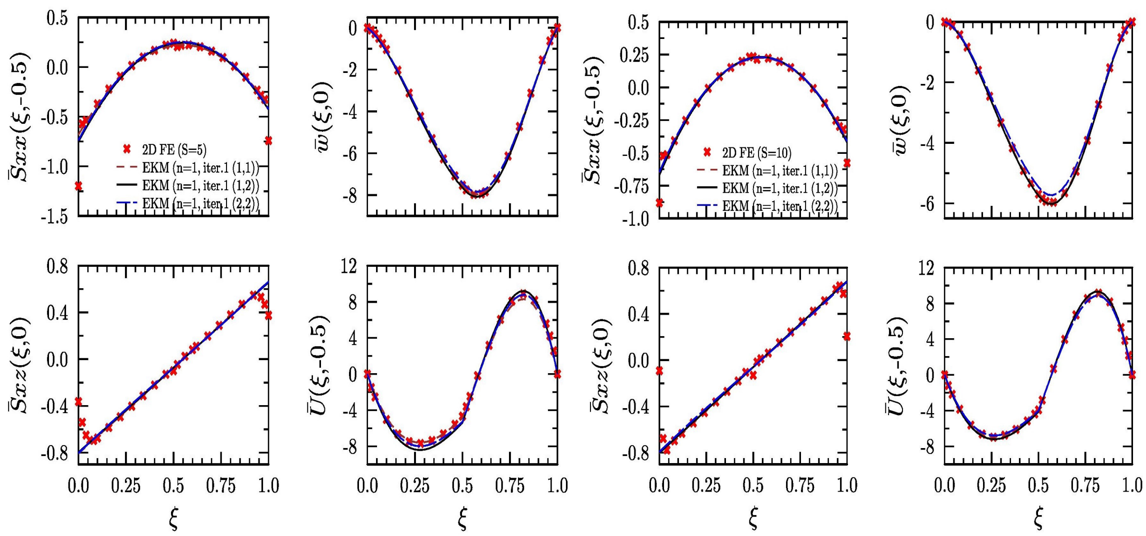

Figure 6: Variation of deflections and stresses for panel with S=5 & 10 under C–C case

4 CONCLUSIONS

The results for thin and thick equally segmented dissimilar material panel for various support conditions have been obtained and analyzed. From the results, it is observed that the effect of thickness in the panel is almost negligible for the S-S case. Whereas, a significant difference is observed in the deformations (u, w) with the increase in thickness of the panel for C-S, C-C and C-F cases. A very least difference in the stresses is observed except at the edges. The present EKM results showing a good agreement with the FE results. The present research can be extended for the bending, buckling and free vibration analysis of more than two segmental dissimilar material panel.

5 ACKNOWLEDGEMENT

This work is supported by funding received from Science and Engineering Research Board India under Women Excellence Award (SB/WEA04/2019).

REFERENCES

[1] I. Joodaky and A. Joodaky, "Bending of edge-bonded dissimilar rectangular plates," *Meccanica*, vol. 54, no. 3, pp. 565–572, 2019.

[2] S. M. Hasheminia, B. C. Park, H.-J. Chun, J.-C. Park, and H. S. Chang, "Failure mechanism of bonded joints with similar and dissimilar material," *Composites Part B: Engineering*, vol. 161, pp. 702–709, 2019.

[3] A. Fereidoon, A. Mohyeddin, M. Sheikhi, and H. Rahmani, "Bending analysis of functionally graded annular sector plates by extended kantorovich method," *Composites Part B: Engineering*, vol. 43, no. 5, pp. 2172–2179, 2012.

[4] A. Gullino, P. Matteis, and F. D'Aiuto, "Review of aluminum-to-steel welding technologies for car-body applications," *Metals*, vol. 9, no. 3, p. 315, 2019.

[5] C.-S. Ban, Y.-H. Lee, J.-H. Choi, and J.-H. Kweon, "Strength prediction of adhesive joints using the modified damage zone theory," *Composite structures*, vol. 86, no. 1-3, pp. 96–100, 2008.

[6] K.-S. Kim, J.-S. Yoo, Y.-M. Yi, and C.-G. Kim, "Failure mode and strength of uni-directional composite single lap bonded joints with different bonding methods," *Composite structures*, vol. 72, no. 4, pp. 477–485, 2006.

[7] V. Kumar and P. Kumari, "2d closed form solution for bending of edge bonded dissimilar beams: An application of ekm," *Composite Structures*, vol. 289, p. 115420, 2022.

[8] A. Singh, P. Kumari, and R. Hazarika, "Analytical solution for bending analysis of axially functionally graded angle-ply flat panels," *Mathematical Problems in Engineering*, vol. 2018, 2018.

[9] A. Singh and P. Kumari, "Two-dimensional elasticity solution for arbitrarily supported axially functionally graded beams," *Journal of Solid Mechanics*, vol. 10, no. 4, pp. 719–733, 2018.

[10] *ABAQUS/Standard User's Manual, Version 6.13*. United States: Dassault Systèmes Simulia Corp, 2013.

[11] S. Kapuria and P. Kumari, "Multiterm Extended Kantorovich Method for Three-Dimensional Elasticity Solution of Laminated Plates," *Journal of Applied Mechanics*, vol. 79, 09 2012. 061018.