PROGRESSIVE COLLAPSE AND ROBUSTNESS OF FRAMED STRUCTURES: SIMULATION AND DESIGN

Masoero E., Calvi A., Chiaia B.M.

Politecnico di Torino, Dept. of Structural and Geotechnical Engineering, Torino, Italy

ABSTRACT

We present a framework to investigate progressive collapse and robustness of 2D framed structures, subjected to multiple column removals. Progressive collapse is first simulated with an algorithm based on the Discrete Elements. The possible collapse mechanisms emerging from the simulations constitute the input for an analytical model. The model takes into account the dynamics of the response to the sudden initial damage and, under the hypotheses of ideally plastic or perfectly brittle ruptures, provides upper and lower bounds for the collapse loads and for the progressive collapse resistance. Closed form expressions are obtained, that can be valuable for robustness-oriented design. A novel concept of hierarchy in robustness capacity design emerges, which partly conflicts with anti-seismic capacity design. Strategies of compartmentalization and the influence of the position of the initial damage within the frame are discussed, also taking into account the impacts between falling rubble and horizontal floor slabs.

Keywords: Progressive collapse, robustness, simulation

1. INTRODUCTION

A robust building ensures satisfactory performance in presence of unforeseen extraordinary events, like explosions, impacts, earthquakes, or design-constructions errors. The definition of "satisfactory" is tricky and the design codes, e.g. [1], mostly refer to the still concept of "proportion" between accidental event and negative consequences. This explain the relevance of *progressive collapse* in robustnesss-oriented design, because this phenomenon can cause disproportion between initially localized damage and widespread final extent of collape. Progressive collapse consists in a series of ruptures of structural elements, like beams and columns, and consequent dynamic load redistribution, which in turn can cause further ruptures and generate a catastrophic "domino effect". Notorious cases of progressive collapse of the Ronan Point Building, in London, 1968, due to a gas explosion [2], the partial collapse of the Alfred P. Murrah Federal Building, Oklahoma, 1995, due to a bomb-truck [3], and the total collapse of the Twin Towers of the WTC, New York City, 2001, due to aircraft impact, explosion, and fire [4, 5].

An intuitive but unacceptably expensive way to provide damage tolerance to a building is by strengthening all its structural elements. Alternative solutions refer to the concepts redundancy and compartmentalization (see, e.g. [6, 7], and the codes incorporate measures that go in this sense, like introducing ties, employing very ductile structural elements, and recommending moment resistant connections (see [8] for a review on measures aimed at improving structural robustness). These solutions generally improve progressive collapse resistance, but in some cases their effectiveness is still debated [9]. Similarly, even though anti-seismic design generally

improves progressive collapse resistance (see, e.g.,[10]), in the conclusions of the present work we show that it does not represent an optimal solution to maximize it.

The limitations of the existing design rules on structural robustness justify an effort to understand the physics of progressive collapse. Straightforward analytical models can be developed only in very particular cases, like the collapse of towers (see, e.g., [4]). Already for simple 2D structures, fully non-linear dynamic simulations are required and usually performed within the standard framework of the Alternate Load Path Method (ALPM) [11]. According to the ALPM, structural elements are suddenly removed from the model structure to represent an unknown initial damage event. Modern simulation algorithms incorporate the ALPM method and can allow very detailed analyses (e.g. [12]). Nevertheless our aim is to obtain general results and to study the effect of several design parameters, like strength and ductility of beams and columns or structural hierarchy, rather than reproducing collapse of a particular structure in detail. Therefore we developed a new simulation code based on the Discrete Element Method (DEM) which is optimal to perform extended parametric studies of collapsing buildings.

A further very important step is to use simulations as numerical experiments, to derive simplified analytical models which can provide convenient expressions to measure and design progressive collapse resistance. In this paper, we propose a strategy to derive formulas for the collapse loads and for the progressive collapse resistance. Our strategy consists in a first exploration stage, where DEM simulations are used to individuate the possible collapse mechanisms. Then, simple kinematic models are defined, which can reproduce the found collapse mechanisms. The kinematic models permit to interpret analytically and generalize the results of the simulations. As an application of our methodology, we study the progressive collapse of regular 2D frames made of reinforced concrete and subjected to the sudden removal of beams and columns from different parts of the structure.

2. METHODS: SIMULATION AND ANALYSIS

We developed a simulation code to treat progressive collapse of structures starting from a Discrete Element Method (DEM) algorithm for the dynamics of cohesive granular media [13, 14]. Detailed descriptions of the code can be found in [6, 15, 16], while in the following we will highlight only the main features. The code enables fully non-linear dynamic analyses of 3D structures, meshed with Spherical Discrete Elements (SDEs) interconnected by Euler Bernoulli Beams (EBEs). The mass of the structural elements is lumped into the SDEs, whose dynamics is reproduced via direct time integration of Newton's equations of motion. The forces acting on the SDEs are due to gravity, external load, and internal forces caused by deformation of the EBEs. Furthermore, SDEs colliding with each other or with a horizontal plane representing the ground are subjected to a normal repulsive force and to tangential forces, according to a damped Hertzian contact scheme. The EBEs are linear elastic in axial direction, shear, bending and torsion. When a yield normal force (N_y in tension or $N_{y,c}$ in compression) or a yield bending moment B_{y} is reached, the EBE enters an ideally plastic regime. The plastic regime is uncoupled in axial direction and bending, with N and B kept constant to the yield value setting a plastic axial strain ε^{pl} and plastic rotations φ^{pl} at the edges of the EBE. When the cumulated plastic strain and rotation reach threshold values, a coupled criterion in terms of ε and φ determines the rupture of the EBE, which is suddenly removed from the structure.

The simulations permit to quantify the collapse load of the intact structure q_u^I and the collapse load, also referred to as critical load, of the suddenly damaged structure q_c , with damage represented through the ALPM. q_u^I is evaluated via a static analysis, increasing the applied load q until a first EBE fails. Differently, q_c is found with several nonlinear dynamic analyses, consisting of a first stage when the structure is equilibrated under q, and a second stage when

damage is applied and the dynamic response shows whether $q < q_c$ (no collapse after damage) or not. Various collapse mechanisms can be activated by structures with same overall geometry, depending on the mechanical parameters of the SDEs and of the EBEs. Each collapse mechanism is associated with specific values of q_u^I and q_c . In some cases (e.g. wide damage or brittle structures) the damaged structure, or even the intact one, not only is unable to carry any external load, but can neither carry its own weight. In these cases q_c is searched reducing the mass of the EBEs, i.e. the self weight of the structure. This leads to a discontinuous definition of the load parameter q, which requires the introduction of the *equivalent load*, as a single measure taking into account both the external load and the self weight. The derivation of some equivalent loads is presented in [6] and in the appendix of [15].

For the analytical interpretation of the results, simple structural schemes of the elements involved in the collapse are defined. The static schemes are such that non-linear static collapse analyses with load proportional to q must result into a sequences of deformed states that reproduce qualitatively the kinematics of the collapse mechanisms obtained from the simulations. Considering the limit cases of perfect brittleness and of ideal plasticity, we can obtain the lower and upper bounds of the collapse loads starting from the static analyses of the simplified schemes. Under the assumption of perfect brittleness, we compute the static internal forces S in the structural schemes of the intact I and of the damaged D structures, to find the point where brittle collapse can be triggered and the evaluate the static internal force that causes it ($S^{I,max}$ and $S^{D,max}$). For the intact structure q_u^I is obtained directly from:

$$S^{I,max}\left(q_{u}^{I}\right) = S_{y} \quad , \tag{1}$$

where S_y is the yield or rupture value for the generic internal force. For the damaged structure we need also the internal force $S^{I-D,max}$ corresponding to $S^{D,max}$ (i.e. in the same point of the structure) but in the static scheme of the intact structure. In this way we can compute q_c from:

$$S^{I-D,max}(q_c) + D_{fac} \left[S^{D,max}(q_c) - S^{I-D,max}(q_c) \right] = S_y \quad , \tag{2}$$

where the dynamic factor D_{fac} accounts for the dynamic overshooting. D_{fac} can generally be set equal to 2 for linear elastic perfectly brittle collapsing structures, even though this assumption is rigorously valid only for single degree of freedom systems (see [6]). On the other hand, under the assumption of ideal plasticity, localized plastic rotations and axial sliding must be allowed in a sufficient number of points to generate mechanisms that resemble the simulated ones. The kinematic theorem is then applied to obtain the collapse loads, with $D_{fac} = 1$ because of the infinite capacity of plastic energy dissipation. Energy arguments can provide D_{fac} for any level of plastic capacity (e.g. in [6, 17, 18]).

The *closed form expressions* of the collapse loads obtained with our approach usually incorporate geometric parameters of the structure, mechanical parameters of the structural elements involved in the collapse, and a measure of the initial damage extent. The knowledge of the collapse loads associated to the various collapse mechanisms permits to determine which mechanism is a specific structure susceptible to, both before and after the damage. The *residual strength* corresponding to the various combinations of collapse mechanism before and after damage can be measured as:

$$R_1 = \frac{q_c}{q_u^I} \quad . \tag{3}$$

 R_1 can not be improved by simply increasing the cross section of the structural elements or the strength of the material, because this would increase both the numerator and the denominator. Structural optimization towards a sudden change of structural scheme is therefore necessary,

which can imply e.g. topological and hierarchical modifications (see [6, 15, 19]). Finally, with the analytical approach that we developed, the designer can quantitatively tune the mechanical and geometric parameters that determine q_u^I and q_c , with the aim of avoiding *a priori* collapse mechanisms that are associated with low R_1 .

3. PROGRESSIVE COLLAPSE OF 2D FRAMES

As an application of the proposed methodology, we consider the 2D framed structures in Figure 1, made of reinforced concrete with high plastic strain and rotation capacity. The structures have same overall size but are made of a different number of structural cells n^2 . The frames with n = 2 and n = 5 can be seen as a hierarchical reorganization of the frame with n = 11, obtained employing a primary structure made of fewer but larger structural elements. To this aim, we set the cross sectional area and reinforcement of beams and columns proportional to their length, respectively L and H. In this way the slenderness of the structural elements is constant for different n. From now on, we will call *hierarchical level* of a frame the ratio 1/n. Details on the parameters of the materials, cross sections, and design strategy can be found in [6, 15]. The frames are subjected to the sudden removal of beams and columns within a damage area, dotted in Figure 1. The damage areas has identical size for frames with different n, to represent accidental events with same destructive energy or spatial extent, like explosions or impacts. The size of the damage areas is such that a constant fraction of 1/3 of the columns at one storey is removed. We consider the effect of 4 different initial damage positions (see Figure 1). The case of damage position CB is treated in detail in [6, 15, 19] and therefore, for this case, we will only summarize the main findings. For the other damage positions, we will show the results of the simulations and provide an analytical interpretation.



Figure 1 Studied frames and damage positions (dashed area). B = bottom, T = top, C = central, E = external.

3.1. Simulations

Two values of the compressive strength of concrete f_c where considered, in order to activate different collapse mechanisms. Employing $f_c = 35$ MPa the coulms turn out to be very strong and collapse occurs due to bending failure of the beams triggering a *bending collapse* mechanism. We studied bending collapse for damage positions CB, EB and EM inFigure 1, and we will also provide some simulation results for a damage position intermediate between CB and EB. On the other hand, $f_c = 0.35$ MPa produces very weak columns and collapse can occur because of teir compressive rupture at one storey (*pancake collapse* mode). We simulated systematically pancake collapse only for damage position CB, but some first results for the other damage positions will also be included in the following sections. Finally, when the initial damage is in position ET, the structural elements within the damage area become rubble that impacts the structural elements below, possibly triggering an *impact-driven* collapse mechanism.

Bending collapse. Bending collapse before damage occurs after the static formation of a triple hinge mechanisms for the beams (see Figure 2-a), which reflects their high plastic capacity. After damage in position CB, the simulations in [6, 15] show that a dynamic triple hinge mechanism occurs for frames with n = 2 (see Figure 2-b), while four hinges form when n = 5 or n = 11 (see Figure 2-c). When n > 2, bending collapse generally involves only the elements



Figure 2 (a) Static bending collapse before damage. Dynamic bending collapse after damage in position (b) CB: n = 2, (c) CB: n = 11, (d) EB: n = 5, (e) EM: n = 5, and (f) EM: n = 2. (g) Pancake collapse with damage in position CB: n = 5 (similar for the intact structure). (h) Impact driven collapse after damage in ET position: n = 5.

directly above the damage area, inducing a partial final collapse that compartmentalize against horizontal spreading of damage. Nevertheless the simultaneous presence of high plastic capacity of the beams and high load (close to q_u^I) can induce a strong catastrophic inertial effect, with falling central portion of the structure that drag down the rest of the structure (see [6, 15]). In case of damage position EB we still observe a double-hinge mechanism, and this is also true for position EM when n = 5 and n = 11 (see Figure 2-d,e). Differently, when the damage is in position EM and n = 2 the beam above the damage area becomes a cantilever that fails in bending after the formation of one single plastic hinge (see Figure 2-f). In case of lateral damage, dynamic dragging effects were not recorded. Further simulations showed that damage position intermediate between CB and EB are equivalent to position CB, as long as the external column is not affected by the initial damage. This implies that the presence of one single intact column at the edge of one beam, in such dynamical conditions, is sufficient to fully constrain rotations.

The collapse loads obtained from the simulation are summarized in Figure 3. q_u^I/L depends neither on the damage position, nor on the hierarchical level, and this latter is a consequence

of the adopted design rules (see [6, 15]). Differently q_c/L decreases with n, because the concentration of bending moment after damage increases with the number of columns removed at one storey. q_c/L also decreases when the damage is moved from position CB to EB, because the double hinge collapse mechanism triggered by damage in the external position corresponds to a four hinge mechanism of a beam with doubled span. Consistently with the qualitative response described previously, the cases of damage positions EM and EB coincide except except when n = 2, due to cantilever rupture mechanism Figure 2-f. Since q_u^I/L stays constant, R_1 in the different cases has the same trend as q_c/L , which indicates that hierarchical structures with small n are better suited to resist bending collapse after damage.



Figure 3 (a) Collapse loads in bending from the simulations. (b) Residual strength.

Pancake collapse Pancake collapse of the intact frames implies simultaneous crushing of the columns at one storey, which occurs as soon as the quasi-statically increased load provokes axial plasticization at the base of the columns. Simulations with damage in CB position (see [6, 15]) show that dynamic collapse occurs with a rapid, almost simultaneous progressive crushing of the columns at the sides of the damage area until total collapse (see Figure 2-g). The collapse loads are summarized in Figure 4, where the inset shows that R_1 does not depend on the hierarchical level. This suggests that R_1 may be related to the fraction of columns removed at one storey (constant and equal to 1/3 in our simulation), which would result from a democratic redistribution of the overload after damage between the survived columns.

First simulations with damage in position EB show a remarkable reduction in q_c , with corresponding decrease of R_1 to about 0.3, independent on the hierarchical level. Moreover, we also observed that a damage in an intermediate position between CB and EB produces a decrease of R_1 from 0.6 to 0.3 which affects first the less hierarchical frames with large n.

Impacts-driven collapse: An initial damage in position ET causes the disintegration of the upper storeys that fall and collide against those below. In this case the intact structures can collapse either in bending or pancake mode, depending on the relative strength of beams and columns (e.g. on f_c). After damage, we simulated frames with strong columns by setting $f_c = 35$ MPa. This lead to the impact-driven beam rupture mechanism in Figure 2-h. Varying the Hertzian contact stiffness E of the discrete elements between the two limit conditions of elements passing through each other and perfectly rigid collisions, we did not observe a significant qualitative difference in the outcome of the simulations. From the quantitative point of



Figure 4 (a) Collapse loads and residual strength (inset) in pancake from the simulations. (b) Collapse loads towards the impact driven mechanism (damage position ET), as a function of the Hertzian contact stiffness.

view, Figure 4-b shows that q_c/L decreases with n, i.e. hierarchical structures are more resistant towards impact-driven collapse. Furthermore, different values of Hertzian contact stiffness lead to a maximum change in q_c of less than 20%, which evidences the robustness of our simulation algorithm with respect to impact events. We still did not perform simulations of impact-driven collapse of frames with low compressive strength of the columns, but we expect that in this case the structure would undergo pancake collapse after the initial impact of the rubble.

3.2. Analytical interpretation

The analytical approach outlined in Section 2 was applied in [19] to the frames with damage in position CB (see Figure 1). The three collapse mechanisms in Figure 5-a,b,c were considered: bending, global pancake, and local pancake collapse. Global and local pancake mechanisms



Figure 5 Collapse modes considered in [19] for frames with damage position CB. (a) Bending collapse, (b) Global and (c) local pancake, (d) whose propagation can trigger bending collapse.

are two limit cases, respectively corresponding to the ideal cases of horizontal beams that are infinitely rigid or compliant in bending. In case of global pancake, the overload after damage is shared in equal parts between the intact columns at one story, while local pancake imply a progressive nearest neighbour redistribution of compression to the first columns immediately aside. For the studied frames the assumptions of plastic collapse and global redistribution of compression provide the best matching between simulations and analytical expressions. The expressions in [19] show that q_c decreases with the fraction of columns removed at one story in case of global pancake collapse, and with the number of removed columns in case of local pancake collapse. Consequently, for the scenario of 1/3 of removed columns in the simulations, the analytical expressions of R_1 (ideal plasticity) return a constant value of about 0.66 under the assumption of global pancake, and a value increasing with the hierarchical level if local pancake is assumed. The R_1 associated to global pancake is also an upper limit for the local pancake, attained at high hierarchical levels. Further numerical effort and additional analytical work are required to tackle damage positions other than CB, because our first simulations in Section 3.1 indicate that the hypotheses of global or nearest neighbour redistribution are insufficient to capture the actual dynamics of collapse in these cases.

The interpretation of the results regarding impacts is also left to further works. Existing models, e.g. [18], generally refer to brittle or plastic collapse with idealized continuous distribution of impact load, but important asymmetries and stress concentration due to the complex dynamics of falling rubble should not be neglected a priori. Energy arguments can be valuable for the development of such models.

The bending moment diagram in Figure 6-a and the kinematic theorem applied to the mechanism in Figure 7-a provide analytical formula for the static collapse loads in bending B before damage, under the assumptions of perfectly brittle rupture in linear elastic regime el or ideally plasticity pl:

$$q_u^{I,B,el} = \frac{12B_y}{L^2}$$
 and $q_u^{I,B,pl} = \frac{16B_y}{L^2}$. (4)

After damage in CB position, Equation 2 and Figure 6-c and 7-c provide the critical loads [19]:

$$q_c^{B,el} = \frac{12B_y}{L^2 (6n_{r,c} + 1)}$$
 and $q_c^{B,pl} = \frac{4B_y}{n_{r,c}L^2}$. (5)

Equation 5 can be employed also for damage position EB (and EM if n > 2), after substituting $n_{r,c}$ with $2n_{r,c}^* - 1$, where $n_{r,c}^*$ is the actual number of removed columns. This is justified by the fact that a structure with damage in EB position and undergoing bending collapse can be regarded as the left half of a twice as big symmetric structure with damage in position CB. This leads to a substantial reduction of the critical loads. Finally, in case of damage in EM position and n = 2, Equation 2 and the cantilever schemes in Figure 6-b and 6-b provide the critical loads:

$$q_c^{B,el} = \frac{12B_y}{11L^2} - \frac{2F_c}{L}$$
 and $q_c^{B,pl} = \frac{2B_y}{L^2 n_{r,c}} - \frac{2F_c}{L}$, (6)

where F_c is the concentrated force due to the self weight of part of the external column that can possibly hang from the edge of the cantilever after damage. Equations 4, 5 and 6 capture very accurately the results of the simulations, using the input parameters (e.g. B_y and L) that can be found in [15] and in the appendix of [19]. These analytical expressions confirm that q_c decreases with the number of columns removed at one story $n_{r,c}$. The expressions of R_1 , that can be obtained straightforwardly (see e.g. [19]), show that bending collapse is the most severe in terms of R_1 , with $R_1 \leq 25\%$ for the removal of 1/3 of the columns at one story that we employed in our simulations.

Activated collapse mechanism Comparing the analytical expressions of q_u^I and q_c for the bending, local pancake and global pancake collapse modes, one can individuate which mechanisms is associated with the smallest collapse load and thus will be activated. Defining the activated collapse mechanism is necessary in order to choose the most appropriate expressions



Figure 6 Simplified static schemes and bending moment diagram for perfectly brittle collapse in bending mode. (a) Generic beam of the intact structure, (b) cantilever scheme for n = 2 and damage in EM position, (c) damage in CB position. The latter scheme is also valid for damage in EB position (and EM position if n > 2) assuming $n_{r,c} = 2n_{r,c}^* - 1$, where $n_{r,c}^*$ is the number of columns that are actually removed.



Figure 7 Simplified static schemes for ideally plastic collapse in bending mode (cf. Figure 6).

to evaluate q_u^I and q_c , and consequently R_1 . Such comparisons between collapse loads are performed in [19] for the initial damage position CB. What emerges is that two factors determine the activated collapse mechanism: the initial damage extent, in terms of number or fraction of removed columns, and the so called *mechanism parameter* m_p :

$$m_p = \frac{R_c L}{B_y n_s} \quad , \tag{7}$$

where R_c is the maximum compressive force that a column can carry, B_y is the yield bending moment of the beams, L is the bay of the beams, and n_s is the number of stories above the initial damage.

4. CONCLUSIONS AND OUTLOOK

In our approach to progressive collapse analysis, we combine simulations and analytical models to obtain closed form expressions and design parameters that can be valuable for robustnessoriented design of complex structures. The simulations can be seen as numerical experiments aimed at individuating the relevant collapse mechanisms that can be activated, depending on geometric and mechanical features of the structures. The qualitative knowledge of the collapse mechanisms serves as a starting point to defined simplified static schemes that can provide closed form expressions of the collapse loads and of the residual strength after damage. The analytical expressions permit to quantify the relevance of design parameters, e.g. the mechanism parameter in Equation 7. One major advantage of our approach resides in enabling to define optimal robustness-oriented structural solutions a priori, contrary to the most common strategy of designing a system for ordinary loading conditions only and then improving progressive collapse resistance with targeted modifications a posteriori.

The application to 2D frames points out three main possible collapse modes due to bending, pancake, and impact-driven mechanisms. Removing a constant fraction of 1/3 of the columns at one storey, we showed that hierarchical structures made of few large structural elements perform better than homogeneous ones in case of bending or local pancake (because the strength loss is related to the absolute number of removed columns), and the results of the simulations seem to indicate that this is valid also for impact-driven collapse. Otherwise, hierarchical and homogeneous structures are equally well performing in case of global pancake, where the strength loss is related to the fraction of columns removed at one storey. Finally, in case of single column loss, which can represent for instance the effect of a gross construction or design error, hierarchical and homogeneous structures would be better against global pancake collapse. Nevertheless, the larger strength loss related to bending and local pancake collapse imply that hierarchical structures should be preferred.

The higher values of R_1 associated to global pancake collapse suggest that structures should be conceived in a way that accidental damage trigger this collapse mode, rather than bending or local pancake. The mechanism parameter m_p that we introduced allows to determine design constraints aimed at "choosing" the collapse mechanism that a structure can exhibit in case of damage. m_p can be used to quantify how stronger the beams must be with respect to the columns in order for global pancake collapse to occur (see [19]). Adopting stronger beams would result in a higher progressive collapse resistance also toward impact-driven mechanism, but on the other hand conflicts with anti-seismic capacity design. And overcoming this contradiction is a present challenge for performance based design. Furthermore, global pancake collapse leads by definition to the rupture of all the columns at one storey, with potentially total final collapse. Nevertheless, one should not forget that global pancake collapse is just an ideal limit scenario, while a local redistribution of compression starting from the damage area and proceeding outwards does always happen in reality. This progressive "more local" dynamics of compressive breakdowns of the columns can therefore be exploited to design new strategies of compartmentalization where bending collapse interrupts the horizontal propagation of damage above a certain threshold (see Figure 5-d).

We observed the well-know fact that a damage affecting external columns leads to a substantial reduction in the R_1 associated to bending collapse, and motivated this with analytical arguments. From first simulations, we also observed that also R_1 towards pancake seems to decrease in case of external damage. This result disagrees with the interpretation of our previous simulations with damage in the middle of the frames, where the results suggested a collapse mechanism close to global pancake, which in principle should return R_1 independent on damage position. Further simulations and analytical model development are therefore required to clarify the influence of damage position on pancake collapse. A last result that is worth recalling is that from our simulations the Hertzian contact stiffness between colliding elements seems to only marginally affect the final outcome of the simulations, which evidences the reliability of our simulation method with respect to impact-driven collapse mechanisms.

REFERENCES

- 1. BS Eurocode 1, "Actions on structures Part 1-7: General actions Accidental actions", EN 1991-1-7, undated (c.2004).
- 2. C. Pearson, N. Delatte, J. Perf.Constr. Fac. ASCE, 19(2):172-177, 2005.
- 3. M.A. Sozen, C.H. Thornton, P.F. Mlakar, and W.G. Corley. J. Perf. of Constr. Fac. ASCE, 12(3):120-136, 1998.
- 4. Z. Bažant, Y.Zhou, J. Eng. Mech. ASCE 2002, 128(1):2-6.
- 5. Federal Emergency Management Authority (FEMA), "World Trade Center building performance study", FEMA Report 403, May 2002.
- 6. E. Masoero, "Progressive collapse and robustness of framed structures", PhD Thesis, Politecnico di Torino, Italy, 2010.
- 7. Starossek, U., Struct. Eng. Int. 16 (2), 113117, 2006.
- 8. Knoll F., Vogel T., "Design for Robustness", IABSE, Struct. Eng.ng. Documents 11, ETH Zürich, CH-8093 Zürich, Switzerland, 2009.
- 9. Vlassis, A., Izzudin, B., Elghazouli, A., Nethercot, D., 2006. Struct. Eng. Int., 16 (2), 129136.
- 10. S.M.Baldridge, F.K.Humay, Concr. Int., vol. 25, No. 11, Nov. 2005, pp. 73-79.
- 11. General Services Administration (GSA), "Progressive Collapse Analysis and Design Guidelines for New Federal Office Buildings and Major Modernization Projects", Washington D.C., 2003.
- 12. J. Hallquist, LS-DYNA keyword users manual. Livermore Software Technology Corporation, 1992-2005.
- 13. H. Carmona, F.K. Wittel, F. Kun, H.J. Herrmann, Phys. Rev. E 2008, 77(5):243-253.
- 14. T Pöschel, T. Schwager, "Computational Granular Dynamics", Springer-Verlag GmbH, Berlin, 2005.
- 15. E. Masoero, F.K. Wittel, H.J. Herrmann, B.M. Chiaia, submitted to J. Eng. Mech. ASCE, 2011.
- E. Masoero, F.K. Wittel, H.J. Herrmann, B.M. Chiaia, J. Eng. Mech. ASCE, 136 (8), 987 995, 2010.
- 17. E. Masoero, P. Vallini, A.P. Fantilli, B.M. Chiaia, Key Eng. Mat., 417-418, 2010.
- A.G. Vlassis., "Progressive collapse assessment of tall buildings", PhD thesis, London Imperial College, UK, 2007.
- 19. E. Masoero, P.Darò, B.M. Chiaia, submitted to Eng. Struct., 2011.