

# ON THE DEVELOPMENT OF DATASET SUPPORTED STRATEGIES FOR THE CONSTITUTIVE PARAMETERS IDENTIFICATION OF METAL SHEETS

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**Abstract.** This work presents an exploratory study for the development of a dataset-supported strategy to identify the plastic behaviour of metal sheets. Datasets were generated from numerical simulation results obtained from the biaxial tensile test on a cruciform-shaped sample, for 4000 hypothetical materials. These datasets were used to simultaneously estimate the yield criterion and hardening law parameters of reference materials, resorting to two types of objective function. Sensitivity analyses showed that the performance of the proposed identification strategy depends on the size of the dataset and the reference material.

## 1 INTRODUCTION

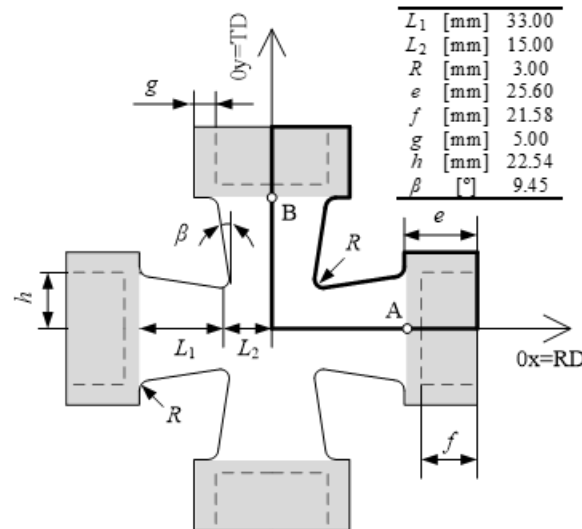
Data driven approaches have become a subject of current interest for identifying constitutive parameters that describe the plastic behaviour of metal sheets, mainly due to the increasing availability of large datasets coupled with the exponential growth of computer performance [1]. However, the use of such dataset supported approaches in the context of material parameters identification is not yet been fully explored. One of the first attempts to elaborate a database supported strategy was undertaken by Zhong et al. [2], to evaluate the yield and ultimate tensile strengths of Cr-Mo steels from small punch test results.

The present work explores the elaboration of a dataset supported strategy, which permits to

identify the plastic behaviour of metal sheets. First, a large dataset is built from numerical simulation results of the biaxial tensile test on a cruciform sample. A substantial set of numerical simulations were performed for hypothetical materials, where the various combinations of constitutive parameters were generated according to the Sobol sequence. For the sake of simplicity, the constitutive model adopted for all numerical simulations follows the Hill'48 yield criterion coupled with the Swift isotropic hardening law, under an associated flow rule. The dataset is populated with local and global variables of the biaxial tensile test (strain field and the evolution of the force during the test, for the two axes of the sample). In this context, the dataset consists of biaxial tensile test results obtained from each hypothetical material and the corresponding constitutive parameters of the model. Afterwards, the dataset is used to process the force vs. displacement and the equivalent strain field results of three reference materials, using different objective functions to estimate their constitutive parameters. Sensitivity analyses were carried out to study the influence of the dataset size and the type of reference material on the performance of the proposed identification strategy.

## 2 NUMERICAL SIMULATION AND MATERIALS

Figure 1 represents the geometry and dimensions of the cruciform sample in the sheet plane. The sample is submitted to equal displacements in the  $0x$  and  $0y$  directions. The  $0x$  and the  $0y$  axes coincide with the rolling direction (RD) and the transverse direction (TD) of the sheet, respectively. The displacements along the  $0x$  and  $0y$  axes are measured at points A and B, respectively. The sheet thickness considered in this study is 1.0 mm.



**Figure 1:** Geometry and dimensions of the cruciform-shaped sample [3].

The material is considered orthotropic. Due to geometrical and material symmetries, only one eighth of the specimen was considered in the numerical simulation model. The sample was discretized with tri-linear 8-node hexahedral solid elements with an average in-plane size of 0.5 mm and two layers through-thickness. Numerical simulations were carried out with DD3IMP in-house finite element (FE) code, developed and optimised to simulate sheet metal forming processes [4].

For simplicity, the constitutive model adopted for building a dataset follows the Hill'48 yield criterion [5] coupled with the Swift isotropic hardening law [6], under an associated flow rule. The Hill'48 yield criterion describes the yield surface for the case of an orthotropic material as follows:

$$F(\sigma_{22} - \sigma_{33})^2 + G(\sigma_{33} - \sigma_{11})^2 + H(\sigma_{11} - \sigma_{22})^2 + 2L\sigma_{23}^2 + 2M\sigma_{13}^2 + 2N\sigma_{12}^2 = Y^2 \quad (1)$$

where  $\sigma_{11}$ ,  $\sigma_{22}$ ,  $\sigma_{33}$ ,  $\sigma_{12}$ ,  $\sigma_{23}$  and  $\sigma_{13}$  are the components of the Cauchy stress tensor ( $\boldsymbol{\sigma}$ ) defined in the principal axes of orthotropy;  $F$ ,  $G$ ,  $H$ ,  $L$ ,  $M$  and  $N$  are the anisotropy parameters and  $Y$  is the yield stress, which evolution is defined by the work-hardening law. In this study, it is assumed the condition  $G + H = 1$  and, therefore, the work-hardening law is represented by the uniaxial tensile stress along the RD, and  $L = M = 1.5$  (von Mises). The work hardening is described by Swift law as follows:

$$Y = C(\varepsilon_0 + \bar{\varepsilon}^p)^n \quad (2)$$

where  $\bar{\varepsilon}^p$  is the equivalent plastic strain and  $C$ ,  $\varepsilon_0$  and  $n$  are the material parameters. The initial yield stress can be written as:  $Y_0 = C(\varepsilon_0)^n$ .

DC06 mild steel was used as reference material. The constitutive parameters that describe the plastic behaviour of this steel are shown in Table 1.

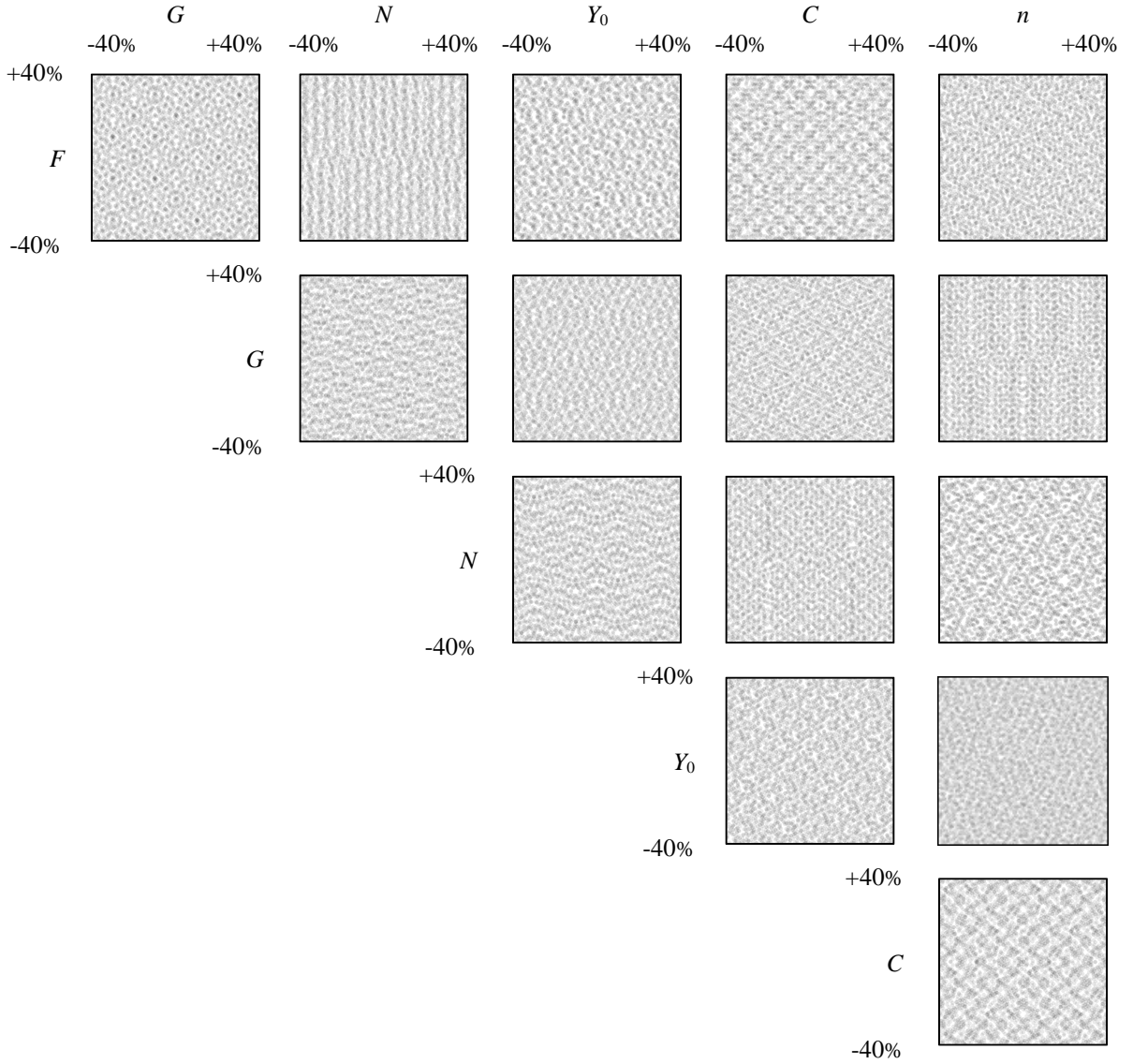
**Table 1:** Constitutive parameters of reference material DC06.

$F$	$G$	$N$	$Y_0$ [MPa]	$C$ [MPa]	$n$
0.2826	0.3584	1.2885	157.12	565.32	0.259

### 3 BUILDING OF A DATASET AND IDENTIFICATION STRATEGY

#### 3.1 Dataset generation

The dataset is populated with numerical simulation results of the biaxial tensile tests, of 4000 hypothetical materials. The values of the constitutive parameters that describe the plastic behaviour of the hypothetical materials were generated based on the constitutive parameters of DC06 (see Table 1), within a variation range of  $\pm 40\%$  of each parameter, using the Sobol Sequence [7]. The Sobol Sequence is a quasi-random sampling approach that allows obtaining a more uniform distribution of the sets of constitutive parameters than the pseudo-random sampling, also preventing the repetition of sets. Figure 2 shows the distribution pattern of the 4000 sets of constitutive parameters, showing that the sets are well dispersed.



**Figure 2:** Distribution pattern of the 4000 sets of constitutive parameters generated using the Sobol Sequence.

### 3.2 Identification strategy

The parameter identification strategy consists on the simultaneous identification of the Hill'48 yield criterion and the Swift hardening law parameters, supported by a dataset of 4000 numerical simulation results of the biaxial tensile test. The identification strategy is focused on the following biaxial tensile test results: (i) force values along the  $0x$  and  $0y$  axes, corresponding to displacements of 0.2mm, 0.5mm, 1mm, 2mm and 3mm; (ii) equivalent strain field at a given moment of the test, for a tool displacement equal to 3mm. Two different objective functions were selected to quantify the differences between the cruciform test results of the reference material and those of each hypothetical material in the database. The first objective function,  $f_1^i$ , expresses the relative difference in the forces, between the reference material and each hypothetical material  $i$ :

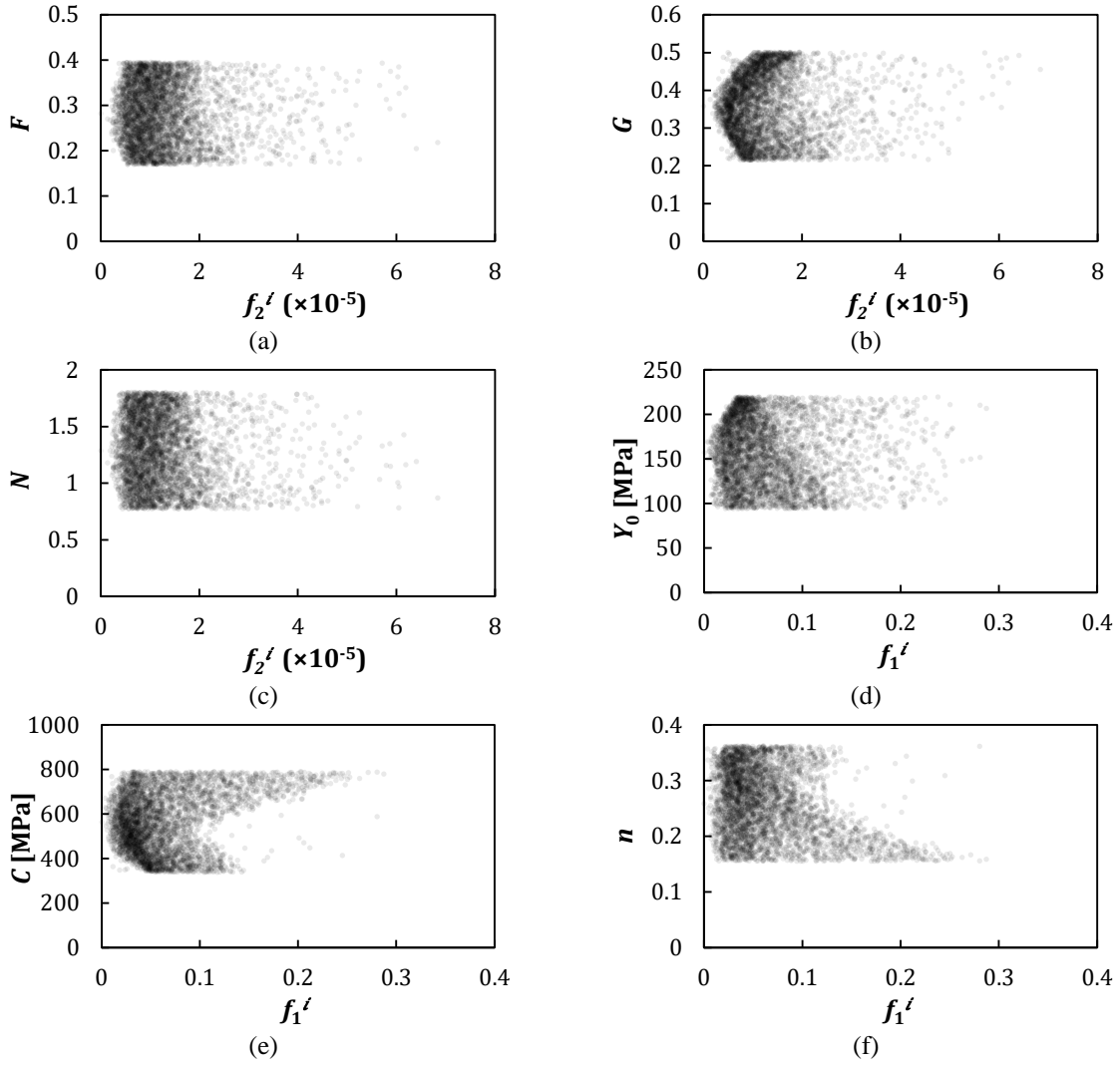
$$f_1^i = \frac{1}{2p} \sqrt{\sum_{j=1}^p \left( \frac{F_{xj}^i - F_{xj}^{ref}}{F_{xj}^{ref}} \right)^2 + \sum_{j=1}^p \left( \frac{F_{yj}^i - F_{yj}^{ref}}{F_{yj}^{ref}} \right)^2}, \quad (3)$$

where  $F_{xj}^{ref}$  and  $F_{yj}^{ref}$  are the force values of the reference material for the displacement value  $j$  along the 0x and 0y axes, respectively;  $F_{xj}^i$  and  $F_{yj}^i$  are the force values of the hypothetical material  $i$ , measured for the displacement  $j$ , along the 0x and 0y axes, respectively, and  $p$  is the total number of displacement values considered in all numerical simulations for each axis, in this case  $p = 5$  (i.e. 0.2mm, 0.5mm, 1mm, 2mm and 3mm). The second objective function,  $f_2^i$ , expresses the differences in the equivalent strain field, between the reference material and each hypothetical material  $i$ :

$$f_2^i = \frac{1}{N_{nodes}} \sqrt{\sum_{k=1}^{N_{nodes}} (\varepsilon_k^i - \varepsilon_k^{ref})^2}, \quad (4)$$

where  $\varepsilon_k^{ref}$  and  $\varepsilon_k^i$  are the equivalent strains calculated for the reference material and the hypothetical material  $i$  at node  $k$ , respectively, and  $N_{nodes}$  is the number of nodes ( $N_{nodes} = 6935$ ) for which the equivalent strains were calculated.

The Swift hardening law parameters  $Y_0$ ,  $C$  and  $n$  of the reference material were estimated from an exploratory analysis on  $f_1^i$ , whereas  $f_2^i$  was used to estimate the parameters  $F$ ,  $G$  and  $N$  of the Hill'48 yield criterion. Fig. 3 shows the relationship between the objective function ( $f_1^i$  or  $f_2^i$ ) and each constitutive parameter ( $F$ ,  $G$ ,  $N$ ,  $Y_0$ ,  $C$  or  $n$ ) of the 4000 hypothetical materials, considering the reference material DC06 (see Table 1). Subsets of hypothetical materials presenting the lowest values of the objective function were used, to estimate the values of the constitutive parameters for the reference material; these values are estimated by calculating the average values of the constitutive parameters of the subsets, composed of 10, 50 and 100 simulations.



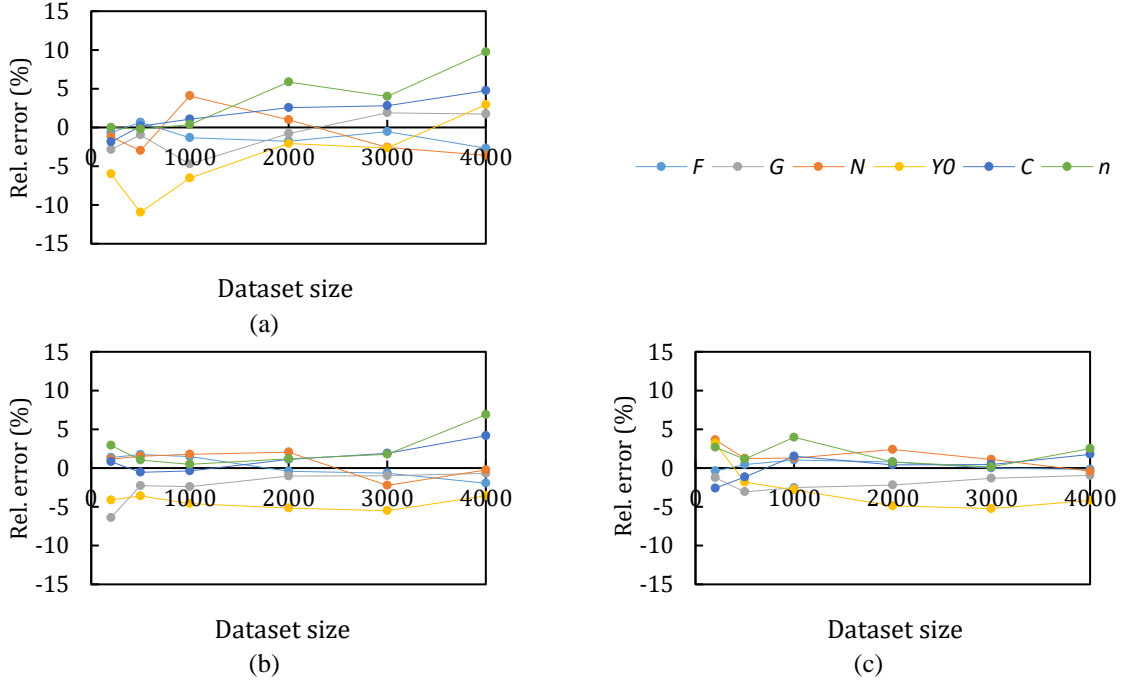
**Figure 3:** Relationship between the objective function and the constitutive parameters of the 4000 hypothetical materials: (a)  $F$ , (b)  $G$ , (c)  $N$ , (d)  $Y_0$ , (e)  $C$  and (f)  $n$ .

## 4 SENSITIVITY ANALYSIS

### 4.1 Sensitivity to the dataset size

The influence of the dataset size, i.e. the number of hypothetical materials (and corresponding numerical simulations) generated by the Sobol sequence, on the efficiency of the proposed identification strategy was investigated. Dataset sizes of 200, 500, 1000, 2000, 3000 and 4000 hypothetical materials were considered. Fig. 4 shows the relative error in the estimation of  $F$ ,  $G$ ,  $N$ ,  $Y_0$ ,  $C$  and  $n$  for the reference material DC06, as a function of the dataset size, for subsets of 10, 50 and 100 simulations with the lowest objective function. In general, the relative error is kept within 5%, whatever the dataset size and subset size; exceptions include

$Y_0$  and  $n$ , with relative errors near 10% for a subset size of 10. For a given dataset size, increasing the subset size leads to a general decrease of the relative error, which is because the hypothetical materials were generated from the constitutive parameters of DC06 (i.e. DC06 corresponds to the “average material” of the dataset).



**Figure 4:** Relative error in the estimation of  $F$ ,  $G$ ,  $N$ ,  $Y_0$ ,  $C$  and  $n$  for the reference material DC06, as a function of the dataset size, for subsets of (a) 10, (b) 50 and (c) 100 simulations with the lowest objective function.

## 4.2 Sensitivity to the reference material

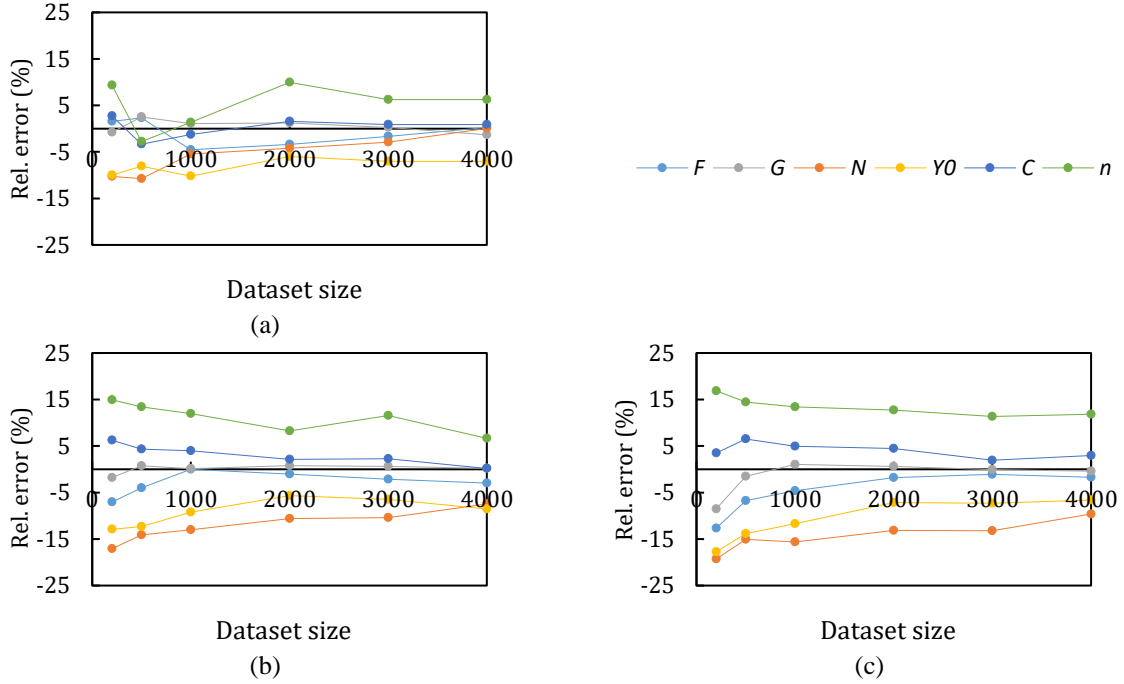
In addition to the reference material DC06 (see Table 1), the parameter identification strategy is now tested on two distinct materials, RM1 and RM2, considered as new reference materials. These new reference materials correspond to hypothetical materials from the generated dataset. The constitutive parameters of RM1 and RM2 are presented in Table 2, and their values correspond to a variation of about 20% (RM1) and 40% (RM2) relatively to those of the reference material DC06 (see Table 1).

**Table 2:** Constitutive parameters of the new reference materials, RM1 and RM2.

Material	$F$	$G$	$N$	$Y_0$ [MPa]	$C$ [MPa]	$n$
RM1	0.3314	0.4415	1.6518	200.20	600.21	0.211
RM2	0.1893	0.4957	1.6594	193.52	790.34	0.158

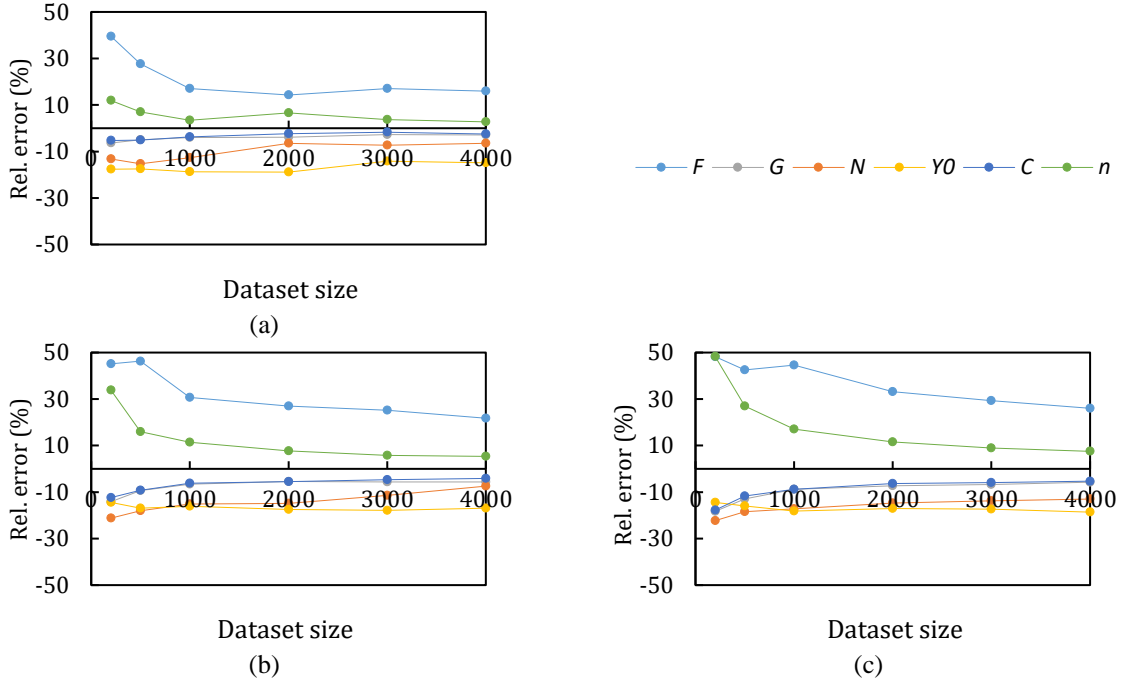
Fig. 5 and Fig. 6 show the relative error in the estimation of  $F$ ,  $G$ ,  $N$ ,  $Y_0$ ,  $C$  and  $n$  for the reference materials RM1 and RM2, respectively, as a function of the dataset size, for subsets of 10, 50 and 100 simulations with the lowest objective function. In general, the increase of the

dataset size leads to a decrease of the relative error in the estimation of the constitutive parameters; moreover, the increase of the subset size leads to a general increase of the relative error. The relative error in the parameter estimation of RM1 and RM2 is about 5%, for subsets of 10 simulations and for dataset sizes of 3000 and 4000 hypothetical materials; exceptions include the relative error in estimating  $n$  and  $Y_0$ , which is about 15%. For a given dataset size, the increase of the subset size leads to parameter estimates closer to the values of the reference material DC06 (i.e. “average material” of the dataset – see Table 1), which increases the relative error in the estimates of RM1 and RM2.



**Figure 5:** Relative error in the estimation of  $F$ ,  $G$ ,  $N$ ,  $Y_0$ ,  $C$  and  $n$  for the reference material RM1, as a function of the dataset size, for subsets of (a) 10, (b) 50 and (c) 100 simulations with the lowest objective function.





**Figure 6:** Relative error in the estimation of  $F$ ,  $G$ ,  $N$ ,  $Y_0$ ,  $C$  and  $n$  for the reference material RM2, as a function of the dataset size, for subsets of (a) 10, (b) 50 and (c) 100 simulations with the lowest objective function.

## 5 CONCLUSIONS

This exploratory work concerns the analysis of a dataset-supported parameter identification strategy proposed to identify the constitutive parameters that describe the plastic behaviour of metal sheets. The numerical simulation results of the biaxial tensile test on a cruciform sample enabled the construction of a dataset of 4000 hypothetical materials, to support the identification of the parameters of the reference materials. Sensitivity analyses were performed on the dataset size and the type of reference material, showing that the performance of the proposed identification strategy depends on both aspects. Future work will focus on exploring alternative approaches to perform database-supported parameter identification.

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## REFERENCES

- [1] Morand, L. and Helm, D. A mixture of experts approach to handle ambiguities in parameter identification problems in material modeling. *Comput. Mater. Sci.* (2019) **167**:85–91.
- [2] Zhong, J., Song, M., Guan, K. and Dymacek, P. Application of a database in the evaluation of strengths of Cr-Mo steels by means of small punch test. *Int. J. Mech. Sci.* (2020) **166**:105195.
- [3] Prates, P.A., Oliveira, M.C. and Fernandes, J.V. Identification of material parameters for thin sheets from single biaxial tensile test using a sequential inverse identification strategy. *Int. J. Mater. Form.* (2016) **9**(4):547–571.
- [4] Oliveira, M.C., Alves, J.L. and Menezes, L.F., Algorithms and strategies for treatment of large deformation frictional contact in the numerical simulation of deep drawing process. *Arch. Comput. Method Eng.* (2008) **15**:113–162.
- [5] Hill, R., A theory of the yielding and plastic flow of anisotropic metals. *Proc. Roy. Soc. London* (1948) **193**:281–297.
- [6] Swift, H.W., Plastic instability under plane stress. *J. Mech. Phys. Solids* (1952) **1**:1–18
- [7] Saltelli, A., Ratto, M., Andres, T., Campolongo, F., Cariboni, J., Gatelli, D., Saisana, M., and Tarantola, S. *Global Sensitivity Analysis. The Primer*. Wiley, (2008).