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COMPARISON OF DIFFERENT ENTROPY STABILIZATION TECHNIQUES FOR DISCONTINUOUS GALERKIN SPECTRAL ELEMENT METHODS

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We review and compare two techniques to get entropy stability for nodal Discontinuous Galerkin Methods (DG) in compressible flows. One technique is based on entropy split forms, e.g., [1, 2, 3] and one is based on a direct algebraic correction [4].

We have implemented the split form methodology for both, Legendre-Gauss-Lobatto (LGL) and Legendre-Gauss (LG) based spectral element basis functions. While the LGL operators belong to the class of diagonal norm summation-by-parts (SBP) operators, the LG operators belong to the generalized class of SBP operators, where it is not necessary that the boundary nodes are included. To reach entropy conservation, respectively guaranteed entropy dissipation, a key ingredient is an entropy conserving numerical flux function. With this ingredient, only the volume integral term of the DG method has to be changed accordingly.

We have also implemented the second technique, which is in general applicable for a wide range of discretisations. Abgrall [4] introduced an algebraic correction term that retains conservation of the primary quantities and is furthermore constructed such, that an entropy (in-)equality can be shown.

The second technique is at first sight a simpler alternative to the split-form based approach. Hence, questions regarding its advantages and disadvantages naturally come up. We thus investigate and compare the performance of both techniques regarding accuracy, robustness and efficiency.

REFERENCES

[1] T. C. Fisher and M. H. Carpenter, High-order entropy stable finite difference schemes for nonlinear conservation laws: Finite domains. *Journal of Computational Physics*, Vol. **252**, pp. 518–557, Elsevier, 2013.

[2] Gregor J. Gassner, Andrew R. Winters and David A. Kopriva. Split form nodal discontinuous Galerkin schemes with summation-by-parts property for the compressible Euler equations. *Journal of Computational Physics*, Vol. **327**, pp. 39–66, Elsevier, 2016.

[3] Jesse Chan, David C. Del Rey Fernandez and Mark H. Carpenter. Efficient entropy stable Gauss collocation methods. *SIAM Journal on Scientific Computing*, Vol. **41**, no. 5, pp. A2938–A2966, SIAM, 2019.

[4] R. Abgrall, A general framework to construct schemes satisfying additional conservation relations. Application to entropy conservative and entropy dissipative schemes. *Journal of Computational Physics*, Vol. **372**, pp. 640–666, Elsevier, 2018.