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STATIC ANALYSIS OF A MASONRY ARCHED AND BUTTRESSED RETAINING WALL

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Abstract: *The wall of Casa Salvans is a masonry arched and buttressed retaining wall which was built in 1909 and located in Terrassa, Catalonia. It contains a series of 11 arches of unique shapes, dimensions and inclinations that are supported on buttresses. Despite being a common retaining wall typology, it can be very challenging to assess the stability of such structures given the variability of their geometry. As such, a systematic methodology for the stability assessment of such walls was developed and applied to the case of the wall of Casa Salvans. Self-produced photogrammetry survey enabled creating accurate three-dimensional models of the front of the wall that were later combined with available information from a topographic survey to build cross sections of the wall for analysis. Modern retaining wall design principles evaluating the stability against overturning and sliding were used to assess the condition of the wall. As a consequence of the uncertainties related to the soil parameters and buried elements of the wall, the stability factors could not comply with modern design criterion. Some parametric analyses were carried out on the shear strength parameters of the backfill, revealing that slight increases in the soil internal friction and wall friction angles enhance the stability factors significantly. Since reasonable soil internal and wall friction angles justifying the stability of the wall were obtained, carrying out a geotechnical survey to determine the actual values was recommended. The in-plane stability of the arches was also verified according to the Lower Bound Theorem by locating a thrust line that fits within all the boundaries despite very conservative geometric assumptions.*

1 INTRODUCTION

The wall of Casa Salvans, located in Terrassa, Catalonia, was built in 1909 by the architect Lluís Muncunill i Parellada. It is a spectacular example of a common retaining wall typology that restrain hard, unsaturated soils through a series of arches connected at buttresses. The masonry retaining wall, made of pebble stones bonded by mortar, has a total length of 144 m and a maximum height of 11.5 m. This study is focused on the section of the wall that contains 11 arches of varying shapes and dimensions, connected by buttresses. The arches have an instrumental role in load transmission to the buttresses, which are significantly thicker than the wall, enabling the use of less material throughout the wall.

The height, the span and the thickness of the arches increase as one follows the descending slope of the adjacent path (Figure 1) The enumeration of the arches adopted throughout the study imply that Arch 1 has the smallest dimensions while Arch 10 has the largest and Arch 11 is the half arch at the end.

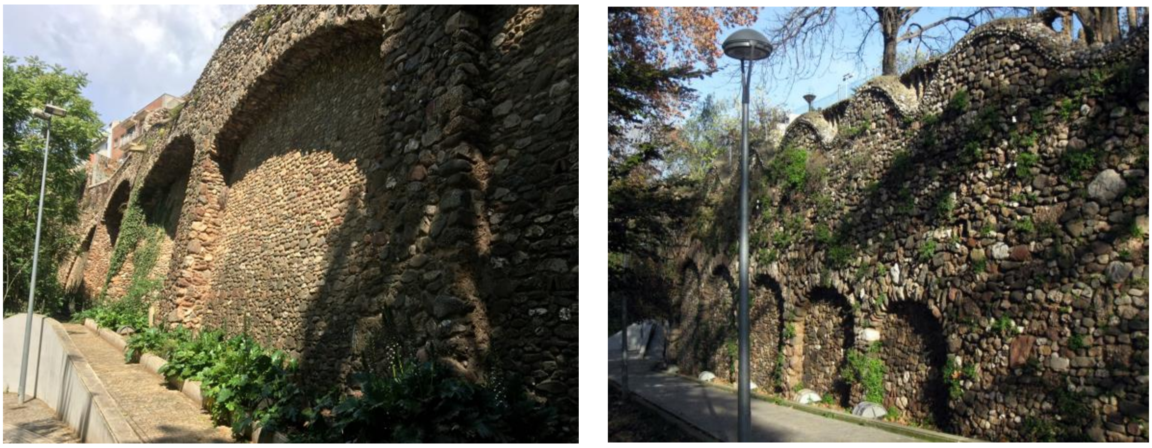


Figure 1: Bigger arches of the wall of Casa Salvans on the left, smaller arches on the right.

As a consequence of the uncertainties related to hidden morphology and soil characteristics, the stability assessment of the wall can be challenging. This study presents a systematic methodology used to address this issue. Firstly, a photogrammetry survey was carried out. Measurements made from the resulting models were then combined with available information to build an idealized geometric representation of the structure. Since no large cracks or detached parts could be found during visual inspections, partial collapse mechanisms were not considered. An initial stability analysis, based on the estimates of the soil parameters, is followed by a sensitivity analysis which aims to comprehend the effect of key geotechnical parameters on the overall stability. Finally, the in-plane stability of the wall is verified according to the Lower Bound Theorem of Limit Analysis.

2 AVAILABLE INFORMATION

2.1 Detailed drawings

Municipality of Terrassa provided detailed plan and elevation drawings obtained through previously performed laser scanning (Figure 2). The precise drawings of the unburied parts of the front and back faces of the wall include elevations and provide useful two - dimensional information.

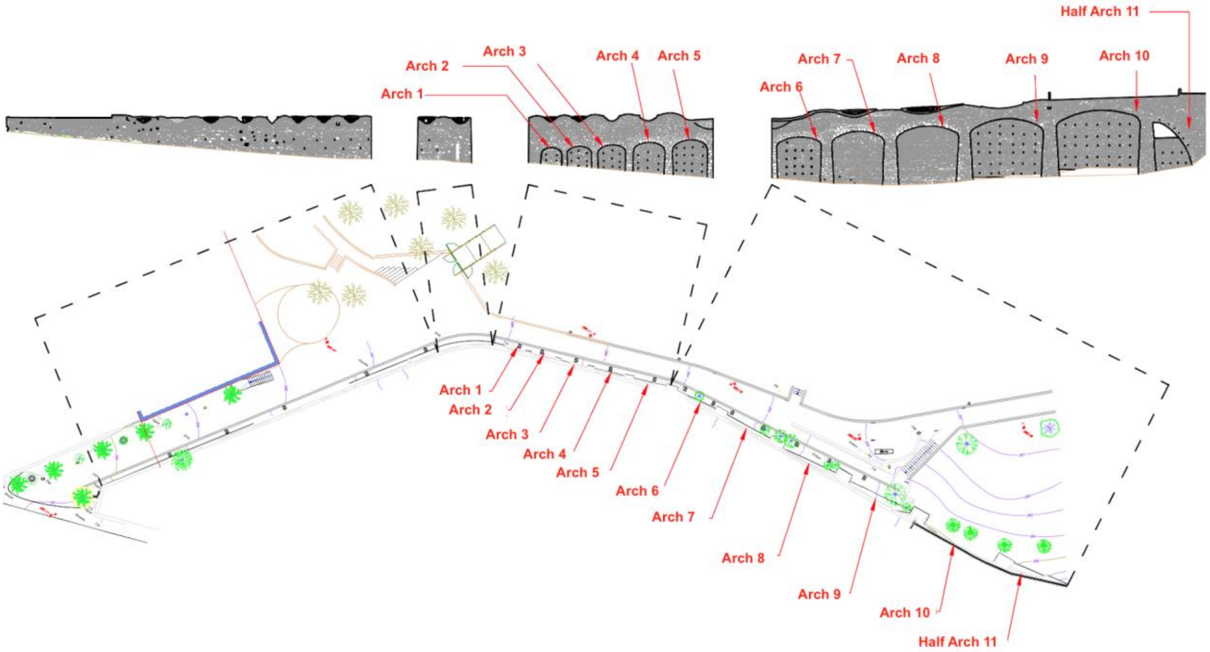


Figure 2: Elevation drawing of the front surface and plan view of the wall of Casa Salvans.

2.2 Topographic survey and exploration pits and holes

In order to gather information on the buried parts of the wall, 20 exploration pits in the ground as well as 5 exploration holes in the wall were made. A topographic survey was also carried out by the Municipality of Terrassa. The 20 exploration pits were located at the base and at the top of the wall on the buttresses and the keys of the arches.

The wall thickness below Arches 8, 9 and 10 was determined to be 0.6 m from the corresponding exploration holes. Therefore, the thickness below arch level was assumed to be constant for Arches 6 – 10 for structural calculations. From the exploration pits at the top of the wall, the extrados of Arches 6 – 9 were located at approximately 0.55 m depth. Based on these measurements, the key of Arch 10 was also assumed to be located at the same depth for the calculations.

A concrete enlargement, acting as a foundation, was located at the base of the wall below Arches 8, 9 and 10. The distance by which the concrete foundation protrudes outwards from the wall was determined to vary between 0.2 m and 0.4 m. At the bases of Arches 1 – 7, the excavations were terminated at shallower depths due to land restrictions.

2.3 Geotechnical survey

Upon request of Municipality of Terrassa, a geotechnical survey was carried out on May 2003 by the company Gesond S.A. Four boreholes of depths 14.5 m to 15 m were executed in the area behind the wall in order to determine the soil profiles, characteristics of the sub-layers and the location of the water table (Figure 3). Nine Standard Penetration Tests (SPT) were carried out at different layers and four samples were taken to be tested in the laboratory.

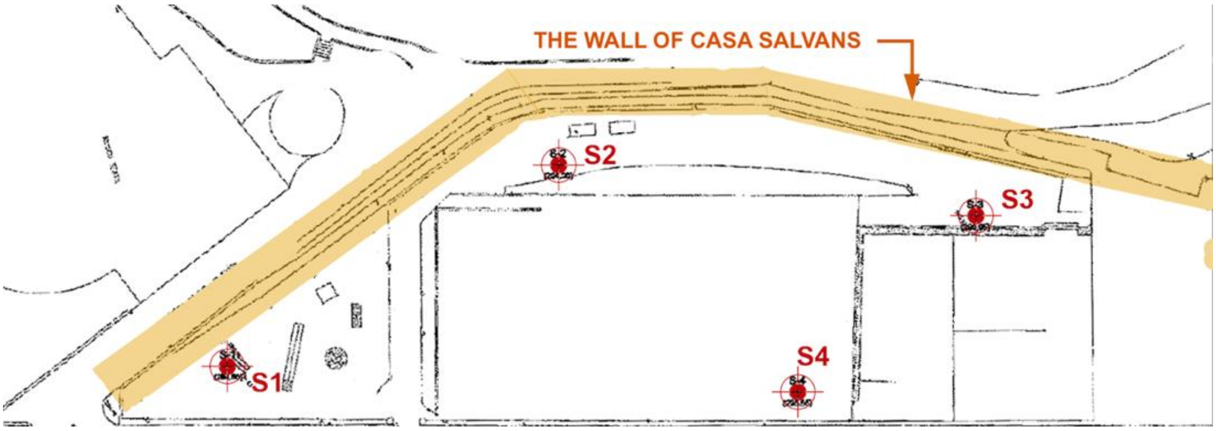


Figure 3: Location of the boreholes relative to the wall of Casa Salvans.

Three predominant layers were determined, of which the characteristics are tabulated in Table 1. Shear strength parameters, cohesion and internal friction angle, ϕ were obtained through correlations with SPT numbers of respective layers.

Table 1: Characteristics and locations of soil layers.

Layer	Characteristics	Depth(m)	γ(kN/m^3)	c_u(kPa)	Φ(°)
R	Backfill material consisting of silty sands	0.8-1.2	19	0	24
A	Alluvial soil consisting of clayey sand	10-10.2	19-21	<10	30
B	Sandy clays with layers of salt and gravel *(saturated below depths of 13.5-14 m)*	10.2 -	21	29	28

The geotechnical cross-sections derived from the stratigraphic profiles were used to estimate the prevalent soil layers throughout the wall. The geotechnical cross-sections were overlaid on the elevation drawing based on relative locations of the boreholes and matching elevations as shown in Figure 4. Considering the assumptions on the extent of the base of the wall, which will be discussed later in Section 5.1, it can be appreciated that the wall under the larger arches is resting on Layer B. At the time of the survey, the water table was located at a lower level than the wall in all boreholes enabling to neglect the effect of pore water pressure.

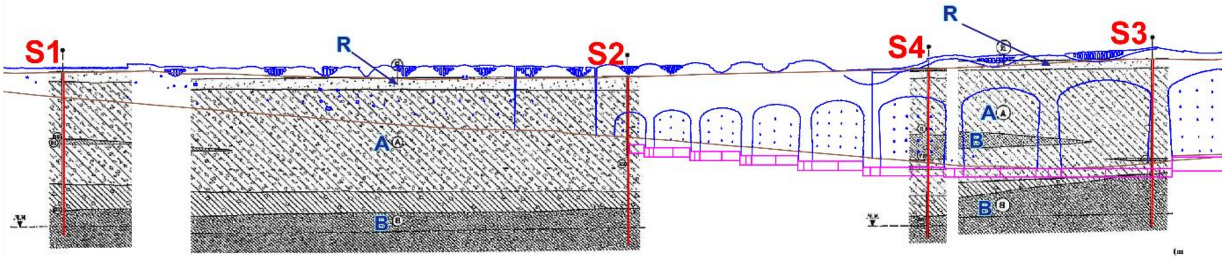


Figure 4: The approximate projection of geotechnical cross-sections on elevation drawing of the wall.

Although it can be suggested that Layer B exists at the bases of Arches 6 – 10, it is not possible to estimate the depth of the layer in contact with the wall. For these arches, Layer B was assumed to be present only at the level of foundation base while for Arches 1 – 5, Layer A was assumed to prevail through the wall height.

3 PHOTOGRAMMETRY

As mentioned earlier, the wall of Casa Salvans has arches and buttresses of unique shapes and dimensions. Another key aspect of the wall is the front surface having an inclination that varies throughout the wall. For the structural analysis to be representative of reality, three-dimensional characteristics of the wall must be considered in the calculations. Since the available drawings only provide two-dimensional information, a photogrammetry survey was carried out to determine the three-dimensional characteristics of the visible geometry.

Photogrammetry is the science of making reliable measurements from photographs. This is usually achieved through the creation of 3D models of objects of interest. There are several factors affecting the accuracy of the final model such as the quality of the photos and the ability to capture large overlapping areas of the structure in the images. Therefore, it is crucial to take measurements during the survey in order to calibrate and validate the resulting model.

3D models of different parts of the wall of Casa Salvans, which were built using ReCap Photo [1], yielded a mean error of less than 1% over 38 distance measurements. Photogrammetry model of the part of the wall including Arch 10 can be seen in Figure 5. During the survey, markers were placed on the sides of the buttresses, providing a straight line in between, to be considered as the global x-axis in the models. In addition, the inclination of the wall at different locations was verified through inclination measurements taken on site using a plumb line.

Figure 5: Photogrammetry model of Arch 10.

4 IDEALISED GEOMETRY

The stability calculations assume that the concrete enlargement detected at several locations at the base of the wall is prevalent throughout the wall, acting as a foundation and protruding outwards by 0.30 m. Considering the common construction practices of foundations, it was deemed as appropriate to assume the same enlargement at the heel of the base.

The buttresses of Arches 8, 9 and 10 were extended by the depths measured on site. The foundation of an arch was assumed to be at the same level as its shallower adjacent buttress (Figure 6). For Arches 1 – 7, where there were no available measurements due to restricted excavations, the part of a buttress buried under ground was assumed to be 0.75 m, corresponding to the shallowest depth measured on site. The aforementioned trend was also assumed for the foundations of smaller arches.

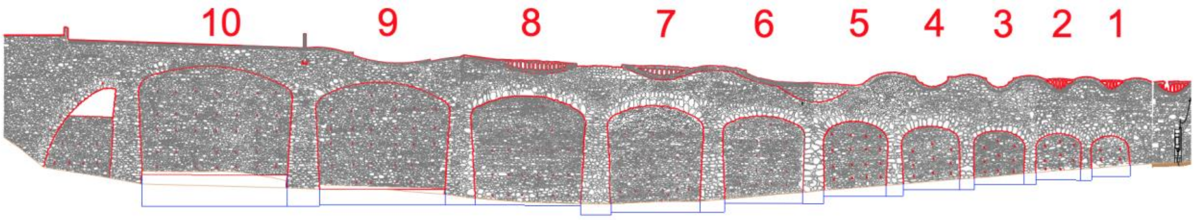


Figure 6: Assumed foundation configuration.

For Arches 1 – 5, the visible arch widths, the distance an arch protrudes from the wall beneath it, are compatible within each other, having a mean of 22 cm with a standard deviation of only 2 cm. Furthermore, the front surface of this part of the wall has a uniform inclination with a mean of 85.6° and a standard deviation of 0.5°. Therefore, it can be suggested that Arches 1 – 5 are non-structural arches that were built for decorative purposes. Thus, it was assumed that this part of the wall behaves as a traditional gravity retaining wall (Figure 7). The back surface of the wall was assumed to have a uniform inclination, as determined at the exploration pit above Arch 3.

The part of the wall consisting of Arches 6 – 10 was idealized based on elevation drawings, the photogrammetry model and the measurements from exploration pits and holes. The excavations revealed that the back surface of the wall is vertical, thus, it was assumed to be vertical until the arch level. The level of the arches as well as their buried widths and thicknesses were obtained from the topographic survey outcomes. Arches were assumed to have rectangular cross-sections.

In order to be conservative, the rear edge of the wall below the arches was assumed to be parallel to the front (Figure 7). Similarly, buttresses were assumed to have parallelogram cross-sections until the arch level. The height of each buttress was taken as the height up to which it already had sufficient thickness to be compatible with the thicker arch connecting to it. The measured buried thickness of 0.6 m was used both for the parts of the wall under the arches and the buttresses.

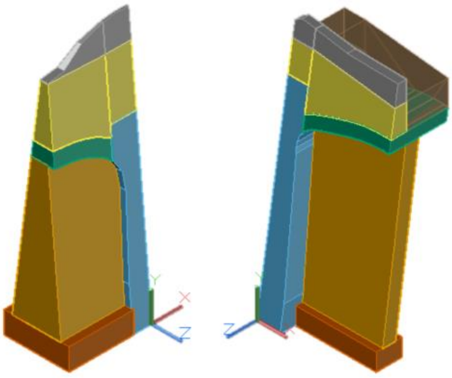


Figure 7: Assumed morphology of the wall for the part consisting of Arches 1 – 5 on the left, for the part consisting of Arches 6 – 10 on the right.

There is a reinforced concrete cantilever section, having a quarter ellipsoid shape, protruding outwards from the wall on the buttress between Arches 5 and 6 (Figure 8). An equivalent of the visible section was assumed to be present at the back of the wall supporting the cantilever. Figure 8 demonstrates the idealized front and back surfaces of the wall.

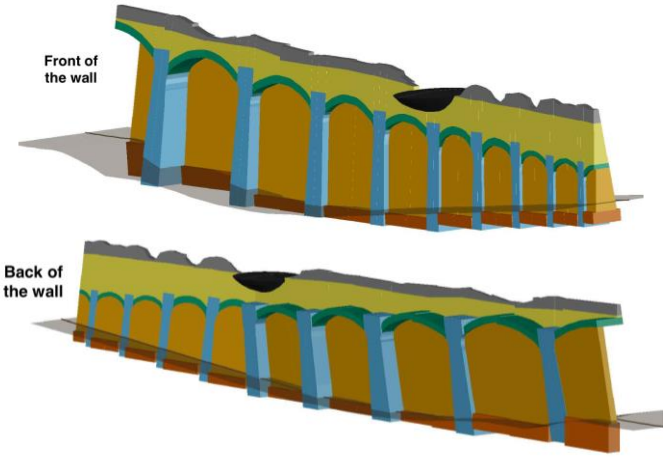


Figure 8: Idealized geometry of the wall from the front view on top, from the rear view on bottom.

5 STABILITY ANALYSIS

5.1 Methodology and assumptions

For the part of the wall consisting of Arches 6 – 10, the failure would be expected due to collapse of individual buttresses as a consequence of the conceivable load transmission path. Loads from the keys of the two adjacent arches are transferred to the buttress, making it the most vulnerable structural element in terms of stability. Hence, the stability of the wall can be evaluated by analyzing sections spanning between the keys of arches on either side of each buttress. Since each arch and buttress are unique in terms of shape and dimensions, assigning a single cross-section to a unit consisting of two half arches and a buttress is inappropriate. By means of the symmetry of an arch and a buttress about their center lines, the section spanning from the key of an arch to the center line of each buttress was defined as the unit section to be analyzed. Considering the geometrical variations throughout the wall, the stability of 18 half-arch sections were evaluated with respect to overturning and sliding (Figure 9). The sections are labelled according to the arch they contain with a suffix of “a” for the right half-arch section and a suffix of “b” for the left half-arch section.

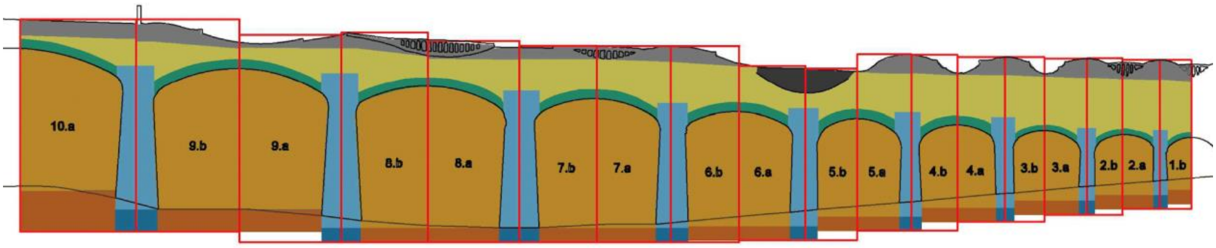


Figure 9: Sections analyzed to assess the stability of the wall of Casa Salvans.

The forces acting on a cross-section of the retaining wall are the weight of the structure, the weight of soil above arch level for Arches 6 – 10 and the forces due to lateral earth pressures. Figure 10 illustrates the forces acting on a section of the wall from the part consisting of Arches

1 – 5. The lateral earth pressures acting on a unit section are the active pressure, P_a applied by the retained soil at the back of the wall and the passive pressure, P_p exerted by the soil at the front of the wall. The forces due to lateral earth pressures were calculated by using Coulomb's Theory of Lateral Earth Pressure. The ground level behind the wall, which was used in the active pressure calculations, was taken as the average ground level within the section. Since pipelines were uncovered at a depth of 0.4 m in some of the pits, the ground level at the front of the wall was assumed to be 0.55 m lower than the actual ground level for the passive pressure calculations.

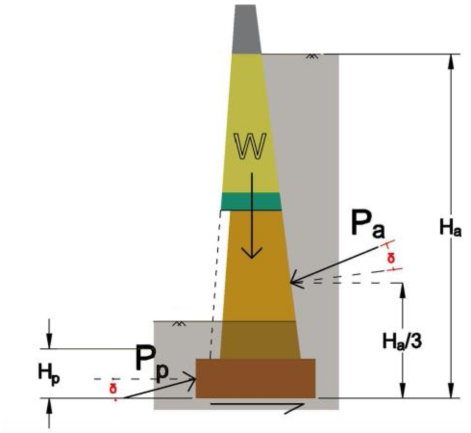


Figure 10: The forces acting on a unit section of the wall from the part consisting of Arches 1 – 5.

The internal friction angle of the soil (ϕ), the friction angle between masonry surface and soil (δ), the ground inclination and the inclination of the back surface of the wall are the parameters used to calculate Coulomb's lateral earth pressure coefficients. The geotechnical survey revealed that most of the retained soil acting on the wall is likely to consist of Layer A. Thus, the parameters defined for the corresponding layer, a specific weight of 19 kN/m^3 and an internal friction angle of 30° were adopted for all computations. According to some literature sources, typical values for the friction angle between stone masonry and soil (δ) varies from 16.4° to 26.3° for clayey earth and from 26.4° to 30.6° for sand and gravel [2]. Since the analysis aims to be representative of the reality, an initial δ value of 20° was assumed instead of correlation to internal soil friction angle, which is the common practice in modern retaining wall design procedures [3].

For Layer A, the undrained cohesion, c_u was determined to be smaller than 10 kPa. In order to be conservative and facilitate the application of Coulomb's Theory, c_u was neglected except for sliding calculations. While estimating the stability against sliding, it was taken as 10 kPa to provide adequate shear resistance. For Arches 6 – 10, a cohesion value of 29 kPa was assumed since it is plausible that the base of the wall rests on Layer B in this part.

After determining the active and passive earth pressures, the total stabilizing vertical load was calculated by multiplying every volume constituting a section by the corresponding specific weight. The specific weight of masonry was taken as 22 kN/m^3 while that of concrete was taken as 23 kN/m^3 [4]. Sections 6a to 10b have an additional stabilizing load due to the weight of the soil lying above the arches and the buttress.

Once all the loads acting on a cross-section have been computed, the factor against overturning, $F_{overturning}$ was determined as the ratio of the stabilizing moments to the overturning ones, around the toe of the buttress. Similarly, the factor against sliding, $F_{sliding}$ was determined as the ratio of the sum of the sliding forces to the sum of the resisting forces. For both of the factors, being greater than or equal to one indicates that a stable equilibrium condition exists against the corresponding phenomena.

5.2 Discussion of results

The factors against overturning and sliding of all sections are reported in Figure 11 and Figure 12 respectively. As mentioned earlier, Sections 1b to 5b compose the part of the wall that limited information was available on the buried structure, thus it was assumed to behave as a traditional retaining wall with non-structural arches. Considering Section 6a shares the load of the reinforced concrete cantilever, from Section 6a to 10a, the overturning stability enhances as the weight of the section increases. Furthermore, the effect of a change of the soil layer to the more cohesive one at the foundation base can be appreciated in Figure 12 through the sharp increase of the sliding factors when wall adhesion is considered.

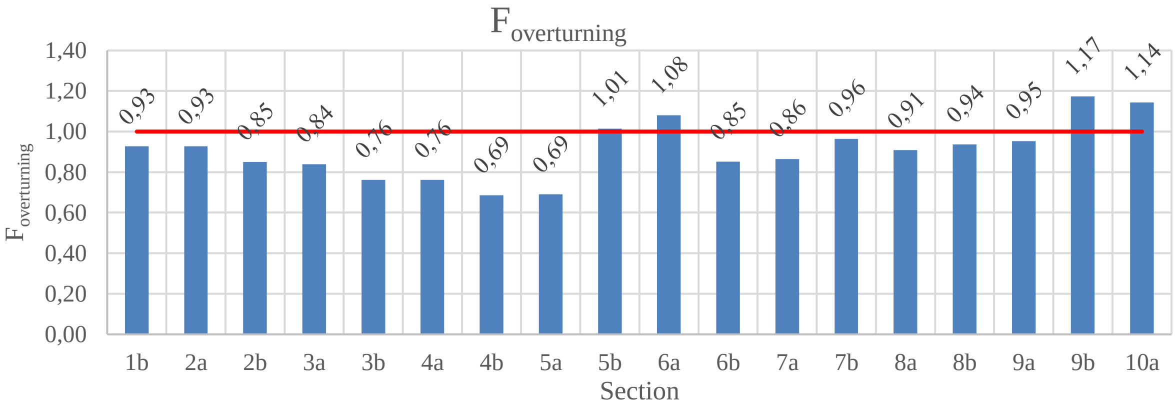


Figure 11: Factors against overturning.

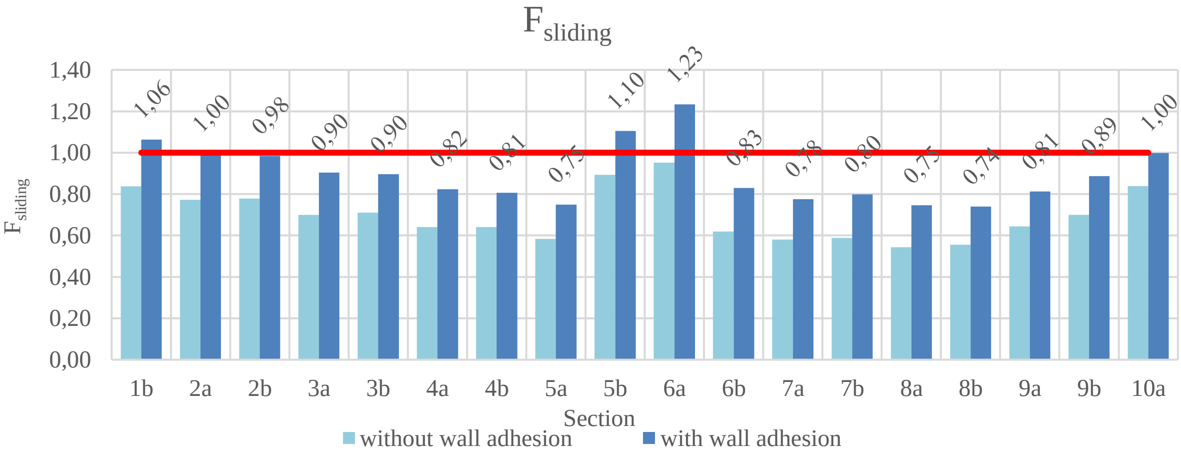


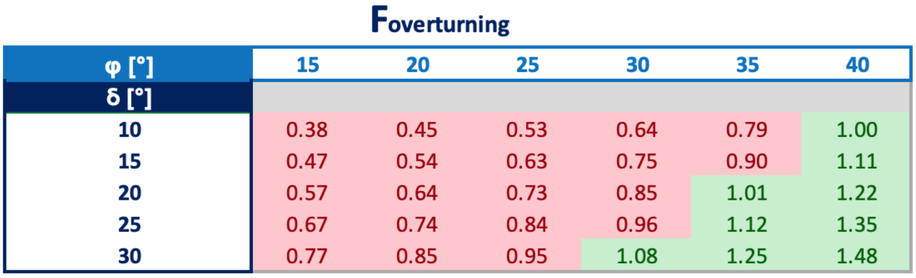
Figure 12: Factors against sliding.

It should be noted that a comprehensive structural verification of a retaining wall would also include the computation of the base pressures and a comparison with the ultimate bearing capacity of the soil layer. However, based on the assumed geotechnical parameters, the analysis results in an unreliable equilibrium state and the base pressures cannot be computed for most of the sections as they are overturning.

The results indicate that the stability of the wall of Casa Salvans cannot be justified based on the current assumptions. Since the wall has been stable for 100 years and does not present any signs of damage, the factors should not necessarily be interpreted as the wall being unsafe. The shear strength parameters of the soil having been obtained through correlations to SPT-N values evokes uncertainties on the parameters. Therefore, some parametric analyses were carried out to determine the sensitivity of the results to key uncertain parameters.

6 SENSITIVITY ANALYSIS ON GEOTECHNICAL PARAMETERS

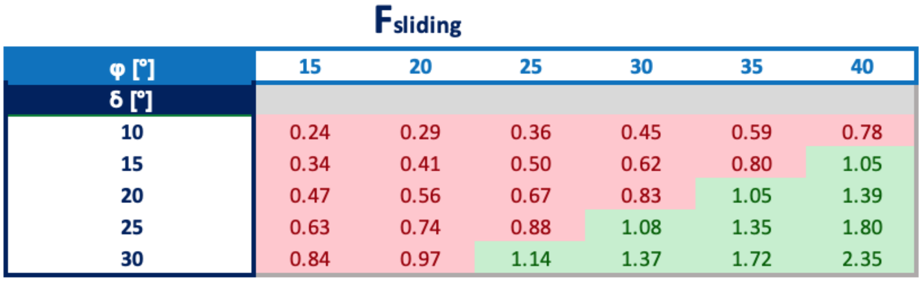
Sensitivity analysis was performed on the factors against overturning and sliding of Section 6b by modifying the internal friction angle of the soil, ϕ, and the wall friction angle between the masonry surface and the soil, δ. Section 6b was chosen as the representative unit as it presents relatively low factors compared to the other sections of the assumed cross-section typology. The analysis verified that the evaluation of stability is highly dependent on the aforementioned geotechnical parameters. The effect of different combinations of ϕ and δ ranging from 10° to 40° on the overturning and sliding factors is shown in Figure 13 and Figure 14.



$F_{overturning}$

φ [°]	15	20	25	30	35	40
δ [°]						
10	0.38	0.45	0.53	0.64	0.79	1.00
15	0.47	0.54	0.63	0.75	0.90	1.11
20	0.57	0.64	0.73	0.85	1.01	1.22
25	0.67	0.74	0.84	0.96	1.12	1.35
30	0.77	0.85	0.95	1.08	1.25	1.48

Figure 13: Sensitivity of $F_{overturning}$ of Section 6b to geotechnical parameters.



$F_{sliding}$

φ [°]	15	20	25	30	35	40
δ [°]						
10	0.24	0.29	0.36	0.45	0.59	0.78
15	0.34	0.41	0.50	0.62	0.80	1.05
20	0.47	0.56	0.67	0.83	1.05	1.39
25	0.63	0.74	0.88	1.08	1.35	1.80
30	0.84	0.97	1.14	1.37	1.72	2.35

Figure 14: Sensitivity of $F_{sliding}$ of Section 6b to geotechnical parameters.

Increasing φ enhances the stability of the section against both actions significantly. The effects of changing δ for different φ values on both factors are illustrated in Figure 15 and Figure 16. Increasing δ alters the factor against overturning by a similar trend regardless of the soil internal friction angle. However, the increase in the factor against sliding due to the increase of δ is higher as the soil friction angle gets higher.

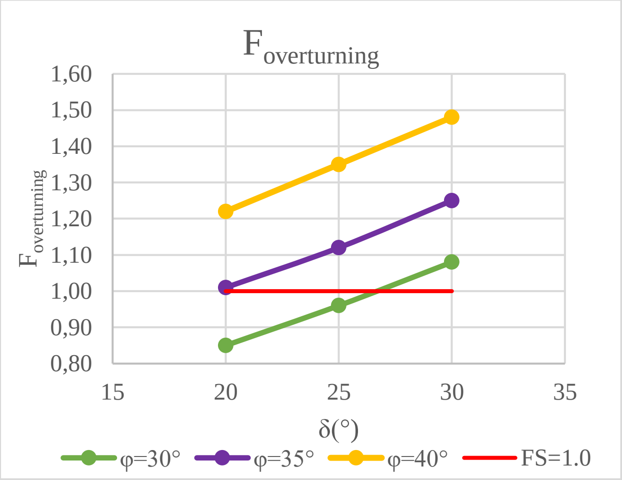


Figure 15: $F_{overturning}$ of Section 6b for different values of φ and δ.

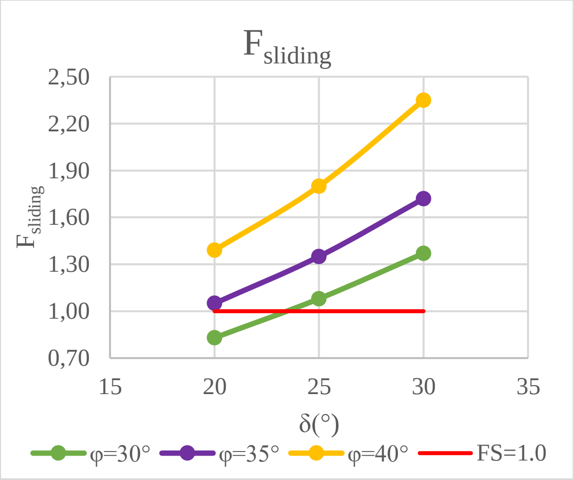


Figure 16: $F_{sliding}$ of Section 6b for different values of φ and δ.

The analysis verified that there are reasonable combinations of ϕ and δ, such as 35° and 20° respectively, which would allow the stability of the wall to be justified in terms of both overturning and sliding. Therefore, it is crucial to determine the actual values of these parameters in order to properly evaluate the stability of the wall.

7 IN-PLANE STABILITY OF ARCHES

In addition to the ability of the wall to resist overturning and sliding caused by lateral pressure of the retained soil, a complete stability verification also involves verifying the in-plane stability of the arches to stand in equilibrium. Unreinforced masonry arches are prone to fail due to stability problems. A structural analysis technique based on plastic theory, Lower Bound Theorem of Limit Analysis, was utilized to evaluate the equilibrium of the arches of the wall of Casa Salvans.

The analysis assumes that masonry has no tensile strength while it has unlimited compressive strength due to stresses being low and that failure may not occur due to sliding. Graphic statics can then be used to locate thrust lines within the boundaries of the arches. A thrust line is a possible load path along the arch that is composed of the resultant compressive forces in equilibrium with the weight of the unit blocks of the arch, in an inverted catenary shape. The Lower Bound Theorem states that an arch can be deemed safe if a thrust line can be found lying entirely within the geometry.

The elevation drawing of the front surface of the wall and the same dimensions used to assess the overturning and sliding stability for the buttresses and arches were used to apply graphic statics on the arch series. The estimated loads acting on the buttresses and the arches were computed based on the estimated volumes of wall elements and soil lying above the arches. The part of the wall below the arches was ignored in order not to enhance the stability of the arches.

In spite of profoundly conservative assumptions on the geometry, a thrust line was found to lie within all boundaries verifying the in-plane stability of the wall (Figure 17). Although

Arches 1 – 5 were assumed to be merely decorative, the initial graphic statics analysis demonstrated that this part of the wall would be more than capable of resisting the thrust transmitted from the bigger arches.

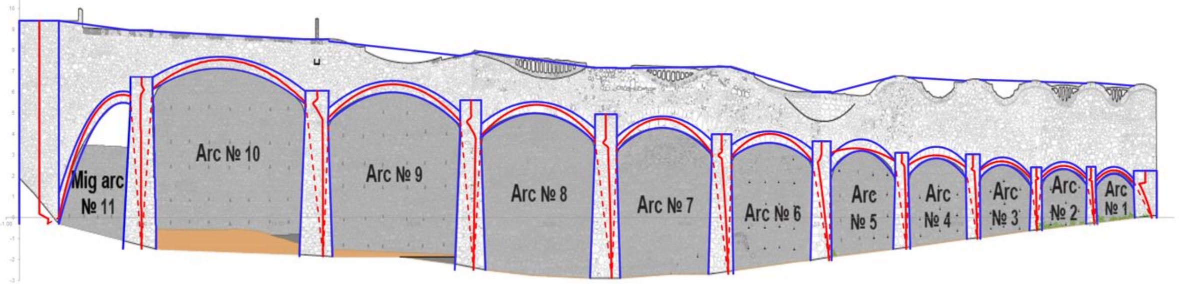


Figure 17: Graphic statics application on the wall of Casa Salvans.

8 CONCLUSIONS

This paper presented the systematic methodology employed for the stability assessment of a masonry arched and buttressed retaining wall. The procedure first involved creating an idealised three-dimensional geometry of the wall by combining information from photogrammetry, excavation pits and holes and detailed elevation drawings. The geometry of the wall is then subdivided into individual analysis sections based on the conceivable load transmission path. After estimates of the lateral earth pressure are computed, the stability of each analysis section against sliding and overturning is verified. Finally, the in-plane stability of the arches is verified using the Lower Bound Theorem of Limit Analysis applied to the idealised geometry. In the case of the wall of Casa Salvans, the stability of the wall against overturning and sliding could not be justified on the basis of the idealized geometry and the estimates of geotechnical parameters obtained through indirect methods. However, the parametric analysis carried out on the effect of the soil internal friction and wall friction angles showed that there are reasonable combinations of the two for which the stability of the wall can be verified in terms of both overturning and sliding. Hence, determining the actual ϕ and δ values is of vital importance to better understand the actual stability condition. In this case, it is therefore highly recommended to carry out a comprehensive geotechnical survey to obtain geotechnical parameters through direct test methods instead of correlations. The in-plane stability of the arches of the wall of Casa Salvans was verified according to the Lower Bound Theorem through graphic statics application.

9 REFERENCES

[1] Autodesk, “ReCap Photo.” Autodesk, 2019.

[2] J. A. J. Dvořák, L. Novák, *Soil conservation and silviculture*. Elsevier, 1994.

[3] R. F. Craig, *Craig’s Soil Mechanics*, 7th Editio. London: Spon Pres, 2004.

[4] B. S. I. BSI, “EC1: Part 1.1: Densities, self-weight, imposed loads for buildings,” *Structural Engineer*, vol. 82, no. 2. European Comittee For Standardization, London, p. 44, 2002.

[5] S. Huerta, “Mechanics of masonry vaults: the equilibrium approach,” in *1st Int. Congr. Struct. Anal. Hist. Constr.*, 2001, pp. 47–70.