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NONLINEAR MACROELEMENT BASED ON BOUC-WEN FORMULATION WITH DEGRADATION FOR THE EQUIVALENT FRAME MODELLING OF MASONRY WALLS

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Abstract. The equivalent frame model takes into account the shear and bending mechanisms that take place in piers and spandrels through plastic hinges. It represents a good compromise between accuracy and computational burden in the analysis of complex masonry walls.

For the purposes of dynamic analysis, in addition to the hysteretic behaviour of the plastic hinges, it is also necessary to introduce their degradation of strength and stiffness.

This study presents a macroelement model for piers and spandrels in which the bending mechanism is described by two hinges at the ends of the macroelement, and the shear mechanism by a shear link. They are characterized by a hysteretic behaviour with progressive plasticity, described by the Bouc-Wen model, and by degradation of strength and stiffness. Degradation is described through a damage parameter, which governs both strength and stiffness decay, and a flexibility increase parameter, which only governs stiffness reduction. In this way it is possible to independently control both strength and stiffness degradation.

The model is applied to simulate experimental tests on panels, highlighting a good agreement with the experimental results.

1 INTRODUCTION

The assessment of masonry structures is a fundamental topic in modern society, especially in seismic prone areas, such as Italy, where a large part of historical buildings is made of masonry. The significant vulnerability of this type of structures must then be properly considered and described when modeling them [1]. This applies especially to the practitioners' field, where simplified methods for the analysis of complex structures are required, in particular when complex loads, such as the seismic excitations, have to be analyzed. For this reason, the development of numerical procedures based on purposely defined macroelements is widely investigated, with the aim of providing tools with a low computational cost and, at the same time, a good accuracy of the results and a low number of input parameters [2].

The equivalent frame models available in [2, 3, 4, 5] satisfy the above requirements and permit an accurate reproduction of the experimental results. Unreinforced masonry elements, such as single arches or panels, as well as entire regular unreinforced masonry structures can be schematized in this framework. In the latter case, the walls, in their in-plane configuration, are usually divided into piers, spandrels and rigid joints. Piers and spandrels are then modeled as beam elements, which can have a 1D, 2D or 3D behavior, while the nodes are reduced to rigid ends of these frames, as they are considered rigid zones. The nonlinear behavior of masonry is represented through lumped hinges, where the in-plane flexural and shear mechanisms can be properly described.

Different nonlinear constitutive laws are commonly used in the structural field to reproduce hysteresis phenomena. Among them, the Bouc-Wen constitutive model has been widely applied, ranging from the classical engineering fields, like steel structures, to the most modern. It has also been recently applied for the description of masonry buildings in a modified version, capable of accounting for damage and energy dissipation, in which also some issues related to the thermodynamic admissibility of some of the required model parameters have been overcome [6, 7, 8]. Furthermore, some issues related to the accuracy of the model regarding the description, during cyclic horizontal loads, of the stiffness degradation in loading and unloading branches of the curves have been investigated and the latter formulation enriched. This study presents a modified and further enriched formulation for the inclusion of a flexibility increase effect observed by comparing the previous model to experimental results available in literature, comparing especially the loading and unloading branches of the cyclic curves.

To reproduce the particular hysteretic cycle's shapes typical of slender panels, which experience flexural mechanisms, the effect of pinching is considered through the introduction of a specific device, as well as a further device to appropriately account for the high initial stiffness.

The capability of the macroelement to model the cyclic behavior of masonry walls subjected to horizontal loads is finally validated considering well-known experimental results available in literature, in order to test its capability to reproduce the typical masonry shear and flexural mechanisms.

2 MACROELEMENT FORMULATION

The one-dimensional beam macroelement relies on a force-based formulation. This requires the elimination of the rigid motions, so that the six in-plane nodal degrees of freedom of the 2D beam are reduced to the basic three. The corresponding basic local displacements are the axial elongation and the two rotations at the ends of the elements, collected into a specific vector: $\boldsymbol{\varepsilon}^{el} = [\delta u_j^{el} \ \phi_i^{el} \ \phi_j^{el}]^T$, to which the axial force and two bending moments are associated and collected into the nodal force vector: $\boldsymbol{\sigma}^{el} = [N_j^{el} \ M_i^{el} \ M_j^{el}]^T$.

The beam macroelement is composed of a series of three different elements: the Euler-Bernoulli elastic beam, the shear hinge, where the entire shear deformation is concentrated, and the two flexural hinges located at the ends of the element.

Specifically for the bending components, the equation that governs the equilibrium between the three elements in series at each end i/j of the element, is the following:

$$M^{el}_{i/j} = M^{el}_{i/j,e} = M^{el}_{i/j,h} \tag{1}$$

It states that the moment at each end of the macroelement $M^{el}_{i/j}$ is equal to the bending moment at the end of the elastic element $M^{el}_{i/j,e}$ and that evaluated at the flexural hinge located at the same end $M^{el}_{i/j,h}$. Regarding the axial force, this distinction is not needed, as no hinges are introduced. Moving to the kinematic description, a similar decomposition can be made regarding the rotational degrees of freedom of the element, leading to the equation:

$$\phi^{el}_{i/j} = \phi^{el}_{i/j,h} + \phi^{el}_{i/j,e} \tag{2}$$

valid at each end i/j of the element. In this case the total rotation $\phi^{el}_{i/j}$ is evaluated as the sum of the rotation at the end of the elastic element and that of the hinge. This latter includes both the contributions of the flexural and shear hinges.

The flexibility matrix of the element accounts for the contributions of all the three components of the macroelement:

$$\mathbf{F}^{el} = \begin{bmatrix} \frac{L}{EA} & 0 & 0 \\ 0 & \frac{L}{3EI} + f_{bhi} + \frac{f_{sh}}{L^2} & -\frac{L}{6EI} + \frac{f_{sh}}{L^2} \\ 0 & -\frac{L}{6EI} + \frac{f_{sh}}{L^2} & \frac{L}{3EI} + f_{bhj} + \frac{f_{sh}}{L^2} \end{bmatrix} \tag{3}$$

$f_{bhi/j}$ is the flexural flexibility of the hinge located at the node i/j, while f_{sh} is the contribution of the shear hinge, that is made consistent with the flexural terms by dividing it by the squared value of the effective length of the element.

The described vectors are used to determine the element state at each iteration, k, of each time step of the nonlinear static analysis. The local displacement increment $\Delta\boldsymbol{\varepsilon}^{el,k}$ and the nodal force increment $\Delta\boldsymbol{\sigma}^{el,k}$ are used to evaluate the rotation and shear deformation increment at each hinge, which are introduced into the constitutive law to evaluate moments and shear, and the tangent flexibility coefficients, $f^k_{bhi/j}$, f^k_{sh}, of each lumped hinge. To satisfy element compatibility equation, a consistent procedure is adopted, where residuals are evaluated in terms of rotations for the flexural hinges and shear deformation for the shear link. These are then collected into a residual vector:

$$\mathbf{r}^k = \left\{0 \quad \rho^k_{ihb} + \rho^k_{hs} \quad \rho^k_{jhb} + \rho^k_{hs}\right\}^T \tag{4}$$

which is null for the axial component, assumed as linear elastic, while accounting for both flexural, $\rho^k_{i/jhb}$, and shear residual, ρ^k_{hs}. These are used to update the element nodal force vector at the current iteration, k, and then transferred at the global level to be used in the Newton-Raphson procedure to evaluate the structural response.

For all the lumped hinges a modified Bouc-Wen hysteretic law accounting for degradation is adopted [6]. Despite this constitutive law is suitable for the shear hinge, it is not capable to capture some of the peculiar aspects of slender panel response, such as pinching and the high initial stiffness. Therefore, the flexural hinges were further enriched by introducing additional devices [7]. A nonlinear elastic device is arranged in parallel with the modified Bouc-Wen with damage and flexibility increase device, and in series with both a linear elastic negative device is added. The resulting curve is flag-shaped, capable of describing pinching, with a high initial stiffness that makes negligible the contribution of the hinges to the initial global stiffness of the system. As a consequence, the kinematic description of the rotation of the flexural hinge must be enriched by accounting for the presence of these additional devices. Regarding the equilibrium, the bending moment at each element end is equal for the two elements in series and is evaluated as the sum of the moment of the modified Bouc-Wen device and that of the nonlinear elastic element. More details on the arrangement of these devices can be found in [7].

3 MODIFIED BOUC-WEN HYSTERESIS

To describe the nonlinear behavior of the lumped shear and flexural hinges, the Bouc-Wen hysteretic model is adopted. Originally formulated by Bouc [9] in 1967, then revised by Wen [10], it was widely used in the literature to describe the behavior of different materials and structural elements. The constitutive law is composed of an elastic and a hysteretic part:

$$F = F^{el} + F^{h} = a\, k_i\, v_y\, u + (1-a)\, k_i\, v_y\, z \tag{5}$$

where k_i is the initial stiffness of the system, a is the ratio between the post- and pre-yielding stiffness, v_y is the yielding displacement, u is the nondimensional displacement and z is the hysteretic variable. The evolution of the hysteretic variable is governed by the following differential equation:

$$\frac{dz}{du} = A - [\beta\, sign(z\dot{u}) + \gamma]|z|^n \tag{6}$$

The dimensionless parameters A, β, γ, n rule the behavior of the cycle: n controls the transition from the elastic to the post-yielding branch, A is a redundant parameter set equal to 1, β and γ affect shape and size of the cycle. According to [11], to ensure thermodynamic admissibility, the condition $\beta \leq \gamma$ must be enforced.

A modification of the Bouc-Wen model was made in [6], with the introduction of a scalar damage variable D, based on continuum damage mechanics, in the evaluation of the hysteretic force:

$$F^h = (1-D)(1-a)\, k_i\, v_y\, z \tag{7}$$

This variable, whose value ranges between 0 for the undamaged state and 1 for the fully damaged state, is capable of reproducing both the strength and stiffness degradation and is evaluated as:

$$D = \delta_D\, U^h \geq 0 \tag{8}$$

where the parameter δ_D has the dimension of the inverse of the energy [J^{-1}] and governs the rate of the evolution of the damage, and U^h is the dissipated energy. The formulation of this latter, accounting for degradation, is:

$$U^h = \frac{1}{\delta_D}\left[1 - \exp\left(-2c\delta_D \int_0^{u^p} \frac{z}{1 - c\,\delta_D z^2} d\tilde{u}^p\right)\right] \tag{9}$$

where:

$$c = \frac{1}{2}\, k_i\, v_y^2\, (1-a) \tag{10}$$

Two terms can be distinguished in the dissipated energy definition, one regarding the strength decrease, $-2c\delta_D$, and one regarding the stiffness decrease, $1 - c\delta_D z^2$. The evaluation of the hysteretic variables remains unchanged with respect to the original Bouc-Wen model.

However, as shown by comparisons of experimental and numerical results, the model described is not capable to completely capture the stiffness degradation. This is the reason why a further modification was made, with the introduction of flexibility increase. According to this formulation, an additional term is introduced in the definition of the elastic displacement, which accounts for the dissipated energy as:

$$v = v_y[(1 + \delta_K U^h)\, z + u^p] \tag{11}$$

The rate of the evolution of flexibility increase is governed by the parameter δ_K, which, similarly to the damage parameter, has the dimension of the inverse of an energy [J^{-1}]. The formulation of the dissipated energy was also updated to account for the presence of the flexibility increase by introducing the δ_K parameter in the term that rules the stiffness decrease:

$$U^h = \frac{1}{\delta_D}\left[1 - \exp\left(-2c\delta_D \int_0^{u^p} \frac{z}{1 - c\,(\delta_D + \delta_K) z^2} d\tilde{u}^p\right)\right] \tag{12}$$

It must be noted that the parameter δ_K can assume either positive or negative values, the latter ranging from 0 to $-\delta_D$ and representing stiffness recovery. The limit value $\delta_K = -\delta_D$, for which the contribution of the stiffness degradation is nullified, represents pure strength degradation.

4 EXPERIMENTAL VALIDATION

The model was implemented in the Finite Element Analysis Program FEAP [12]. The validation was made considering two panels tested in 1995 at the Joint Research Centre of Ispra, whose well-known results are available in literature [13]. The panels are made of unreinforced masonry, and the experimental set-up is meant to reproduce the real conditions that piers experience during cyclic loads when located into a building, having applied a constant vertical load and a horizontal quasi-static cyclic displacement history, with rotation constrained at the ends. The two walls differ only for the aspect ratio: the slender wall has a height of $L = 2$ m, while the squat wall has a height of $L = 1.35$ m, while $B = 1$ m and $w = 0.25$ m are respectively the base length and the width. Extended considerations regarding the evaluation of the yielding displacement for both the shear and flexural hinges can be found in [7]. The mechanical parameters of the masonry can be found in Table 1, regarding the Young's modulus E, the shear modulus G, the compressive strength f_c, the shear strength f_{v0}. A vertical load P equal to 150 kN is initially applied and then kept constant.

The same set of constitutive parameters was used for both the panels and can be found in Table 2.

In both cases the model is able to reproduce the main characteristics of the experimental curves, as shown in Fig. 1. In the case of the slender wall, the numerical curve shows the main

Table 1: Mechanical parameters for masonry walls

E	G	f_c	f_{v0}
kN/m^2	kN/m^2	kN/m^2	kN/m^2
1700x10^3	300x10^3	6200	140

Table 2: Parameters of Bouc-Wen hysteretic model

Flexural Hinges			Shear Hinges		
a	δ_D	δ_K	a	δ_D	δ_K
–	kJ^{-1}	kJ^{-1}	–	kJ^{-1}	kJ^{-1}
0.1	0.2	0.3	0.1	0.2	0.3

features observed during the experimental tests, that is pinching, hardening, and high stiffness. Regarding the squat wall, the strength and stiffness degradation, as well as the energy dissipation mechanisms, are well captured. The introduction of the flexibility increase also permits to better follow the unloading branches of the experimental curve.

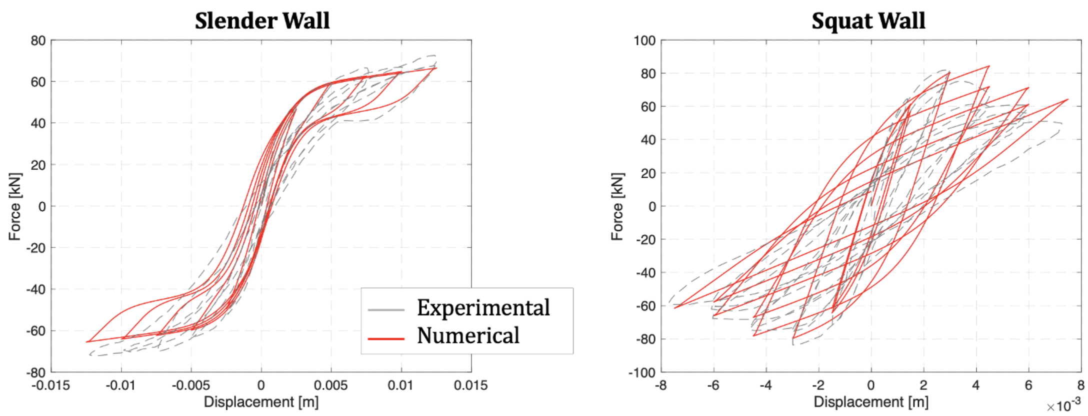

Figure 1: Comparison of the experimental and numerical results for Ispra panels

5 CONCLUSIONS

A one-dimensional force-based beam element for the analysis of masonry structures is presented in this paper within the framework of the equivalent frame approach for the structural analysis of masonry walls. The equivalent frame approach represents a good compromise between accuracy of the results and low computational burden. The proposed force-based macroelement is obtained by arranging an elastic Euler-Bernoulli beam in series with a shear hinge and two flexural hinges, in which the complex nonlinear behavior of the material is concentrated. The model, based on the well-known Bouc-Wen model, is enriched with the introduction of damage, and is able to well reproduce the strength and stiffness degradation and the hysteretic energy dissipation. Thanks to the further introduction of the flexibility increase, a higher accuracy in reproducing the experimental results is reached. Other complex behaviors typical of slender panels, such as pinching and the high initial flexural stiffness, are included by introducing additional simple devices. By adding a limited number of physical parameters is then possible to describe a wider and more detailed range of nonlinear performances.

A validation is proposed comparing the results of well-known experimental tests on a slender and a squat panel under cyclic loadings with the numerical response of the macroelement. A good agreement between the two curves can be observed, and the numerical results correctly reproduce the experimental outcomes in terms of both strength and stiffness degradation.

Following developments of the model aim at implementing the possibility to analyze dynamic loads and, in order to have a complete picture of the performance of masonry structures, to describe the out-of-plane collapse mechanisms of the walls.

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