Teletraffic Performance Analysis of Multi-class OFDM-TDMA Systems with AMC

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Abstract— **In traditional channelized multiple access systems, e.g., TDMA and FDMA, each user is assigned a fixed amount of bandwidth during the whole service time, and the teletraffic performance in terms of time congestion, call congestion and traffic congestion can be easily obtained by using the classical Erlang-B formula. However, with the introduction of adaptive modulation and coding (AMC) scheme employed at the physical layer, the allocation of bandwidth to each user is no longer deterministic, but in a dynamic manner based on the wireless channel conditions. Thus a new connection attempt will be blocked with certain probability depending on the state of the system and the bandwidth requirement of that new connection. In this paper, we present an integrated analytical model of multi-rate loss system with state-dependent blocking to evaluate the performance of multi-class OFDM-TDMA systems with AMC scheme with some numerical examples.**

I. Introduction

Future mobile communication systems will provide not only speech and low-speed data services, but also high-speed data services such as wireless multimedia applications ranging from several kilobits to megabits per second. This can be achieved by operating with Orthogonal Frequency Division Multiplexing (OFDM) over the air interface, which is immune to intersymbol interference and frequency selective fading, as it divides the frequency band into a group of mutually orthogonal subcarriers, each having a much lower bandwidth than the coherence bandwidth of the channel. Recently, OFDM-based systems have become a popular choice for such an endeavor. The IEEE 802.16 standard, for instance, has adopted OFDM-TDMA and OFDMA as two transmission schemes at the 2-11 GHz band.

The economical usefulness of a system is effectively measured by the Erlang capacity, which is generally defined as the maximum traffic load that the system can support when the blocking probabilities at the call admission control (CAC) level do not exceed certain thresholds. Many models have been proposed at separate layers, e.g., the Rayleigh, Rician and Nakagami fading models at the physical layer [8], and queuing models at the data link layer [6]. In traditional channelized multiple access systems, e.g., TDMA and FDMA, each user is assigned a fixed amount of bandwidth during the whole service time, and the Erlang capacity can be easily obtained by using the well-known Erlang-B formula. However, traditional queuing models do not consider the time-varying nature of wireless channels due to multipath fading and Doppler shift. Unlike wired networks, even if large bandwidth is allocated to a certain wireless connection, the QoS requirements may not be satisfied when the channel experiences deep fades.

In order to enhance the spectrum efficiency while maintaining a target packet error rate (PER) over wireless links, adaptive modulation and coding (AMC) scheme has been widely adopted to match the transmission rate to time-varying channel conditions. With AMC, the allocation of bandwidth to each user is no longer deterministic (e.g., a fixed amount of bandwidth), but in a dynamic behavior. Outage is defined to occur when the total number of time slots required by the admitted users exceeds the total available time slots. Therefore, a new connection might be blocked with certain probability depending on the state of the system and the bandwidth requirement of that new connection.

An analytical model to investigate the performance of transmissions over the wireless links is developed in [1], where a finite-length queuing is coupled with AMC. However, the author only concentrates on a single user case. Reference [2] calculates the Erlang capacity of WiMAX systems with fixed modulation scheme, where two traffic streams, streaming and elastic flows, are considered. In reference [3], the author evaluates the Erlang capacity of a multi-class TDMA system with AMC by separating the calculation of blocking and outage probabilities. Performance analysis of OFDM systems has been conducted primarily by simulations in the past. An analytical framework to evaluate the teletraffic performance in terms of time congestion, call congestion and traffic congestion of multi-user multi-class OFDM-TDMA systems with AMC scheme is still missing. In this paper, we propose an integrated analytical model of multi-rate loss system with state-dependent blocking to achieve this goal.

The rest of the paper is organized as follows. In Section II, we introduce the system model, which includes the OFDM transmission with AMC and the calculation of state dependent blocking probability. In Section III, an analytical model of multi-rate loss system with state-dependent blocking is presented with relevant performance measures. Numerical results are given in Section IV. Finally, a conclusion is drawn in Section V.

II. System Model

We consider an infrastructure-based wireless access network, where connections are established between base station

(BS) and mobile stations (MSs). Several service classes with different data rates are supported in the system. Users from each service class arrive at the cell in a random order. The call admission control (CAC) module decides whether an incoming call should be admitted or not based on the current state of the system and the bandwidth requirement of that new call. Specifically, if a new call attempt would bring the system into the state that the required number of time slots exceeds the total available time slots with probability p, it is accepted with probability $1-p$.

We assume that the BS has perfect knowledge of channel state information (CSI) of each subchannel. We further assume that each subchannel is frequency flat, and the channel quality remains constant within a frame, but may vary from frame to frame.

A. OFDM Transmission with AMC

We consider an OFDM-TDMA system with M subchannels. At the physical layer, the time axis is divided into frames. A frame is further divided into K time slots, each of which may contain one or several OFDM symbols. Users transmit in the assigned time slots over all subchannels. Adaptive modulation and coding scheme is employed to adjust the transmission mode in each subchannel dynamically according to the time-varying channel conditions.

We assume that each subchannel follows a Rayleigh fading. For flat Rayleigh fading channels, the received SNR on subchannel m is a random variable with probability density function (pdf) given as follows [1]:

$$p_\gamma(\gamma_m) = \frac{1}{\overline{\gamma}_m}\exp\Big(-\frac{\gamma_m}{\overline{\gamma}_m}\Big) \tag{1}$$

where $\overline{\gamma}_m$ is the average SNR over subchannel m.

The design objective of AMC is to maximize the data rate by adjusting the transmission parameters according to channel conditions, while maintaining a prescribed packet error rate (PER) P_0. Let N denote the total number of transmission modes available (i.e., $N = 5$). Assuming constant power transmission, we partition the entire SNR range into $N+1$ non-overlapping consecutive intervals with boundaries denoted as $\{\Gamma_n\}_{n=1}^{N+1}$. Specifically, mode n is chosen when $\gamma_m \in [\Gamma_n, \Gamma_{n+1})$. Therefore, with Rayleigh fading, mode n will be chosen on subchannel m with probability:

$$\mathrm{P_m}(n) = \exp\Big(-\frac{\Gamma_n}{\overline{\gamma}_m}\Big) - \exp\Big(-\frac{\Gamma_{n+1}}{\overline{\gamma}_m}\Big) \tag{2}$$

Let $\overline{\mathrm{PER}}_{m,n}$ denote the average PER corresponding to mode n on subchannel m. It can be obtained in closed-form as [1]:

$$\overline{\mathrm{PER}}_{m,n} = \frac{1}{\mathrm{P_m}(n)}\int_{\Gamma_n}^{\Gamma_{n+1}} a_n \exp(-g_n\gamma)p_\gamma(\gamma)d\gamma \tag{3}$$

where a_n, g_n are the mode dependent parameters shown in Table I. The average PER of AMC can then be computed as the ratio of the average number of packets in error over the total average number of transmitted packets:

$$\overline{\mathrm{PER}} = \frac{\sum_{m=1}^{M}\sum_{n=1}^{N} R_n \mathrm{P_m}(n)\overline{\mathrm{PER}}_{m,n}}{\sum_{m=1}^{M}\sum_{n=1}^{N} R_n \mathrm{P_m}(n)} \tag{4}$$

The algorithm for determining the threshold $\{\Gamma_n\}_{n=1}^{N+1}$ with the prescribed $\overline{\mathrm{PER}} = P_0$ is described in details in [1].

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
Modulation	BPSK	QPSK	QPSK	16QAM	64QAM
Coding rate	1/2	1/2	3/4	3/4	3/4
R_n (bits/sym)	0.5	1.0	1.5	3.0	4.5
a_n	274.7229	90.2512	67.6181	53.3987	35.3508
g_n	7.9932	3.4998	1.6883	0.3756	0.0900

TABLE I

TRANSMISSION MODES WITH CONVOLUTIONALLY CODED MODULATION [1]

Let us define $\mathcal{R}_m$ be a random variable with probability mass function f_m, denoting the number of bits that can be transmitted over subchannel m in one time slot.

$$\begin{aligned}\mathcal{R}_m &\in \{sR_0, sR_1, \cdots, sR_N\}\\ f_m(n) &= \mathbb{P}(\mathcal{R}_m = sR_n) = \mathrm{P_m}(n)\end{aligned} \tag{5}$$

where s is the number of symbols per time slot, and R_n is the number of bits carried per symbol in transmission mode n shown in Table I.

Let random variable $\mathcal{R} = \sum_{m=1}^{M}\mathcal{R}_m$ denote the number of bits that can be transmitted over all subchannels in one time slot. Based on the assumption that the subchannels are independent to each other, the probability mass function (pmf) of $\mathcal{R}$, denoted as $f_\mathcal{R}$, can be obtained by convolving[1] the pmf of each subchannel f_m as follows:

$$f_\mathcal{R} = f_1 \otimes f_2 \cdots \otimes f_M \tag{6}$$

B. State Dependent Blocking Probability

Assume that there are L service classes accommodated in the system, each of which requires a constant bit rate of r_i bits per frame. In multi-class systems, different service class with different bit rate requirement needs different channel bandwidth in terms of time slots. Thus it would be beneficial for the teletraffic calculations if we could specify a common channel bandwidth which we may call it a *unit channel*. The higher the required accuracy (bandwidth granularity), the smaller the unit channel we have to specify. Let us define r_{unit} be the constant bit rate of a unit channel and $d_i = \frac{r_i}{r_{\mathrm{unit}}}$ be the number of unit channels needed to establish one connection of service class i. The number of time slots occupied by a unit channel can be denoted by a random variable $\mathcal{D}_{\mathrm{unit}} = \frac{r_{\mathrm{unit}}}{\mathcal{R}}$ with probability mass function $f_\mathcal{R}$ obtained in Exp. (6).

[1] $a \otimes b$ denotes discrete convolution.

In AMC scheme, the modulation and coding rate is chosen according to time-varying channel conditions. As a consequence, the number of time slots allocated to each user is varying on a frame by frame basis. Outage is defined to occur when the total number of time slots required by the admitted users exceeds the total available time slots. Therefore, the blocking probability of a new connection is a random variable depending on the state of the system and the bandwidth requirement of the new connection.

Specifically, assume that a new single-slot call arrives at the system at the time instance that x unit channels are currently occupied by the admitted users. If it is accepted, it will bring the system into state $(x+1)$, and the total number of time slots required by the $(x+1)$ unit channels can be modeled by a random variable $\mathcal{D}_{x+1} = \sum_1^{x+1} \mathcal{D}_{\text{unit}}$ with probability mass function $f_{\mathcal{D}_{x+1}}$:

$$f_{\mathcal{D}_{x+1}} = \underbrace{f_{\mathcal{R}} \otimes f_{\mathcal{R}} \cdots \otimes f_{\mathcal{R}}}_{x+1 \text{ times}} \tag{7}$$

Let us define the acceptance probability of a new single-slot call in state x as $a_x = 1 - b_x$, where b_x is the blocking probability in state x, calculated as the probability that the outage probability in state $(x+1)$ is greater than a certain threshold:

$$\begin{aligned} b_x &= \mathbb{P}\{\mathrm{P}_{\text{outage}}(x+1) > \text{Outage}_{\text{Th}}\} \\ &= \max\{0, 1 - F_{\mathcal{D}_{x+1}}(K) - \text{Outage}_{\text{Th}}\} \end{aligned} \tag{8}$$

where $\mathrm{P}_{\text{outage}}(x+1) = \mathbb{P}(\mathcal{D}_{x+1} > K)$ is the outage probability in state $(x+1)$, $\text{Outage}_{\text{Th}}$ is the outage probability threshold, and $F_{\mathcal{D}_{x+1}}(n) = \sum_{i:n_i \le n} f_{\mathcal{D}_{x+1}}(n_i)$ is the probability distribution function (pdf) of random variable $\mathcal{D}_{x+1}$.

For a d-slot call, we have to choose the acceptance probability in state x as a function of the number of unit channels currently occupied and the bandwidth request as follows [5]:

$$\begin{aligned} 1 - b_{x,d} &= \prod_{j=x}^{x+d-1} (1 - b_j) \\ &= (1 - b_x)(1 - b_{x+1}) \cdots (1 - b_{x+d-1}) \end{aligned} \tag{9}$$

Notice that $b_x = b_{x,1}$. This corresponds to that a d-slot call chooses one unit channel d times, and it is only accepted if all d unit channels are successfully obtained. This is a quite natural requirement as we assume full accessibility. In the next section, we will show that it is a necessary and sufficient condition for maintaining the reversibility of the process.

III. Analytical Model

We may evaluate the performance of the above mentioned multi-class OFDM-TDMA systems with AMC scheme by using the classical teletraffic model of multi-rate loss system with state-dependent blocking.

A. Traffic Model

We use the *BPP* (Binomial, Poisson & Pascal) traffic model in our analysis [6]. This model is insensitive to the service time distributions, thus is very robust for applications. Each traffic stream i is characterized by the offered traffic A_i, the peakedness Z_i and the number of unit channels d_i needed for establishing one connection. The offered traffic A_i is usually defined as the average number of call attempts per mean holding time. Peakedness Z_i is the variance/mean ratio of the state probabilities when the system capacity is infinite, and it characterizes the arrival process. For $Z_i = 1$, we have a Poisson arrival process, whereas for $Z_i < 1$, we have a finite number of users and more smooth traffic (Engset case). Engset traffic can alternatively be characterized by the number of sources S and the offered traffic per idle source β. We have the following relations between the two presentations [4]:

$$\begin{aligned} A &= S \cdot \frac{\beta}{1+\beta} \qquad & Z &= \frac{1}{1+\beta} \\ \beta &= \frac{1-Z}{Z} \qquad & S &= \frac{A}{1-Z} \end{aligned} \tag{10}$$

For $Z_i > 1$, it corresponds to a more bursty Pascal arrival process.

B. Algorithms for Calculating Global State Probabilities

The call-level characteristics of multi-class OFDM-TDMA systems with AMC described in Section II can be modeled by a multi-dimensional Continuous Time Markov Chain (CTMC). As an example, we consider a system supporting two service classes with different data rates. Fig. 1 shows the state transition diagram for a system with limited accessibility, where state (i, j) denotes the state of the system (i.e., i and j are the number of unit channels occupied by stream one and two respectively), and $1 - b_{x,d} = \prod_{j=x}^{x+d-1}(1 - b_j)$ is the state dependent blocking probability derived above. From the figure, we can see that diagram is reversible as the flow clockwise is equal to the flow counter-clockwise (Kolmogorov's criteria), but the product form has been lost. Due to reversibility, we can apply the local balance equations to calculate the relative state probabilities expressed with reference to state $(0, 0)$, then normalize the relative state probabilities to obtain the absolute state probabilities and the relevant performance measures.

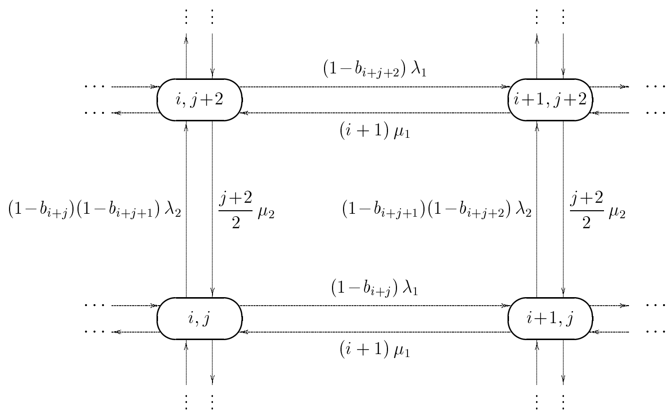

Fig. 1. State-transition diagram with state-dependent blocking probabilities for a multi-rate loss system. The process is reversible as the flow clockwise equals the flow counter-clockwise [5].

Delbrouck [7] developed a general algorithm for calculating the global state probabilities for multi-rate loss systems with BPP-traffic, which is insensitive to service time distribution, i.e., the state probabilities of the system only depend on the holding time distribution through its mean value. Reference [5] extended the Delbrouck's algorithm to include state-dependent blocking as shown in Fig. 1. If we consider a system with C unit channels and L traffic streams. The relative global state probabilities $q(x)$ for multi-rate loss systems with state-dependent blocking can be calculated in a generalized recursion formula expressed as follows [5]:

$$q(x) = \begin{cases} 0 & x < 0 \\ 1 & x = 0 \\ \sum_{i=1}^{L} q_i(x) & x = 1, 2, \cdots, C \end{cases} \tag{11}$$

where

$$q_i(x) = \left\{ \frac{d_i}{x} \cdot \frac{A_i}{Z_i} \cdot q(x - d_i) - \frac{x - d_i}{x} \cdot \frac{1 - Z_i}{Z_i} \cdot q_i(x - d_i) \right\} \cdot (1 - b_{x-d_i,d_i}) \tag{12}$$

In the above equations, $q_i(x)$ is the contribution from traffic stream i to global state $q(x)$. The initialization values of $q_i(x)$ are $\{q_i(x) = 0, x < d_i\}$. The absolute global state probabilities $p(x)$ and $p_i(x)$ can be obtained after normalization.

$$\begin{aligned} p(x) &= \frac{q(x)}{\sum_{j=0}^{C} q(j)} \qquad 0 \le x \le C \\ p_i(x) &= \frac{q_i(x)}{\sum_{j=0}^{C} q(j)} \qquad 1 \le x \le C \end{aligned} \tag{13}$$

C. Performance Measures

Based on the global state probabilities derived above, we are able to get the performance measures of the system in terms of time congestion, call congestion, and traffic congestion.

1) Time Congestion: this is by definition equal to the proportion of time the system is blocked for new call attempts. In multi-class OFDM-TDMA systems with AMC scheme, if the system is in state x, a call attempt of stream i will experience congestion with probability b_{x,d_i}. Thus the time congestion E_i of stream i is calculated as follows:

$$E_i = \sum_{x=0}^{C} b_{x,d_i} \cdot p(x) \qquad i = 1, 2, \cdots, L \tag{14}$$

2) Traffic Congestion: this is by definition equal to the proportion of offered traffic which is blocked. It should be noticed that traffic congestion is the most important performance measure. The carried traffic Y_i of stream i measured in unit channels is given by [5]:

$$Y_i = \sum_{x=1}^{C} x \cdot p_i(x) \qquad i = 1, 2, \cdots, L \tag{15}$$

The offered traffic of stream i measured in unit channels is $d_i \cdot A_i$. Thus the traffic congestion C_i of stream i becomes:

$$C_i = \frac{d_i \cdot A_i - Y_i}{d_i \cdot A_i} \qquad i = 1, 2, \cdots, L \tag{16}$$

3) Call Congestion: this is by definition equal to the proportion of call attempts which are blocked. It is said in reference [4] that the call congestion B_i of stream i can always be obtained from the traffic congestion C_i as follows:

$$B_i = \frac{C_i}{Z_i + (1 - Z_i)C_i} \qquad i = 1, 2, \cdots, L \tag{17}$$

where Z_i is the peakedness of stream i.

IV. Numerical Results

V. Conclusion

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