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A Physics-Informed Deep Learning Approach to Computing Solutions of Hyperbolic Problems

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The recent popularity of physics-informed deep learning models (e.g., [1]) to numerically approximate the solution of partial differential equations (PDEs) represents an astonishing advancement in scientific computing, especially considering the relatively simple neural networks employed for this purpose. We propose an extension of the method in [2] to the case of 2D (e.g., Buckley-Leverett and inviscid Burgers' equations). Here we are showing:

2D Inviscid Burgers' model : $u_t + (u^2/2)_x + (u^2/2)_y = 0, \ (x, y, t) \in [0, 1]^2 \times (0, 0.5], \ (1)$

with Riemann initial conditions $u(x, y, 0) = 2, x < 0.25, y < 0.25$, $u(x, y, 0) = 3, x > 0.25, y > 0.25$, $u(x, y, 0) = 1$, otherwise. To make the deep learning model computationally feasible we investigate scenarios where the number of layers, neurons and epochs are significantly reduced. With the same objective we also verify different optimizers, given that here a nearly optimum minimization of the PDE functional is usually sufficient to provide us with the general structure of the solution. Here we employ 10^6 points on the input domain, 6 hidden layers with 30 neurons and 10000 epochs, at time $T = 1/12$. The optimizer is Adam, instead of the LBFGS algorithm in [1].

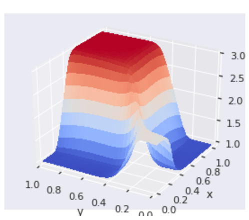

Figure 1: Deep learning solution to (1).

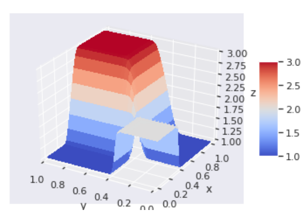

Figure 2: Reference solution (1) via [3].

Despite still having constraints concerning both computational time and accuracy, as illustrated in Figure 1, the results obtained here using deep learning are promising.

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