ACTIVE CONTROL OF NONLINEAR BASE-ISOLATED BUILDINGS

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ABSTRACT: A hybrid seismic control system for building structures is considered, which combines a class of passive nonlinear base isolator with an active control system. The active control forces are applied to the structural base with the objective of reducing its absolute displacements. An adaptive control law is formulated to compute the control forces to assure a form of stable behavior of the system under seismic excitation and in the presence of uncertainties in the characteristics of the building and the base isolator. Numerical simulations are performed to assess the effectiveness of the hybrid control system. The global behavior of the structure-base-isolator system is such that the absolute base displacement is significantly reduced, the price paid being a slight increase of the response of the structure.

INTRODUCTION

The main objective in the seismic design of a structure is to keep its response within limits defined by safety, service, and human comfort conditions. This can be achieved by applying traditional seismic design principles, which assume that earthquakes act upon the structure across its fixed base, to assure partial dissipation of the induced energy. Plastic deformation of certain members can occur and, as a result, the structure is damaged to a certain degree.

This disadvantage can be avoided by using passive control systems such as base isolators or a variety of energy dissipation devices. Base-isolation attempts to uncouple the structure from the seismic ground motion by means of replaceable devices, placed between the building and the foundation, capable of absorbing part of the energy induced by earthquakes. Both elastomeric bearings, which are hysteretic systems, and sliding bearings, which are frictional systems (Kelly 1986; Skinner et al. 1993), are used as seismic isolators. Base-isolation systems can provide, under certain circumstances, a level of performance beyond normal design requirements. The most important disadvantage of such systems is the dependence of their effectiveness on the frequency of the excitation. Moreover, they cannot be applied in the case of tall or heavy structures, because of the size of the dynamic forces involved and the risk of endangering the global stability of the structure.

The seismic behavior of buildings can be improved by incorporating into the structure mechanisms equipped with external actuators capable of producing control forces with the aim of reducing their seismic response. These mechanisms are included in a closed loop governed by a computer and they calculate in real time the control forces to be applied to the structure as a function of the measured structural response. Such active control systems are able to adapt themselves to the frequency characteristics of the seismic action (Soong 1990), but they may require large amounts of energy to govern the actuators.

Another class of structural control systems, referred to as

hybrid systems, has been suggested. They combine different passive and/or active systems with the objective of reducing some of the limitations. One of the hybrid systems with most interesting perspectives for buildings consists of the combination of a base isolation with an active system applying control forces to the base.

The base-isolation component can reduce by itself both the interstory drift and the absolute accelerations of the structure. Thus, the structure tends to behave like a rigid body, the price paid being a significant absolute displacement of the base. The objective of the active control component is to reduce this displacement by means of forces applied on the base. The active component of the system is introduced to complement the base isolation, reacting to the absolute motion, producing a resistance scheme not attainable by purely passive means when the structure is subjected to ground excitation (Inaudi and Kelly 1990). From a practical point of view, this hybrid scheme is appealing since it is possible to achieve the aforementioned objective by means of a single force, which does not exceed some acceptable limits due to the high flexibility of base isolators. From a theoretical point of view, the development of a control law to calculate the active force involves difficulties associated with the nonlinear behavior of the base isolators, with the uncertainties in the models describing the structure-base-isolator system, and in the seismic excitation. A robust control law for linear systems was proposed by Kelley et al. (1987). For linear systems, the application of predictive control was considered (Inaudi et al. 1992), as well as a form of the bang-bang control (Inaudi and Kelly 1991). The nonlinearity of the isolators was considered by Yang et al. (1992), assuming no uncertainties in the structure-base model. Some experimental works with small-scale hybrid systems were also reported (Feng et al. 1993; Riley et al. 1992; Nagarajaiah et al. 1992).

In this paper we present a new active control law for base-isolated structures. In subsequent sections the control problem is stated, a model of the system is described, the control strategy is presented, and some issues concerning its implementation are discussed. Numerical examples are presented to illustrate the effectiveness of the control strategy.

ACTIVE CONTROL PROBLEM

Consider the building with the hybrid control system illustrated in Fig. 1. The passive component consists of a base isolator, while the active component provides control forces applied on the structural base. We address the problem of designing a feedback control law to generate the active forces. Since the effect of these forces is additional to that of the base isolation, it is important to understand the essential behavior of the structure under purely passive conditions before defining the objective of the active control law.

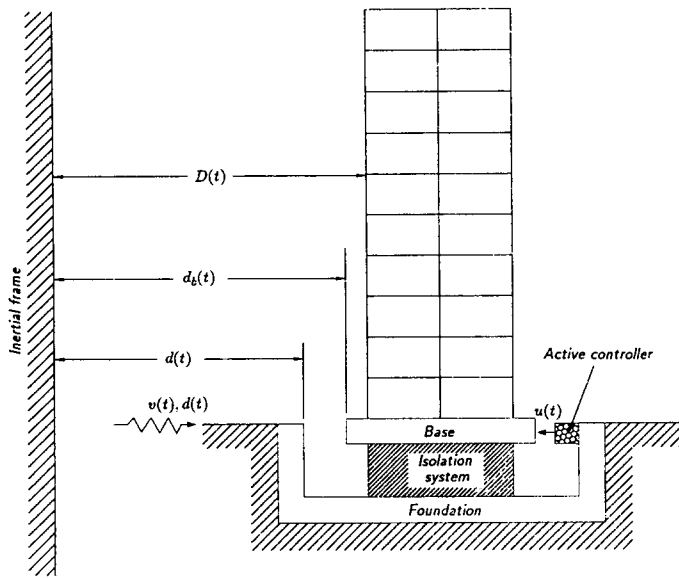When the parameters of the isolator are well tuned to the

FIG. 1. Building Structure with Hybrid Control System

characteristics of the earthquake, good performance of the structure, with a reduced interstory drift, may be expected. Nevertheless, this desirable interstory behavior can be associated with unacceptably high base displacements. Therefore, the main purpose of the active control forces is to reduce these base displacements. However, since the base and the stories are coupled, the application of forces at the base level can produce a negative effect, increasing the interstory drift as compared with that of the structure with purely passive control. Thus, the active control law has to limit this effect as well.

Some essential difficulties in the active control problem arise from the nonlinear behavior of the isolation system, the uncertainties in the models describing the structure and the isolator, and the uncertainties inherent to the seismic actions. These difficulties are magnified by the fact that the overall structure-base system is to be controlled by applying forces only to the base.

In an earlier paper (Rodellar and Ryan 1993) a control method was proposed for a class of uncertain mechanical systems that can be decomposed into two coupled subsystems, with feedback control on one of them. Under appropriate assumptions, this control assures a form of so-called practical stability. The control law has an adaptive nature that does not require a priori knowledge of either the system parameters or the external excitations. The hybrid structural control problem we are dealing with here can be cast within the framework of this class of systems, the base being the subsystem with control, and the structure the residual subsystem. The objective of the active control law is essentially to drive the displacement and velocity of the base asymptotically to an arbitrarily small neighborhood of the equilibrium position, while keeping the interstory drift within acceptable bounds.

SYSTEM DESCRIPTION

The structure's dynamic behavior with the hybrid control system can be described by means of a model composed of two coupled systems: Σ_r (the building or structure) and Σ_c (the base). It is assumed that the structure behaves linearly due to the effect of the base isolation. The behavior of the isolator is assumed to be nonlinear, with hysteretical and/or frictional characteristics. The position of the structure (at time t) is described by a vector $\mathbf{D}(t) \in R^n$, which represents the horizontal displacements of the n degrees of freedom respect

to an inertial frame, while the displacement of the structural base is described by a single degree of freedom with horizontal displacement $d_b(t) \in R$ relative to the abovementioned frame. The dynamic excitation is produced by a horizontal seismic ground motion, characterized by an inertial displacement function $t \to d(t)$: the function $d(\cdot)$ is assumed to be absolutely continuous on the compact subintervals of $(0, \infty)$ and so is differentiable almost everywhere (with derivative $\dot{d} = v$). A single horizontal control force $u(t)$ acts on the structural base. Thus, the equations of motion are

$$\textstyle\sum_r: \mathbf{M\ddot{D}} + \mathbf{C\dot{D}} + \mathbf{KD} = \mathbf{CJ}\dot{d}_b + \mathbf{KJ}d_b;$$

$$\mathbf{D}(0) = \mathbf{D}^0 \in R^n, \dot{\mathbf{D}}(0) = \mathbf{V}^0 \in R^n \qquad (1a)$$

$$\textstyle\sum_c: m_b\ddot{d}_b + [c_b + \mathbf{J}^T\mathbf{CJ}]\dot{d}_b + [k_b + \mathbf{J}^T\mathbf{KJ}]d_b$$

$$- \mathbf{J}^T\mathbf{C\dot{D}} - \mathbf{J}^T\mathbf{KD} - c_b v - k_b d + f(d_b, \dot{d}_b, d, v) = u;$$

$$d_b(0) = d_b^0 \in R, \dot{d}_b(0) = v_b^0 \in R \qquad (1b)$$

where $\mathbf{M}, \mathbf{C},$ and $\mathbf{K}$ = mass, damping, and stiffness matrices of the structure, respectively, each of dimension $n \times n$. The vector $\mathbf{J}$ = rigid body motion according to the degrees of freedom of the model (in this case it is the unit vector $\mathbf{J} = [1, 1, \ldots, 1, 1]$); and scalars $m_b, cx_b,$ and k_b = mass, damping, and stiffness of the base. The last two parameters correspond to the elastic and damping forces that appear on the base due to the linear effects of the isolator; f is an additional horizontal force produced on the structural base by nonlinearities in the isolator.

Before formulating the control law, the description of the system is completed based on the following assumptions:

1. The matrices $\mathbf{M}, \mathbf{C},$ and $\mathbf{K}$ are positive definite with $\mathbf{M}$ and $\mathbf{K}$ symmetric.
2. The displacement d (respectively, velocity $v = \dot{d}$) of the seismic ground motion is bounded (respectively, essentially bounded) and so

$$|c_b v(t) + k_b d(t)| \le \nu \qquad (2)$$

for almost all t, ν being an unknown scalar.
3. The function f is such that, for some known continuous function γ', the following holds for some (unknown) scalar α'

$$|f(d_b, v_b, d(t)v(t))| \le \alpha'\gamma'(d_b, v_b) \qquad (3)$$

for almost all t and all d_b and v_b.

Remarks: Assumption 1 ensures that the equilibrium $\{0\}$ of the unforced system $\mathbf{M\ddot{D}} + \mathbf{C\dot{D}} + \mathbf{KD} = 0$ is (globally) asymptotically stable. Assumption 3 is weak. For example, if it is known only that f depends polynomially (of any degree) on its first two arguments, then 3 holds with $\gamma'(d_b, v_b) = \exp(\sqrt{d_b^2 + v_b^2})$.

CONTROL STRATEGY

Fix $\eta > 0$, $\lambda > 0$ and $\bar{k} > 0$ (design parameters). Write

$$p_b(t) = \eta d_b(t) + \dot{d}_b(t) \qquad (4)$$

and define

$$\gamma(\mathbf{D}, \dot{\mathbf{D}}, d_b, p_b) = [1 + [\gamma'(d_b, p_b - \eta d_b)]^2 + d_b^2 + p_b^2$$

$$+ \|\mathbf{D}\|^2 + \|\dot{\mathbf{D}}\|^2]^{1/2} \qquad (5)$$

The proposed control strategy is

$$u(t) = -k(t)[p_b(t) + \gamma(\mathbf{D}(t), \dot{\mathbf{D}}(t), d_b(t), p_b(t))s_\lambda(p_b(t))] \qquad (6a)$$

$$\dot{k}(t) = \bar{k}[|p_b(t)| + \gamma(\mathbf{D}(t), \dot{\mathbf{D}}(t), d_b(t), p_b(t))]\rho_\lambda(p_b(t)) \qquad (6b)$$

$$k(0) = k^0 \text{ (initial condition)} \tag{6c}$$

where the continuous functions s_λ, ρ_λ are given by

$$s_\lambda(p_b) = \begin{cases} |p_b|^{-1}p_b, & \text{if } |p_b| \geq \lambda \\ \lambda^{-1}p_b, & \text{if } |p_b| < \lambda \end{cases} \tag{7a}$$

$$\rho_\lambda(p_b) = \begin{cases} |p_b| - \lambda, & \text{if } |p_b| \geq \lambda \\ 0, & \text{if } |p_b| < \lambda \end{cases} \tag{7b}$$

The rationale for this control strategy lies in its capacity to ensure a form of practical stability, as we show in the next section. The intuition underpinning the strategy is as follows. If the (continuous) saturation function s_λ is replaced by the (discontinuous) signum function sgn, then, for sufficiently large $\bar{k}$, the control

$$u = -\bar{k}[p_b + \lambda(\mathbf{D}, \dot{\mathbf{D}}, d_b, p_b)\text{sgn}(p_b)]$$

ensures that the subspace $\{(\mathbf{D}, \dot{\mathbf{D}}, d_b, p_b) \in R^{2(n+1)}|p_b = 0\}$ is attained in finite time, and the motion is subsequently constrained thereon: by the differential equation $\dot{d}_b = -\eta d_b + p_b$, we conclude that $d_b(t)$ tends exponentially to zero, with the decay rate determined by the parameter η. By the differential equation defining subsystem Σ_r and its stability in the absence of force (by assumption 1), it follows that $\mathbf{D}(t)$ and $\dot{\mathbf{D}}(t)$ are also asymptotic to zero, from which the stability of the overall system results. Thus, the discontinuous strategy with fixed gain $\bar{k}$ is closely akin to concepts in variable structure control [see, for example, Utkin (1978)]. Here, we circumvent some of the practical and technical drawbacks associated with discontinuous feedback through the adoption of the saturation function in place of the signum function. Synthesis of a fixed gain strategy requires a priori information on structure parameters and disturbance sufficient for the computation of gain $\bar{k}$. Here, we circumvent this requirement by adaptation.

STABILITY ANALYSIS

The overall controlled system representation on R^N, $N: = 2n + 3$, now becomes

$$\mathbf{M}\ddot{\mathbf{D}}(t) + \mathbf{C}\dot{\mathbf{D}}(t) + \mathbf{K}\mathbf{D}(t) = \mathbf{C}\mathbf{J}[p_b(t) - \eta d_b(t)] + \mathbf{K}\mathbf{J}d_b(t);$$

$$\dot{d}_b(t) = -\eta d_b(t) + p_b(t);$$

$$\dot{p}_b(t) = P_\lambda(t, \mathbf{D}(t), \dot{\mathbf{D}}(t), d_b(t), p_b(t), k(t));$$

$$\dot{k}(t) = \bar{k}[|p_b(t)| + \gamma(\mathbf{D}(t), \dot{\mathbf{D}}(t), d_b(t), p_b(t))]\rho_\lambda(p_b(t));$$

$$(\mathbf{D}(0), \dot{\mathbf{D}}(0), d_b(0), p_b(0), k(0)) = (\mathbf{D}^0, \mathbf{V}^0, d_b^0, p_b^0, k^0)$$

$$= :\xi^0 \in R^N \tag{8}$$

where the function P_λ is given by

$$P_\lambda(t, \mathbf{D}, \dot{\mathbf{D}}, d_b, p_b, k): = -\eta^2 d_b + \eta p_b$$

$$+ m_b^{-1}[g(t, \mathbf{D}, \dot{\mathbf{D}}, d_b, p_b)$$

$$- k[p_b + \gamma(\mathbf{D}, \dot{\mathbf{D}}, d_b, p_b)s_\lambda(p_b)]]$$

with

$$g(t, \mathbf{D}, \dot{\mathbf{D}}, d_b, p_b): = \mathbf{J}^T[\mathbf{C}\dot{\mathbf{D}} + \mathbf{K}\mathbf{D}]$$

$$+ c_b v(t) + k_b d(t) - f(d_b, p_b - \eta d_b, d(t), v(t))$$

$$- [c_b + \mathbf{J}^T\mathbf{C}\mathbf{J}][p_b - \eta d_b] - [k_b + \mathbf{J}^T\mathbf{K}\mathbf{J}]d_b$$

Observe that, for almost all t and all $\mathbf{D}$, $\dot{\mathbf{D}}$, d_b, p_b

$$|g(t, \mathbf{D}, \dot{\mathbf{D}}, d_b, p_b)| \leq \alpha\gamma(\mathbf{D}, \dot{\mathbf{D}}, d_b, p_b)$$

for some (unknown) constant $\alpha > 0$.

Writing $\xi(t) = (\mathbf{D}(t), \dot{\mathbf{D}}(t), d_b(t), p_b(t), k(t))$, system (8) may be expressed as

$$\dot{\xi}(t) = F_\lambda(t, \xi(t)), \xi(0) = \xi^0 \tag{9}$$

where

$$F_\lambda(\xi): = (\dot{\mathbf{D}}, \mathbf{M}^{-1}[-\mathbf{C}\dot{\mathbf{D}} - \mathbf{K}\mathbf{D} + \mathbf{C}\mathbf{J}[p_b - \eta d_b]$$

$$+ \mathbf{K}\mathbf{J}d_b], p_b - \eta d_b, P_\lambda(t, \xi), K_\lambda(\xi))$$

with

$$K_\lambda(\xi): = \bar{k}[|p_b| + \gamma(\mathbf{D}, \dot{\mathbf{D}}, d_b, p_b)]\rho_\lambda(p_b)$$

This system satisfies the classical Carathéodory conditions and so, for every $\xi^0 \in R^N$, the initial-value problem has a solution and every solution can be extended into a maximal solution (Coddington and Levinson 1953).

Theorem: Let $\lambda > 0$ and $\xi^0 \in R^N$. For every maximal solution

$$\xi(\cdot) = (\mathbf{D}, \dot{\mathbf{D}}, d_b, p_b, k)(\cdot): [0, \omega] \to R^N$$

of the initial-value problem (9) [equivalently (8)]: (1) $\omega = \infty$; (2) $\lim_{t\to\infty} k(t)$ exists and is finite; (3) $\rho_{\lambda\eta^{-1}}(d_b(t))$ and $\rho_\lambda(p_b(t)) \to 0$ as $t \to \infty$; (4) for some positive scalar c, $\rho_{c\lambda}(\sqrt{\|\mathbf{D}(t)\|^2 + \|\dot{\mathbf{D}}(t)\|^2}) \to 0$ as $t \to \infty$. Proof: See Appendix I. Remarks: The preceding result may be paraphrased as follows: (1) Forward-time solutions are globally defined; (2) the monotone gain function does not grow unbounded; (3) the state of the base, characterized by the coordinates (d_b, p_b), tends asymptotically to a ball of any prescribed radius $\Lambda = \lambda\sqrt{1 + \eta^{-2}} > 0$ centered in zero; (4) the state of the structure, characterized by the vectors $\mathbf{D}$ and $\dot{\mathbf{D}}$, tends asymptotically to a ball (centred at zero), with radius proportional to λ, however, the proportionality constant depends on unknown bounds on the system uncertainties and cannot be calculated a priori.

It is important to emphasize the paucity of knowledge on the system that is required a priori. From the control law, it is apparent that parameters of the system such as masses, damping, and stiffness need not be known to the designer. The external seismic excitation is unknown, assuming only that it is bounded by an unknown constant, as in (2). Regarding the nonlinear force f produced by the isolators, the control strategy allows it to be unknown but bounded, modulo arbitrary scaling, by a known continuous function as in (3). This function enters into the control law in the definition of the function γ in (5). The adaptive nature of the control law, associated with the time-varying gain $k(t)$, guarantees the stability properties for any realization of the unknown parameters satisfying assumptions 1–3.

For the implementation of the control law, the absolute displacement and velocity responses of the base and the structure are required as feedback information. With this information, (6) and (7) are used to calculate the value of the control $u(t)$. The parameters (positive-valued) η, λ, k_0, and $\bar{k}$ are open to choice by the designer. $\Lambda = \lambda\sqrt{1 + \eta^{-2}}$ is the most significant quantity. It defines the guaranteed stability ball and has a primary influence on achieving the control objective.

NUMERICAL EXAMPLES

To assess the effectiveness of the aforementioned control law, numerical simulations are presented for two types of base-isolated structures. In the first example, the structure is modeled as a single-degree-of-freedom system. The second example considers a 10-story shear building.

For the force f, we need to specify the model describing
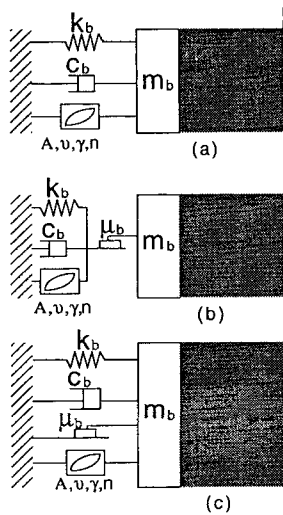
FIG. 2. Base-Isolation Systems: (a) New Zealand (NZ); (b) NZ with Frictional System Connected in Series (NZ + F–SER); (c) NZ with Frictional System Connected in Parallel (NZ + F–PAR)

the nonlinear behavior of the isolating devices. Two types of behavior are considered, one frictional and the other hysteretical. In Fig. 2, three isolators are defined for different combinations of the two types of behavior. These are the New Zealand (NZ) isolator [Fig. 2(a)] (Su et al. 1989), characterized by a purely hysteretical performance, the NZ with a purely frictional system connected in series (NZ + F–SER) [Fig. 2(b)], and the NZ with a purely frictional system connected in parallel (NZ + F–PAR) [Fig. 2(c)].

For frictional isolation devices, the formulation of a purely frictional force f_1 was proposed by Nagarajaiah et al. (1989, 1991) by an equation

$$f_1 = -\text{sgn}(x)[\mu_{max} - \Delta \mu e^{\beta|x|}]Q \tag{10}$$

where Q = force normal to the friction surface; μ = friction coefficient; β = a constant; μ_{max} = coefficient for high sliding velocity; $\Delta\mu$ = difference between μ_{max} and the friction coefficient for low sliding velocity, which is considered to be constant; and x = displacement of the base relative to the ground, i.e., $x = d_b - d$.

One way to formulate a hysteretical isolation device is to use constitutive models defined by differential equations. In this paper, with the purpose of illustrating the application of the proposed control law, Wen's (1976) uniaxial model is adopted. The hysteretic force f_2 is expressed by the following equation:

$$f_2 = f^y z \tag{11}$$

where f^y = yielding force; and z = an auxiliary variable defined by the differential equation

$$\frac{dz}{dx} = A \pm (v_1 \pm v_2)z^m \tag{12}$$

The parameters A, v_1, v_2, and m allow the description of the hysteretical cycles for a wide class of materials, ranging from elastic to elastoplastic materials (Wen 1976).

To implement the control law, the force f is required to be bounded in the form expressed in (3). In the cases we consider here, f is a combination (as illustrated in Fig. 2) of forces of types defined by (8)–(10). Therefore, we consider that the force f is bounded by a constant. This implies that the function γ' in (3), also appearing in the control law in (5), is just equal to 1. It simplifies the implementation of the control law and, as ultimately verified through the numerical simulations, gives satisfactory results.

Example 1

The purpose of this example is to analyze the effectiveness of the three hybrid control systems with the proposed control law for a frequency range covering a wide class of building structures. This effectiveness is compared with that of the corresponding purely passive systems. To do this, a single-degree-of-freedom model with a mass of 6×10^5 kg is considered. Its stiffness is varied in such a way that the natural period lies in the range between 0.1 s and 1.0 s. The mass of the base is also 6×10^5 kg, and its stiffness is 1.184×10^7 N/m. The characteristics of the frictional bearings are: $\mu =$
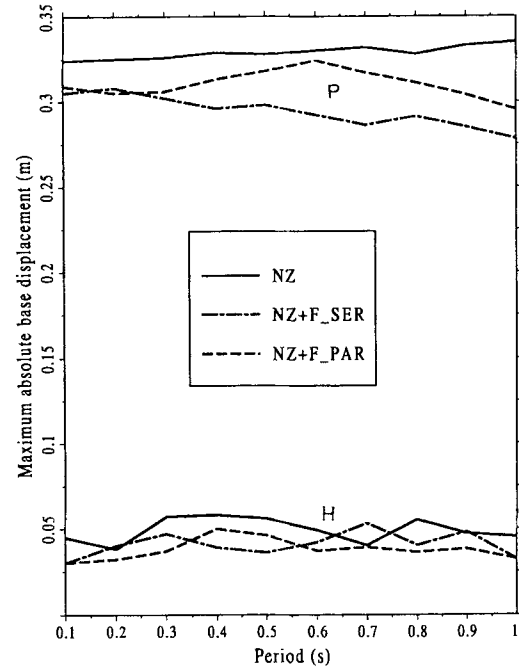

FIG. 3. Maximum Absolute Base Displacement against Structural Period for Single-Degree-of-Freedom Model with Passive (P) and Hybrid (H) Control Systems
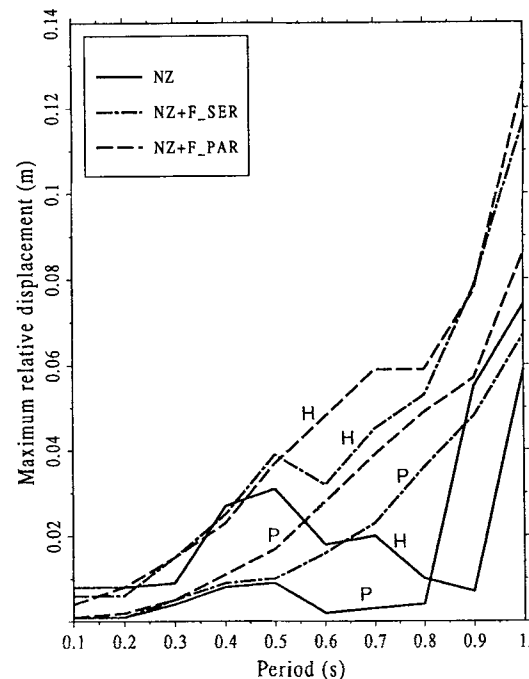

FIG. 4. Maximum Relative Displacement against Structural Period for Single-Degree-of-Freedom Model with Passive (P) and Hybrid (H) Control Systems
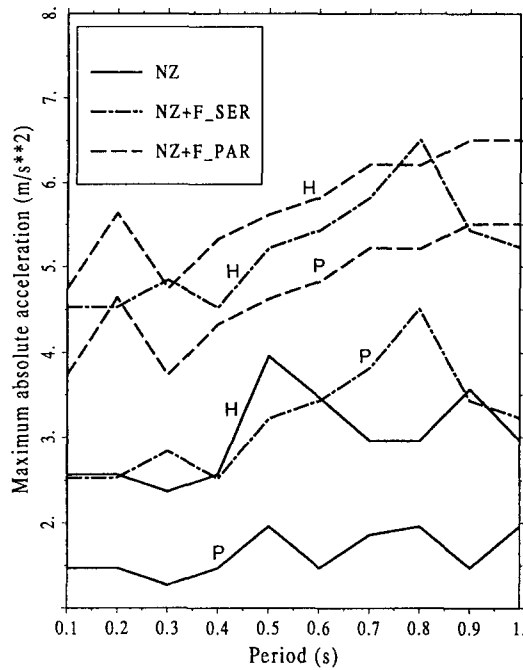
FIG. 5. Maximum Absolute Acceleration against Structural Period for Single-Degree-of-Freedom Model with Passive (*P*) and Hybrid (*H*) Control Systems
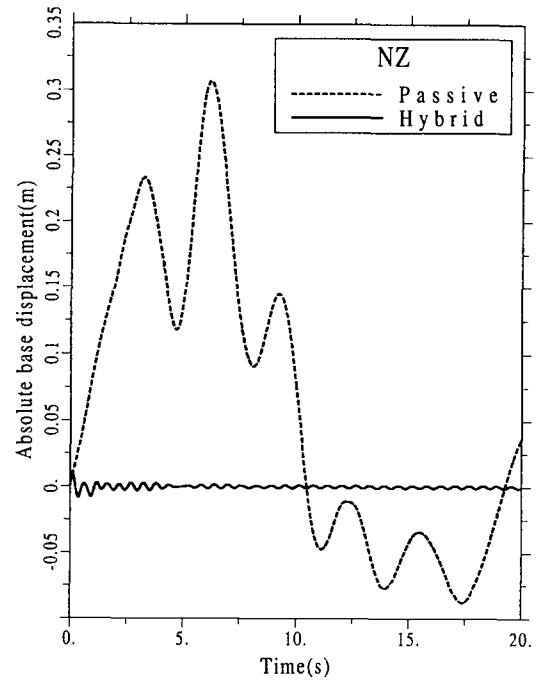

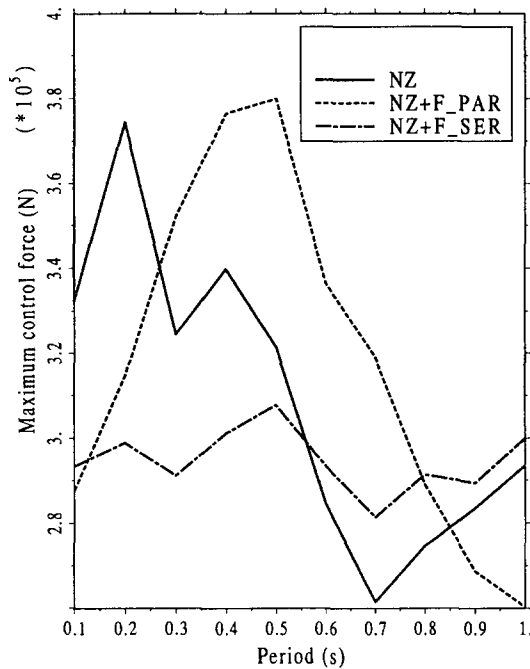FIG. 7. Absolute Base Displacement of 10-Degrees-of-Freedom Model

FIG. 6. Maximum Control Force against Structural Period for Single-Degree-of-Freedom Model with Hybrid Control Systems

FIG. 8. Top Floor Displacement Relative to Base of 10-Degrees-of-Freedom Model

0.1, $\mu_{max} = 0.185$, $\Delta\mu = 0.09$, $\beta = 2.0$. For the hysteretic bearings, the following parameters are used: $f^y = 1.5 \times 10^3$ N, $A = 1.0$, $\nu_1 = \nu_2 = 0.5$, and $m = 1$.

The model is subjected to the 1940 El Centro earthquake. For each natural period considered in the analysis, the structural response is simulated solving the equations of motion (1) by a Newmark integration method, and (10) by Runge-Kutta scheme of the fourth order. This simulation is performed for the three isolators under purely passive conditions (active control is disconnected) and for the hybrid case (active control is operating). The parameters associated with the con-
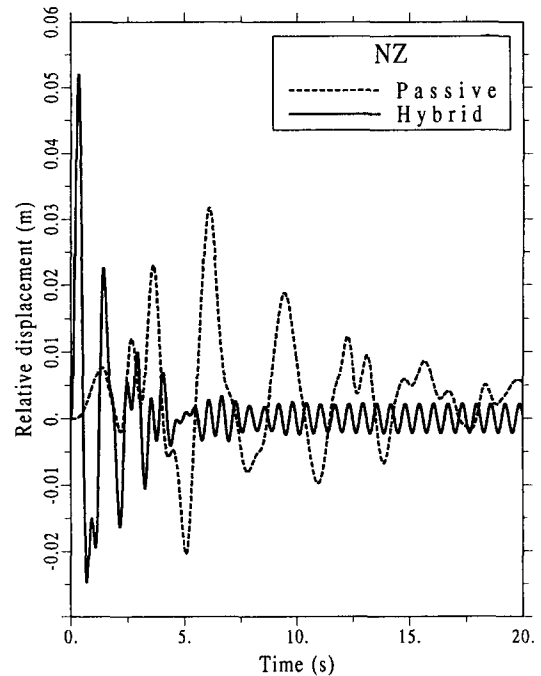
trol law have been chosen: $\eta = 1$, $\lambda = 0.42$, $\bar{k} = 10^7$, and $k_0 = 0$.

The maximum response values are plotted in Figs. 3–5 against the natural period: the absolute base displacement in Fig. 3, the displacement of the structure relative to the base in Fig. 4, and the absolute acceleration of the structure in Fig. 5. These figures show that the maximum absolute displacement is significantly reduced by the hybrid system. This was the main objective of the proposed control law. The price paid is the increase of the maximum relative displacement and the maximum absolute acceleration of the structure, which still remain within adequate bounds. In Fig. 6 the maximum
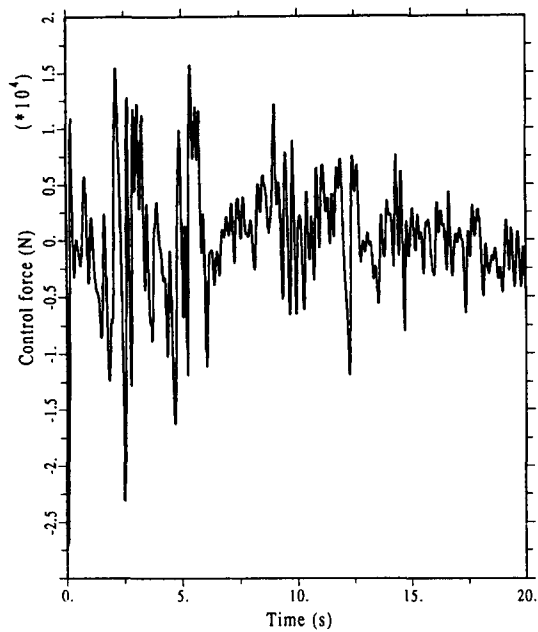
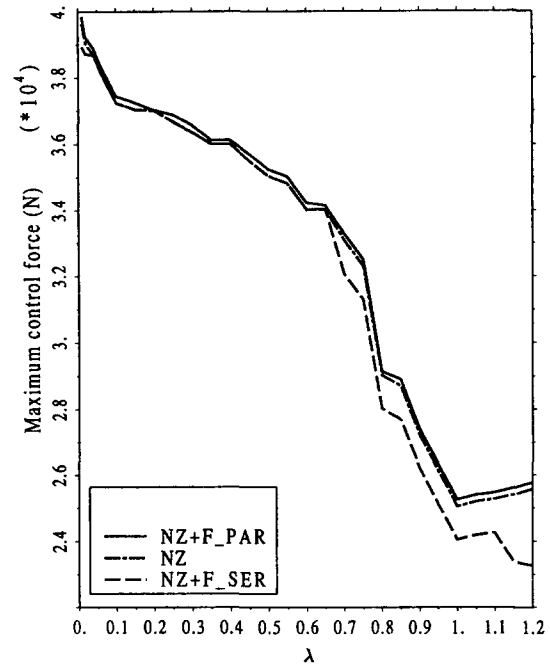FIG. 9. Active Control Force for 10-Degrees-of-Freedom Model

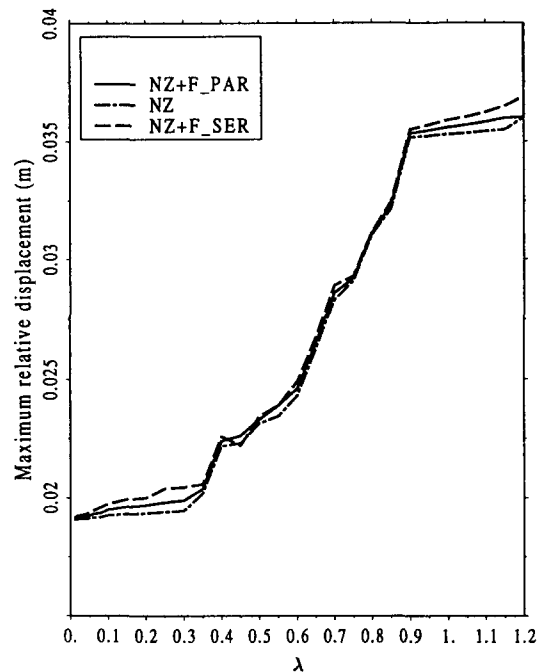FIG. 11. Maximum Control Force for 10-Degrees-of-Freedom Model for Different Values of λ

FIG. 10. Maximum Top Floor Relative Displacement for 10-Degrees-of-Freedom Model for Different Values of λ

FIG. 12. Variation of Maximum Interstory Drift Ratio with Floor Level

value of the control force is plotted against the period for the three isolators, showing an acceptable magnitude.

Example 2

To assess the behavior of the hybrid system in a more demanding situation, consider a 10-story building structure as illustrated in Fig. 1. The mass of each level, including that of the base, is 6×10^5 kg. The stiffness varies in 5×10^7 N/m between levels, from 9×10^8 N/m at the first level to 4.5×10^8 N/m at the top level. The damping ratio is 0.05. The characteristics of the base-isolation devices are the same as those in the single-degree-of-freedom case.

The control law has been applied to the New Zealand base isolator and the 1940 El Centro earthquake, considered as
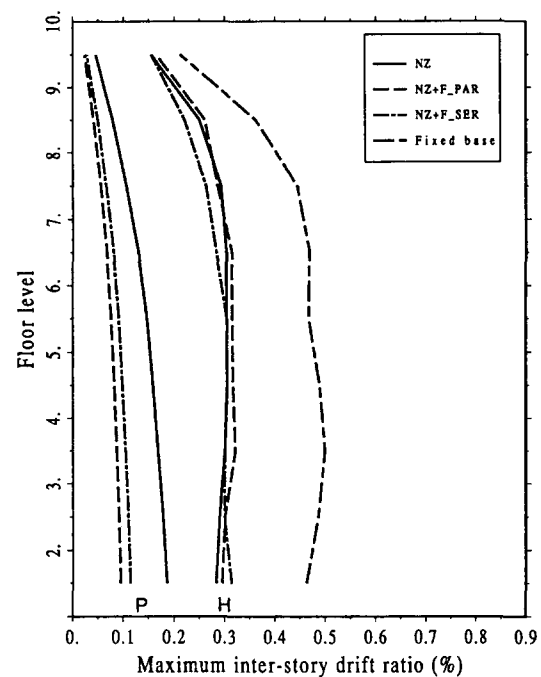
excitation. Figs. 7 and 8 show the time histories of the absolute base displacement and the displacement of the top floor relative to the base for the passive and hybrid cases. Fig. 9 shows the corresponding control force for the hybrid case. In these three figures the parameter λ is equal to 0.9. Fig. 7 shows that the absolute base displacement in the hybrid case rapidly enters into a neighborhood around zero, with a response level much lower than the corresponding response to the passive case. Fig. 8 shows that there is an initial period in which the hybrid relative displacement of the top floor is higher than the passive one, but it reduces more rapidly than in the passive case to a neighborhood centered at zero. The behaviors observed in Figs. 7 and 8 fulfill the theoretical objective of the
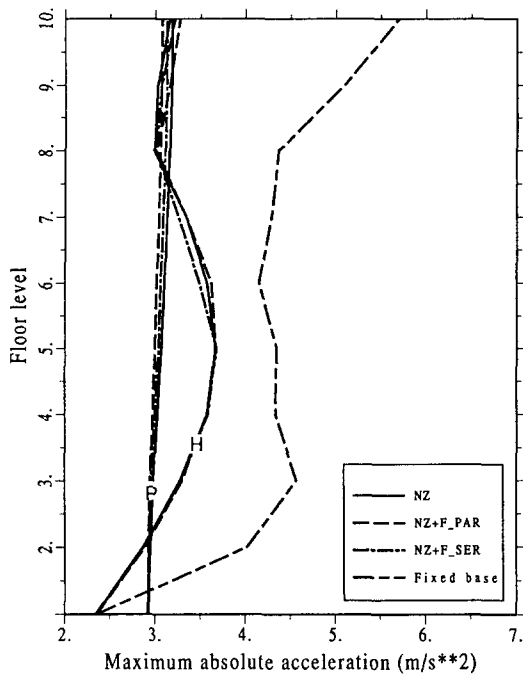
FIG. 13. Variation of Maximum Absolute Acceleration with Floor Level

control law. Fig. 9 shows that the corresponding control force is reasonable.

As mentioned before, λ is the most important parameter in the control law, because it defines the size of the neighborhood to which the controlled base response has to approach. To evaluate its influence, Figs. 10 and 11 show the maximum values of the relative top-floor displacement and the control force for different values of λ. An increase of λ produces higher maximum values of the relative displacement and lower maximum control forces. This is as expected, because an increase of λ implies a less demanding control.

For a more detailed insight on the structural behavior, it can be useful to examine the interstory drift ratio, which is defined as the difference between the displacements of two adjacent floors divided by the floor height. Fig. 12 describes the variation of the maximum interstory drift ratio with the floor level for the three base isolators in the purely passive and hybrid cases. For comparison purpose, the same is shown for the structure with the fixed base. An increase of the maximum values for the hybrid cases is observed as compared with the passive cases. A similar pattern can be seen in Fig. 13, in which the maximum floor accelerations are plotted for the same cases. Nevertheless, when considering both the structure and the base, an improvement in the global behavior can be recognized.

CONCLUSIONS

The hybrid control system discussed in this paper combines a nonlinear base-isolation component with an active one applying forces at the base level. The main purpose was to propose a new feedback control law for the active component, and to explore its potential through simulation examples.

The objective for the control law was posed in terms of stability, and two very important features present in hybrid systems have been taken into account: (1) uncertainties in knowledge on the structure, the isolators, and the excitation; and (2) the nonlinear behavior of the isolator devices. Regarding the uncertainties, knowledge of the parameters of the base and structure model are not required in the control law, because of its adaptive nature. Concerning the nonlinearities

in the isolator, for illustrative purposes, a variety of hysteretic and frictional characteristics were considered in the examples.

In the evaluation of the structural behavior, the following variables were checked: the absolute base displacement, the interstory drift, and the absolute acceleration of the structure. The numerical results show that the active control force significantly reduces the base displacement of the structure, producing a slight increase in the structural response, with respect to the purely passive case. However, the global structural behavior improves when the hybrid system is used. This can be particularly significant to excitations with predominant frequencies in the range in which the purely passive base-isolated structure has its maximum response.

The emphasis of this paper was on the adaptive control law, with the examples illustrating its features. The results presented show a clear potential for this control law. Its capabilities need to be more extensively assessed by means of a systematic numerical application in a variety of operating conditions: different structural models, isolator nonlinearities, seismic excitations, etc., completed with experimental verification.

APPENDIX I. PROOF OF THEOREM

Let V be the C^1 function $R \rightarrow [0, \infty)$ given by

$$V(p_b): = \frac{1}{2} \rho_\lambda^2(p_b) \tag{13}$$

For almost all $t \in R$ and all $\xi = (\mathbf{D}, \dot{\mathbf{D}}, d_b, p_b, k) \in R^N$, we have

$$\langle \Delta V(p_b), P_\lambda(t, \xi) \rangle \leq \rho_\lambda(p_b)[\eta^2|d_b|$$
$$+ m_b^{-1}(\alpha - k)\gamma(\mathbf{D}, \dot{\mathbf{D}}, d_b, p_b) + (\eta - m_b^{-1}k)|p_b|] \tag{14}$$

Defining $k^*: = \alpha + m_b\eta$, it follows that

$$\frac{d}{dt} V(p_b(t)) \leq -m_b^{-1}(k(t) - k^*)\dot{k}(t) + \eta^2\rho_\lambda(p_b(t))|d_b(t)| \tag{15}$$

for almost all $t \in [0, \omega)$. Integration yields, with $c_0: = V(p_b^0) + 1/2m_b^{-1}(k^0 - k^*)^2$

$$0 \leq V(p_t(t)) \leq c_0 - \frac{1}{2} m_b^{-1}(k(t) - k^*)^2$$
$$+ \eta^2 \int_0^t \rho_\lambda(p_b(s))|d_b(s)|ds \tag{16}$$

which is valid for all $t \in [0, \omega)$.

We briefly digress to prove a technicality. Proposition: For some positive scalar c_1

$$\int_0^t \rho_\lambda(p_b(s))|d_b(s)|ds \leq c_1 \int_0^t [\rho_\lambda(p_b(s)) + \rho_\lambda^2(p_b(s))]ds \tag{17}$$

for all $t \in [0, \omega)$. Proof: First observe that by the differential equation $\dot{d}_b = -\eta d_b + p_b$

$$|d_b(s)| \leq |d_b^0| + \int_0^s e^{-\eta(s-\sigma)}|p_b(\sigma)|d\sigma \leq |d_b^0|$$
$$+ \int_0^s e^{-\eta(s-\sigma)}[\rho_\lambda(p_b(\sigma)) + \lambda]d\sigma \leq |d_b^0| + \lambda\eta^{-1}$$
$$+ \int_0^s e^{-\eta(s-\sigma)}\rho_\lambda(p_b(\sigma))d\sigma \tag{18}$$

Therefore

$$\int_0^t \rho_\lambda(p_b(s))|d_b(s)|ds \le (|d_b^0| + \lambda\eta^{-1}) \int_0^t \rho_\lambda(p_b(s))ds$$

$$+ \int_0^t \rho_\lambda(p_b(s)) \int_0^s e^{-\eta(s-\sigma)}\rho_\lambda(p_b(\sigma))d\sigma ds \quad (19)$$

Applying Hölder's inequality to the second term on the right

$$\int_0^t \rho_\lambda(p_b(s)) \int_0^s e^{-\eta(s-\sigma)}\rho_\lambda(p_b(\sigma))d\sigma ds \le \left(\int_0^t \rho_\lambda^2(p_b(s))ds\right)^{1/2}$$

$$\cdot \left(\int_0^t e^{-2\eta s}\left(\int_0^s e^{\eta\sigma}\rho_\lambda(p_b(\sigma))d\sigma\right)^2 ds\right)^{1/2} \quad (20)$$

Integrating by parts in the last term on the right

$$\int_0^t e^{-2\eta s}\left(\int_0^s e^{\eta\sigma}\rho_\lambda(p_b(\sigma))d\sigma\right)^2 ds$$

$$\le \eta^{-1} \int_0^t \rho_\lambda(p_b(s)) \int_0^s e^{-\eta(s-\sigma)}\rho_\lambda(p_b(\sigma))d\sigma ds \quad (21)$$

We may now conclude that

$$\int_0^t \rho_\lambda(p_b(s)) \int_0^s e^{-\eta(s-\sigma)}\rho_\lambda(p_b(\sigma))d\sigma ds \le \eta^{-1} \int_0^t \rho_\lambda^2(p_b(s))ds \quad (22)$$

from which the claim.

Returning to the proof of the theorem and noting that

$$\rho_\lambda(p_b(t)) + \rho_\lambda^2(p_b(t)) \le (1 + \lambda^{-1})\rho_\lambda(p_b(t))|p_b(t)|$$

$$\le \bar{k}^{-1}(1 + \lambda^{-1}\dot{k}(t) \quad (23)$$

we now have

$$0 \le V(p_b(t)) \le c_0 - \frac{1}{2} m_b^{-1}(k(t) - k^*)^2$$

$$+ \eta^2 c_1 \bar{k}^{-1}(1 + \lambda^{-1})(k(t) - k^0) \quad (24)$$

for all $t \in [0, \omega)$. Therefore, we see that the monotone increasing function $k(\cdot)$ is bounded. This, in turn, implies boundedness of $V[p_b(\cdot)]$ and so $p_b(\cdot)$ is bounded. It immediately follows that $d_b(\cdot)$ is bounded. By assumption 1, we see that $\{0\}$ is an asymptotically stable equilibrium of the unforced subsystem

$$\mathbf{M\ddot{D} + C\dot{D} + KD = 0} \quad (25)$$

By the boundedness of $d_b(\cdot)$ and $p_b(\cdot)$, it follows from the forced equation

$$\mathbf{M\ddot{D} + C\dot{D} + KD = CJ}[p_b - \eta d_b] + \mathbf{KJ}d_b \quad (26)$$

that $\mathbf{D}(\cdot)$ and $\mathbf{\dot{D}}(\cdot)$ are bounded. We have now shown that the solution $\xi(\cdot)$ is bounded and so $\omega = \infty$. Assertion (2) of the theorem is now a consequence of the boundedness and monotonicity of $k(\cdot)$.

To prove assertion (3), we argue as follows. Observe that

$$\rho_\lambda(p_b(t))|d_b(t)| \le \rho_\lambda(p_b(t))\lambda^{-1}|p_b(t)|\|d_b(t)|$$

$$\le (\bar{k}\lambda)^{-1}|d_b(t)|\dot{k}(t) \quad (27)$$

and so, by the boundedness of $d_b(\cdot)$, there exists positive scalar c_2 such that

$$\eta^2 \rho_\lambda(p_b(t))|d_b(t)| \le c_2\dot{k}(t) \quad (28)$$

for almost all $t \ge 0$. Writing $c_3 := m_b^{-1}(k^* - k^0) + c_2$, we conclude that

$$\frac{d}{dt} V(p_b(t)) \le -\dot{k}(t) + (c_3 + 1)\dot{k}(t) \quad (29)$$

for almost all $t \ge 0$. Therefore the function W given by

$$W(p_b, k) := V(p_b) - (1 + c_3)k \quad (30)$$

is such that

$$\frac{d}{dt} W(p_b(t), k(t)) \le -\dot{k}(t) \le -\bar{k}_{\rho_\lambda}(p_b(t))|p_b(t)| \quad (31)$$

for almost all $t \ge 0$. The boundedness of the solution $\xi(\cdot)$ ensures that it has a nonempty ω-limit set Ω. Since the solution approaches its ω-limit set, we prove that

$$\rho_\lambda(p_b(t)) \to 0 \text{ as } t \to \infty \quad (32)$$

by showing that $\rho_\lambda(\bar{p}_b) = 0$ for all $\bar{\xi} = (\bar{\mathbf{D}}, \bar{\mathbf{V}}, \bar{d}_b, \bar{k}) \in \Omega$. Suppose otherwise. Then there exists $\bar{\xi} = (\bar{\mathbf{D}}, \bar{\mathbf{V}}, \bar{d}_b, \bar{p}_b, \bar{k}) \in \Omega$ and $\varepsilon > 0$, such that $\rho_\lambda(\bar{p}_b)|\bar{p}_b| > 2\varepsilon$. By continuity, there exists $\delta > 0$ such that

$$|p_b - \bar{p}_b| < \hat{\delta} \Rightarrow \bar{k}_{\rho_\lambda}(p_b)|p_b| > \varepsilon \quad (33)$$

Since $\bar{\xi}$ is an ω-limit point, there exists a sequence (t_j) with $t_j \to \infty$ and

$$\xi(t_j) = (\mathbf{D}(t_j), \dot{\mathbf{D}}(t_j), d_b(t_j), p_b(t_j), k(t_j)) \to \bar{\xi} \quad (34)$$

as $j \to \infty$. Using assumptions 2 and 3, it is readily verified that $\tilde{\delta} > 0$ and $R > 0$ exist such that

$$\|\xi - \bar{\xi}\| < \tilde{\delta} \Rightarrow \|F_\lambda(t, \xi)\| < R \quad (35)$$

We may assume $\tilde{\delta} < \hat{\delta}$. By the continuity of W

$$W(p_b(t_j), k(t_j)) - W(\bar{p}_b, \bar{k}) < \frac{\varepsilon\tilde{\delta}}{4R} \quad (36)$$

for all sufficiently large j. Let j^* be such that

$$\|\xi(t_j) - \bar{\xi}\| < \frac{1}{2}\tilde{\delta} \ \forall j > j^* \quad (37)$$

By (8) and (35), it follows that

$$\|p_b(t_j) - \bar{p}_b\| < \tilde{\delta} \ \forall t \in \left[t_j, t_j + \frac{\tilde{\delta}}{3R}\right] \quad (38)$$

which holds for all $j > j^*$. Therefore, using (31), we have for all $j > j^*$

$$W(p_b(t_j),k(t_j)) - W(\bar{p}_b, \bar{k}) > \int_{t_j}^{t_j + (\tilde{\delta}/3R)} \bar{k}\rho_\lambda(p_b(t))|p_b(t)|dt \ge \frac{\varepsilon\tilde{\delta}}{3R}$$

$$(39)$$

which contradicts (36). Therefore, $\rho_\gamma(p_b(t)) \to 0$ as $t \to \infty$. Since $\dot{d}_b(t) = -\eta d_b(t) + p_b(t)$, we also have $\rho_{\lambda\eta}(d_b(t)) \to 0$ as $t \to \infty$. This establishes assertion 3.

Finally, the asymptotic stability of the unforced system (25), with (26) and assertion 3, yields assertion 5.

APPENDIX II.   REFERENCES

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APPENDIX III.   NOTATION

The following symbols are used in this paper:

A = parameter of hysteretical model (12);
C = damping matrix of structure;
c_b = damping of base;
D = absolute displacement vector of structure;
d = ground displacement;
d_b = absolute displacement of base;
f = nonlinear force in isolator;
f_1 = frictional force in isolator;
f_2 = hysteretic force in isolator;
f^y = yielding force;
J = unit vector;
K = stiffness matrix of structure;
k = time-varying gain in control law;
k_b = stiffness of base;
k_o = initial value of gain;
$\bar{k}$ = scale factor in gain equation (6b);
M = mass matrix of structure;
m = parameter of hysteretical model (12);
m_b = mass of base;
p_b = linear combination of base displacement and velocity in control law;
Q = force normal to the friction surface in (10);
s_λ = function in (7a);
t = time;
u = active control force;
v = ground velocity;
x = base displacement relative to the ground;
z = auxiliary variable in hysteretic model (12);
α' = unknown scalar in (3);
β = coefficient in frictional model (10);
γ = function in control law;
γ' = bounding function in (3);
ε = bound for the excitation in (2);
η = weighting parameter in (4);
λ = parameter in control law;
μ = friction coefficient;
μ_{max} = friction coefficient for high sliding velocity;
ν_1, ν_2 = parameters in hysteretic model (12);
ρ_λ = function in (7b);
Σ_c = base; and
Σ_r = structure.