

# EXPERIMENTAL INVESTIGATION OF A MOVING VEHICLE FOR IDENTIFICATION BRIDGE DYNAMIC PARAMETERS

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**Abstract.** The interaction between a moving vehicle and a bridge was used by Yang and colleagues [1] to develop an indirect method for determining bridge frequencies. In our current investigation, we have established an experiment to indirectly measure the frequency of a simply supported beam using a passing test vehicle. This approach also involves in-situ measurements, in which the vibrations of the bridge are recorded as the vehicle passes over it. Subsequently, the bridge frequencies can be obtained from the dynamic response of the mobile test vehicle. Based on this concept, our research has designed an experimental configuration in which a test vehicle travels along a bridge to measure the frequency of the structure. In the laboratory and in-situ experiments, the test vehicle was created as a system with a single degree of freedom (SDF), enabling vertical vibrations. In the laboratory it was guided by a system of tension cables that allowed the vehicle to move along the beam while staying in full contact with it. The presented experimental results demonstrate that the indirect bridge inspection method is applicable to frequency monitoring of bridges.

## 1 INTRODUCTION

In comparison to using a large number of stationary sensors placed on a bridge, the method proposed by Yang and his colleagues in 2004 for determining bridge frequencies from the response of a moving test vehicle offers several advantages: flexibility in sensor placement, cost savings in sensor maintenance and labor, and efficiency in frequency measurement. In addition, a mobile test vehicle equipped with vibration sensors can record all spatial vibration data along the bridge as it is traveled, allowing for the extraction of frequencies, modal shapes, and other bridge properties (see [2, 3, 4]). Building on this theoretical foundation, Yang and his team conducted a series of experimental and theoretical studies on the indirect measurement method [5, 6, 7, 8, 9, 10, 11, 12, 13]. In one such experiment, Lin and Yang [2] used a two-

wheel cart towed by a light truck to evaluate the fundamental vibration frequency of a sustaining bridge, confirming the viability of the indirect measurement method for monitoring bridge frequencies.

Siringoringo and Fujino [14] used an instrumented moving vehicle based on the indirect method to extract the fundamental frequency of a short-span bridge. They found that when a test vehicle moves at a speed  $v$  over a bridge with a span length  $L$ , the response is influenced by frequencies such as the vehicle frequency  $f_v$ , the bridge frequency  $f_b$ , and the driving frequency  $(\pi v/L)$ . Lower speeds of the moving vehicle allow it to extract the dynamic properties of the bridge with greater spectral resolution and less interference from road surface roughness [13]. Additionally, the effect of tire characteristics on the transmission of bridge dynamic properties to the moving test cart was investigated by Yang et al. [9] using a hand-drawn test cart. Zhang et al. [15] used the global filtering method in a damage detection approach based on the operational deflection shape curvature extracted from the vehicle response. Kim et al. [16] detected changes in the dynamic properties of a bridge by comparing dynamic parameter patterns in a scaled laboratory test.

In addition, the indirect measurement approach has been applied to detect the flexural-torsional coupling of slender suspension footbridges with inclined wind [17]. One of their findings is that the use of a moving test vehicle allows engineers to capture span-wise vibration signals from a bridge, including dynamic information related to flexural-torsional coupling frequencies. This feature provides greater mobility and an easier way to monitor the health of bridges compared to the fixed-sensor approach.

## 2 INDIRECT BRIDGE-FREQUENCY MEASUREMENT

As an example, the simplified model shown in Figure 1 represents a mass supported by a beam. In the case of the vehicle-bridge interaction system, as the beam is excited by the moving vehicle, the vehicle is also subject to the vibration of the beam beneath it. In this figure, the following parameters have been used for the beam:  $m$  is the mass per unit length,  $c$  is the damping,  $L$  is the span,  $EI$  is the bending stiffness and  $u(x,t)$  is a flexural displacement of the beam. For the mass unit on the suspension, we consider  $v$  as the moving velocity,  $M_v$  as the lumped mass,  $m_w$  as the wheel mass,  $k_v$  as the spring stiffness,  $R_w$  as the radius of the wheel and  $\theta_w$  as the rotation of the rolling wheel. The equations of motion for both the beam and the mass moving over it can be expressed as described in reference [18].

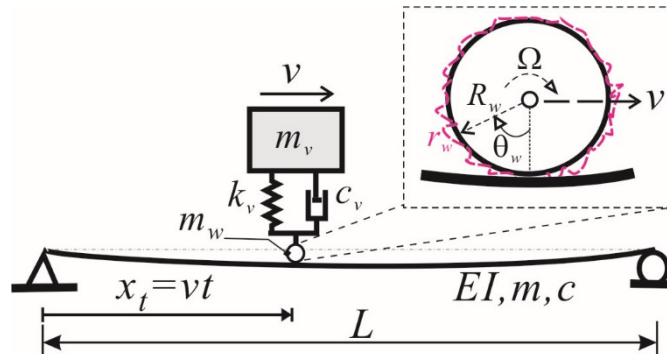


Figure 1: Schematic of a beam subject to a moving sprung mass unit with rolling wheel

$$m\ddot{u} + EIu'''' = -(p_0 - M_v\ddot{u}_v - m_w\ddot{u}(x,t))\delta(x-vt) \quad 0 \leq t \leq L/v \quad (1)$$

$$M_v\ddot{u}_v + k_v u_v = k_v [u(x_t, t) - r_w(x_t)] \quad (2)$$

where  $(\bullet)' = \partial(\bullet) / \partial x$ ,  $(\bullet)\dot{=} \partial(\bullet) / \partial t$ ,  $r(x)$  = surface roughness,  $u_v$  = vertical displacement of the sprung mass,  $p_0 = M_v g$  = weight of the sprung mass,  $g$  = acceleration of gravity,  $\delta(\bullet)$  = Dirac's delta function and  $x_t = vt$  = the position of the sprung mass on the beam. For a simply supported beam, the following boundary conditions and shape functions are adopted:

$$u(0, t) = u(L, t) = 0, \quad EIu''(0, t) = EIu''(L, t) = 0 \quad (3)$$

As shown in equation (2), in addition to the motion  $u(x, t)$  of the beam directly in contact, the motion of the moving sprung mass is influenced by the roughness ( $r_w$ ) of the rolling wheel. In practice, the mass of a vehicle is significantly smaller than that of a bridge, which allows us to neglect the inertial forces ( $M_v\ddot{u}_v, m_w\ddot{u}(x_t, t)$ ) caused by the vibration of the sprung mass without affecting the dynamic response of the beam [1]. By making this approximation and considering an undamped system, we can derive an analytical solution for the acceleration response ( $\ddot{u}_v$ ) of the vehicle as it moves to position  $x_t (=vt)$  on the beam, expressed as  $\ddot{u}_v(t) = \ddot{u}_{v1}(t) + \ddot{u}_{v2}(t)$ , where the acceleration responses ( $\ddot{u}_{v1}(t), \ddot{u}_{v2}(t)$ ) are given by

$$\ddot{u}_{v1}(t) = \frac{k_v}{m_v} \sum_{j=1} A_j \frac{S_{wj}^2 \sin(jvt/R) - S_{wj} \sin(\omega_v t)}{1 - S_{w0}^2} \quad (5)$$

$$\ddot{u}_{v2}(t) = \sum_{n=1}^{\infty} \frac{\omega_v^2 \Delta_{st,n}}{2(1 - S_n^2)} \times \left[ C_{1n} \cos(\omega_v t) + C_{2n} \cos(2n\pi vt/L) \right. \\ \left. + C_{3n} \cos\left(\omega_{b,n} t - \frac{n\pi vt}{L}\right) + C_{4n} \cos\left(\omega_{b,n} t + \frac{n\pi vt}{L}\right) \right] \quad (6)$$

where the following symbols are defined

$$\Delta_{st,n} = \frac{-2p_0 L^3}{(n\pi)^4 EI}, \omega_{b,n} = \left(\frac{n\pi}{L}\right)^2 \sqrt{\frac{EI}{m}}, S_n = \frac{n\pi v}{\omega_{b,n} L}, S_{v,n} = \frac{2n\pi v}{\omega_v L}, S_{wj} = \frac{jv}{\omega_v R} \quad (7)$$

$$C_{1n} = \frac{S_{v,n}^2}{1 - S_{v,n}^2} + \frac{-\omega_v^2 S_n}{\omega_v^2 - \omega_{b,n}^2 (1 - S_n)^2} + \frac{\omega_v^2 S_n}{\omega_v^2 - \omega_{b,n}^2 (1 + S_n)^2}, C_{2n} = \frac{S_{v,n}^2}{1 - S_{v,n}^2} \quad (8)$$

$$C_{3n} = \frac{S_n (1 - S_n)^2 \times \omega_{b,n}^2}{\omega_v^2 - \omega_{b,n}^2 (1 - S_n)^2}, C_{4n} = \frac{-S_n (1 + S_n)^2 \omega_{b,n}^2}{\omega_v^2 - \omega_{b,n}^2 (1 + S_n)^2} \quad (9)$$

Regarding the response shown in equation (6), the dynamic response of the moving test vehicle includes the bridge frequency ( $\omega_{b,n}$ ), so it is possible to detect the bridge frequencies by analysing the response of the test vehicle in the frequency domain. Obviously, the beam

frequency ( $\omega_{b,n}$ ) is also present in the response of the test vehicle. This approach is known as the indirect method of bridge measurement because it requires only one or a few vibration sensors on the test vehicle to capture the dynamic characteristics of the bridge over which it is passing.

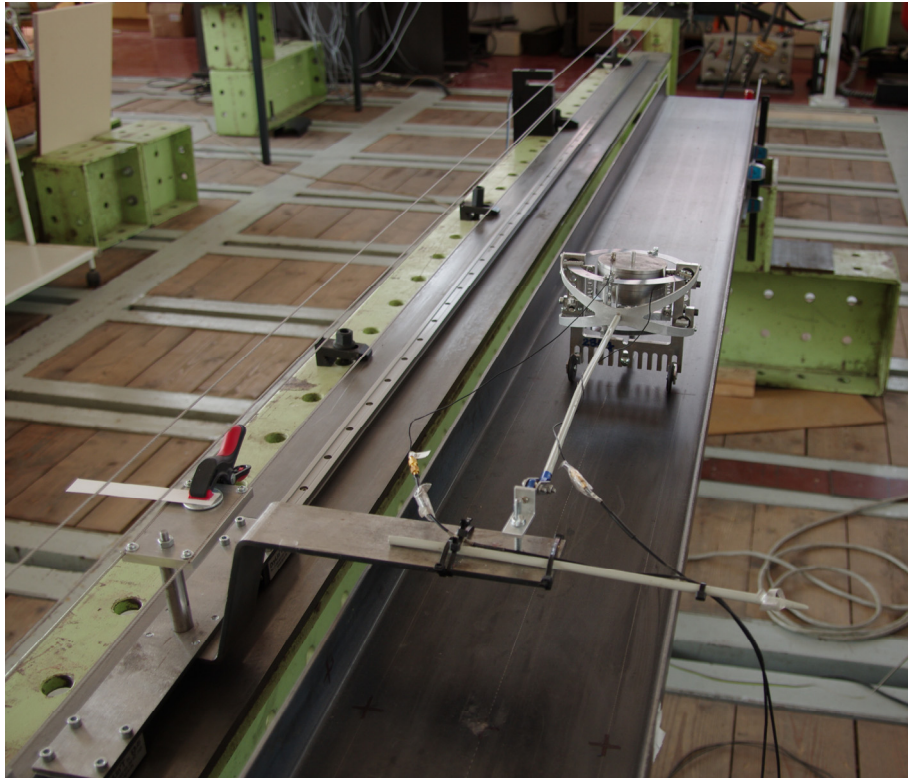
### 3 EXPERIMENTAL EXAMPLES

The single axle test vehicle was used to measure the single beam frequencies in the laboratory and on the bridge during its passage.

#### 3.1 Laboratory test

Indirect measurements were carried out in laboratory experiments on an instrumented SDF vehicle moving along a test beam. These experiments involved driving the test vehicle along a 4 metre steel beam (see Figure 2). The test beam and the instrumented vehicle possess the following properties:

**Test Steel Beam Model:** The U-shaped steel beam, has a total mass of 33.3 kg. Accelerometers are positioned at the one-quarter and middle points of the beam. The test beam receives support at both of its ends from partially rotational constraint supports. The test beam first and second natural frequencies are 7.02 Hz and 27.64 Hz respectively. Detailed information regarding the design of an SDF test vehicle with adjustable frequencies is discussed in previous research [19].



**Figure 2:** Experimental test beam

A Brüel-Kjaer 4374 piezoelectric accelerometer was attached to the concentrated mass of the test vehicle to register the vertical motion of the test vehicle as it passed along the beam. The signals were collected from the accelerometers at a sampling rate of 1000 Hz and sent to a computer using charge amplifiers. In order to maintain steady speed on the test beam, a speed converter became a part of the drive motor, controlling the speed of the test vehicle. The moving speed of the test vehicle is chosen to be 0.2 m/s

As demonstrated in Figure 3, the results of the experiment confirm that the frequency response analysis of the vehicle's moving mass corresponds to the first and second natural frequencies of the beam. The frequency analysis also includes multiple wheel frequencies, which are easy to identify.

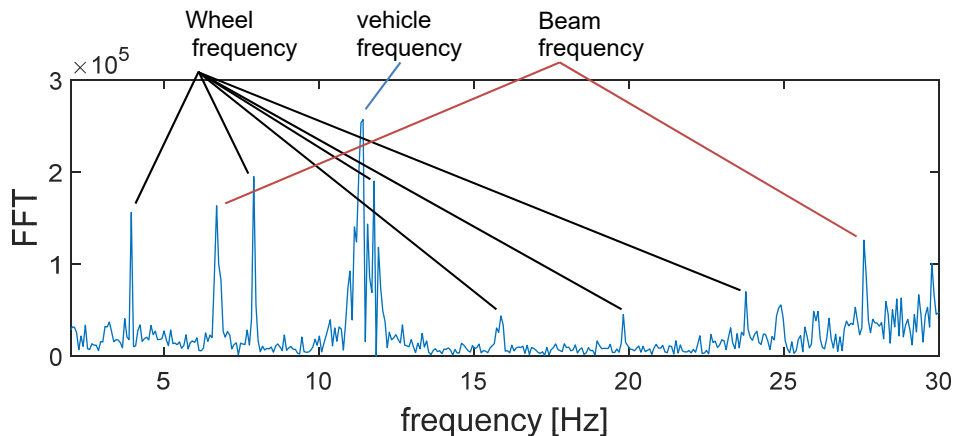


Figure 3: Experimental results for beam first natural frequency

### 3.2 Hanged pedestrian bridge tests

To test pedestrian footbridges, ITAM designed a scanning vehicle (SV) weighing 200 kg (see Figure 4). The vehicle is electrically powered and has the driver's seat located at the front. The spring mass is situated above the rear wheels and comprises of nine removable weights. There is a frame for the installation of the masses, connected to the front part by a hinge and supported by spiral springs. The spring mass moves in a direction perpendicular to the road surface. Field tests of the 200 kg SV were carried out on the footbridge at Lužec-Bukol (see Figure 5). The pedestrian and cyclist footbridge crosses the Vltava River between Lužec and Bukol. It is a suspension bridge with a composite deck with a total length of 138 m. The concrete suspension bridge deck has two span widths of 99.18 m and 31.90 m. The width of the bridge deck is 4.5 m. The SV, equipped with Wilcoxon Model 786LF-500 sensors, was moved across the bridge deck at a speed of 1 m/s while the vertical acceleration response of the spring mass was recorded. The SV was equipped with three stiff springs and all removable weights. Consequently, the SV had a resonance frequency of 8.12 Hz. The additional accelerometers mounted to the left and right of the frame supporting the spring.

Acceleration sensors were placed along the length of the bridge deck to determine the natural frequencies and natural shapes of the bridge. Short ambient vibration measurements were also



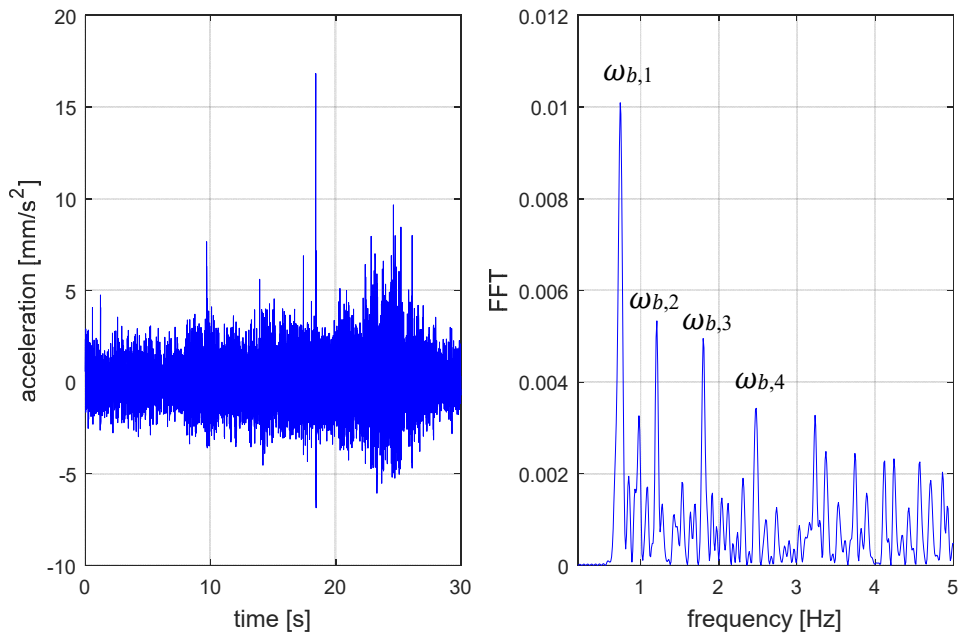
carried out in order to check the bridge frequencies. Frequencies below 0.5 Hz were filtered out from the spectrum, as they are mainly generated by road roughness and ambient noise. Four peaks can be identified from the Fourier spectrum, i.e.  $\omega_{b,1} = 0.74$  Hz,  $\omega_{b,2} = 1.19$  Hz,  $\omega_{b,3} = 1.73$  Hz and  $\omega_{b,4} = 2.47$  Hz. All of them are associated with the flexural vibration modes of the bridge, which can be easily identified. The acceleration responses recorded by SV and corresponding spectrum have been plotted in Figure 6.



**Figure 4:** The 200 kg scanning vehicle



**Figure 5:** Suspended footbridge at Lužec-Bukol



**Figure 6:** Bridge responses from indirect measurement and corresponding spectrum

## 12 CONCLUSIONS

The purpose of this paper was to experimentally investigate the possibility of identifying the fundamental frequency of a bridge by analysing the dynamic response of a vehicle as it traverses the bridge. In order to increase the flexibility and adaptability of a mobile data receiver (sensor) for indirect measurements, an instrumented test vehicle with adjustable frequencies was developed. Indirect bridge frequency measurement was carried out using this adaptable test vehicle and the experimental results were consistent with those obtained using the traditional direct method of bridge vibration monitoring. VSM measurement method can simplify the labor-intensive classical measurement and monitoring of bridge structures.

## 13 FORMAT OF REFERENCES

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