A POD-Galerkin Model for Convection-Diffusion-Reaction Equations with Parametric Data based on Adaptive Snapshots

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ABSTRACT

We consider convection-diffusion-reaction equations with parametrized random and deterministic inputs. For fixed values of the deterministic parameters, the problem reduces to a linear elliptic PDE with random input data and statistical moments of its solution such as mean and variance can be approximated by a stochastic Galerkin finite element (SGFE) method. There are scenarios, like robust optimization or real-time evaluation, where these statistical information must be computable for numerous different values of the deterministic parameter in a short period of time. In these particular cases, it can be computationally beneficial to conduct a certain number of expensive preliminary computations in order to set up a reduced order model (ROM). The reduction of the overall computational costs than results from the fact that this ROM is low dimensional and can thus be evaluated cheaply for each point in the domain of the deterministic parameters. We construct a ROM for our problem using a proper orthogonal decomposition (POD) of SGFE snapshots [1]. As a consequence, there is no need for an additional sampling procedure in order to evaluate the statistics of the solution of the reduced order model.

Computing the snapshots for the ROM means that several different SGFE problems have to be solved, each associated with a large block-structured system of equations. Since the computational costs of solving these systems are high, we use adaptive discretization techniques to find favorable discrete spaces and lower the computational burden of the preliminary computations. Using adaptive approaches leads, however, to a setting where the snapshots belong to different SGFE subspaces. This fact interferes the standard POD procedure. It is still possible to construct a reduced order model based on adaptive snapshots [2] but there are different theoretical and numerical issues that have to be addressed. We derive an upper bound for the error of the ROM that contains one error term for every approximation step we make. To balance the different error sources we derive a computable upper bound for every error contribution. Eventually, we illustrate our approach for a convection-diffusion-reaction test case where the convective velocity is the deterministic parameter and the parametrized reactivity field is the random input.

REFERENCES

[1] S. Ullmann, C. Müller and J. Lang, Stochastic Galerkin reduced basis methods for parametrized linear convection-diffusion-reaction equations, *Fluids*, Vol. **6**(8), pp. 543-561, 263, (2021).

[2] S. Ullmann, M. Rotkovic and J. Lang, POD-Galerkin reduced-order modeling with adaptive finite element snapshots, *J. Comput. Phys.*, Vol. **325**, pp. 244-258 (2016).