Model Order Reduction for Viscoelastic fluid flows

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ABSTRACT

The study of viscoelastic fluid flows are very complexdue to the nature of the governing equations making numerical simulations a complex task.A key challenge in such flows is the stabilisation of numerical techniques which is quite often difficult to meetspecially when the Weissenberg number is high.The momentum and continuity equations form an elliptic saddle point problem for velocity and pressure and the constitutive equation is hyperbolic in nature [1]. In such flows, due to the dependency of the viscoelastic stress tensor on velocity gradients, there arises some compatibility issues in the discretisation space between velocity field and viscoelastic stress tensor. There are many improvements in the numerical techniques to treat and maintain the elliptic nature of the momentum equation. Here, we implement discrete elastic viscous split stress-Galerkin (DEVSS-G) [1,2] for smooth interpolation of velocity gradient in the constitutive equation by considering velocity gradient as an additional dependent variable. Also, due to the presence of convection term in the constitutive equation, there can arise spurious numerical oscillations in convection dominated case. So stabilisation techniques like SUPG [2,3] is applied where the test function is modified and more weightage is being put in the upwind direction.

Solving such high fidelity computational models for a wide range of parameter is computationally exhausting, so motivation of this work is to buildrobust reduced order models such that it can accurately represent the large scale model over a wide range of parameters at a fraction of the CPU cost. Projection based technique like Proper Orthogonal Decomposition (POD) is utilised.We build a framework where the concerns of stabilisation is intrinsically taken care off. We show here three different types of projection based POD for the constitutive equation using Galerkin, Petrov-Galerkin and SUPG test function and observe that all three give similar accuracy while there is an excellent speed up using Galerkin and PetrovGalerkinprojection.

Also, the reduced basis obtained due to the reduced order modelling automatically satisfies the continuity equation implying no extra equation is required for thepressure term. A framework is developed such that without utilising SUPG or defining an extra equation for velocity gradient as in case of DEVSS-G, we are able to build a robust reduced order model within the problem definition such that it is stable and retains the same accuracy as the full order model in a very less computational time.Another challenging issue of the model order reduction of such highly nonlinear system is addressed. POD is not computationally

efficient in reducing the nonlinear terms or non affine terms as it still depends on the complexity of high dimensional system. So we adapt an interpolation based approach DEIM and a projection based cubature approach also known as hyper reduction. We observe that DEIM produces non converging results after a certain number of modes which could be due to the loss of symmetry of the finite element matrices but hyper reduction gives similar results of same orderof accuracy as full order model with a computation speed up factorof 10^4. In order to numerically study the reduced models we solve two benchmark problems: Flow past a Sphere and 4:1 contraction problem. We study the behaviour and address the issues that arise in these two different problems.

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