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On the modeling of pedestrian motion

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ABSTRACT

A model for the simulation of pedestrian flows and crowd dynamics has been developed. The model is based on a series of forces, such as: will forces (the desire to reach a place at a certain time), pedestrian collision avoidance forces, obstacle/wall avoidance forces; pedestrian contact forces, and obstacle/wall contact forces. Except for the will force, it is assumed that for any given pedestrian these forces are the result of only local (nearest neighbour) situations. The near-neighbour search problem is solved by an efficient incremental Delaunay triangulation that is updated at every timestep. In order to allow for general geometries a so-called background triangulation is used to carry all geographic information. At any given time the location of any given pedestrian is updated on this mesh. The results obtained to date show that the model performs well for standard benchmarks, and allows for typical crowd dynamics, such as lane forming, overtaking, avoidance of obstacles and panic behaviour.

1. Introduction

The accurate prediction of pedestrian motion can be used to assess the potential safety hazards and operational performance at events where many individuals are gathered. Examples of such situations are sport and music events, cinemas and theatres, museums, conference centers, places of pilgrimage and worship, as well as street demonstrations. Evacuation from airplanes, ships [1], trains and buildings [2] also represent cases where the prediction of pedestrian motion can be used advantageously. For cases where the prediction of many ($>10^3$) individuals over a long time is considered (e.g. pilgrimage centers), optimal data structures must be used in order to carry out a simulation in a reasonable time.

If we consider the motion of a single individual, at any given time, the direction and speed will be the result of a long list of possible (and very likely conflicting) forces and circumstances. Among these, we mention:

- Motivation to reach a certain place at a certain time (often referred to as will force, and influenced by a variety of factors such as time constraints, importance of punctuality, location constraints, importance of reaching a place and staying there long enough, etc.).
- Physical fitness (or level of exhaustion).
- Material obstacles in the way or close to the pedestrian.
- Pedestrians surrounding a pedestrian (closeness of neighbours, density of crowd, size and velocity of neighbours, etc.).
- Geographical knowledge (i.e. dependence on signs, general flow of pedestrians, etc.).
- Climatic conditions.
- Terrain conditions (splippery, climbing, stairs, escalators, etc.).
- Signs or individuals that steer the flow of pedestrians (e.g. traffic lights, policemen, etc.).

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As one can see, this list combines elements from such diverse disciplines as applied mathematics, computational geometry, computer science, psychology, medical sciences, and others. Moreover, the origin of these forces or circumstances is not always very clear. For example, the motivation to reach a certain place may be influenced by the current climate conditions (we all want to get to a warm place as fast as possible when it is cold and/or raining). As another example, consider a pedestrian with excellent knowledge of the route to be taken. In this case, the effect of signs may be neglected altogether.

Over the last decade a considerable amount of research has been devoted to the computational prediction of pedestrian motion and crowd dynamics [3,4,2]. The methods used may be subsivided into the following categories:

(a) *Models Based on Continuum Theory*: in this case one formulates partial differential equations for the 'pedestrian flow'. The resulting conservation laws are then solved via lattice gas, finite difference, finite volume or finite element methods [5,6]. The modeling aspect that is problematic is the flux function, i.e. the relationship between local pedestrian density and the resulting velocities. For 1-D situations (e.g. tunnels, halls) with no counterflow this approach is viable (there is ample data from empirical measurements). For complex 2-D cases with many cross-streams and a rich variety of paths and individuals, it is difficult to see how a continuum approach such in this may work. Indeed, very few, if any, complex 2-D cases have been solved to date with this approach.
(b) *Models Based on Cellular Automata*: in this case pedestrians can only be located on the (finite number of) points of an adaptive (typically Cartesian) grid. During each timestep, pedestrians jump from one position to the next depending on the local arrangement of neighbours and individual objectives [7–12].
(c) *Models Based on Newtonian Dynamics*: in this case each pedestrian is treated as a mechanical object subjected to forces. Given the forces at any given instant, the acceleration is obtained and from it the trajectory. These so-called continuous models offer a richer variety of options to model pedestrian behaviour. The distinction of 'social forces' and 'external forces' is often made. Perhaps the best known of these models is the Helbing–Molnár–Farkas–Vicsek Social Force Model [13–15]. There are two basic shortcomings of this model. The first is the symmetry of social forces, which is inconsistent with human behaviour: just because pedestrian 1 is trying to avoid pedestrian 2 does not imply that pedestrian 2 is trying to avoid pedestrian 1, and even is this were the case the forces in all likelihood are not the same. The second is that every pedestrian affects all others. The forces diminish considerably with distance, but the principle is inconsistent with human behaviour. We tend to be influenced by nearest neighbours, not every other human in the neighbourhood (many of which we can not even see).

The third category of models has been followed here, attempting to combine as much generality as possible with minimal information and results that are consistent with human behaviour. The starting point is given by Newton's law of motion:

$$m_p \frac{d\mathbf{v}}{dt} = \mathbf{f}, \quad \frac{d\mathbf{x}}{dt} = \mathbf{v}. \tag{1}$$

Here m_p, $\mathbf{v}$ and $\mathbf{x}$ denote the mass, velocity and position of the pedestrian, and $\mathbf{f}$ the sum of all forces exerted by it or acting on it. The basic unknown that requires intensive modeling efforts in these equations is the force-vector. In the sequel, we will discuss the main forces that can be identified as acting on a pedestrian.

2. Pedestrian forces

The forces that accelerate a pedestrian are the result of internal (will) and external (collision, signals, etc.) forces. An alert individual will try to avoid collisions before they happen, resulting in vanishing external forces. However, if the pedestrian density increases, collision forces will appear, and the ability to move freely will be impaired. The difference between external and internal forces is admittedly not too rigorous, and only serves descriptive purposes in the present context. The forces considered are:

- Internal forces:
  - Will force.
  - Pedestrian collision avoidance forces.
  - Obstacle/wall avoidance forces.

- External forces:
  - Pedestrian contact forces.
  - Obstacle/wall contact forces.

Additionally, we consider:

- Kinematic constraints:
  - Motion inhibition forces.

2.1. Will force

We denote by will force the force that will accelerate (or decelerate) a pedestrian to achieve the velocity it desires. Given a desired velocity $\mathbf{v}_d$ and the current velocity $\mathbf{v}$, this force will be of the form

$$\mathbf{f}_{will} = g_w(\mathbf{v}_d - \mathbf{v}). \tag{2}$$

The modeling aspect is included in the function g_w, which, in the non-linear case, may itself be a function of $\mathbf{v}_d - \mathbf{v}$. Suppose g_w is constant, and that only the will force is acting. Furthermore, consider a pedestrian at rest. In this case, we have:

$$m\frac{d\mathbf{v}}{dt} = g_w(\mathbf{v}_d - \mathbf{v}), \quad \mathbf{v}(0) = 0, \tag{3}$$

which implies:

$$\mathbf{v} = \mathbf{v}_d\left(1 - e^{-\beta t}\right), \quad \beta = \frac{g_w}{m}. \tag{4}$$

We can thus obtain g_w by measuring the time required to reach a percentage (e.g. 90%) of the desired velocity, starting from rest. This time is typically in the range of 0.5–1.0 s, but obviously depends on the pedestrian's current state of fitness and stress, desire to reach a goal, climate, signals, noise, etc. The unit of β is given by the inverse of time, i.e. $[\beta] = 1/s$. The initial (and maximum) acceleration is given by:

$$\left.\frac{d\mathbf{v}}{dt}\right|_0 = \beta\mathbf{v}_d = \alpha. \tag{5}$$

This 'characteristic acceleration' of the individual α can be used to relate all other forces. The direction of the desired velocity

$$\mathbf{s} = \frac{\mathbf{v}_d}{|\mathbf{v}_d|}, \tag{6}$$

will depend on the type of pedestrian and the cases considered. A single individual will have as its goal a desired position $\mathbf{x}_d(t_d)$ that is to be reached at a certain time t_d. If there are no time constraints, t_d is simply set to a large number. Given the current position $\mathbf{x}$, the direction of the velocity is given by

$$\mathbf{s} = \frac{\mathbf{x}_d(t_d) - \mathbf{x}}{|\mathbf{x}_d(t_d) - \mathbf{x}|}. \tag{7}$$

For members of groups, the goal is always to stay close to the leader. Thus $\mathbf{x}_d(t_d)$ becomes the position of the leader. In the case of an evacuation simulation, the direction is given by the gradient of the distance function to the closest perceived exit:

$$\mathbf{s} = \frac{\nabla d_e}{|\nabla d_e|}. \tag{8}$$

The magnitude of the desired velocity $|\mathbf{v}_d|$ depends on the fitness of the individual, and the motivation/urgency to reach a certain place at a certain time. Pedestrians typically stroll leisurely at 0.6–0.8 m/s, walk at 0.8–1.5 m/s, jog at 1.5–4.0 m/s, and run at 4.0–10.0 m/s. Measured speeds at airports for freely moving passengers are of the order of 1.33 m/s with a standard deviation of 0.25 m/s [16,17].

2.2. Pedestrian–pedestrian forces

Besides the will to reach a location at a certain time, a second important desire is to avoid collisions with other pedestrians. When walking, we perform this avoidance subconciously, making it difficult to model. The implicit assumption that will be made in the sequel is that only nearest neighbours influence these 'collision avoidance forces'.

2.2.1. Intermediate range force

We denote by intermediate range forces those that change the motion of an individual in order to avoid an encounter with another. The observation employed here is that all of us will try to avoid a collision by moving away from an encounter long before it happens. The nature of these forces is such that they act mainly in a direction normal to the current motion of the individual, and therefore do not lead to a significant decrease in velocity. Moreover, this force is only active if a collision is sensed.

Using as a starting point the situation depicted in Fig. 1, given two pedestrians with current coordinates and velocities $\mathbf{x}_1, \mathbf{v}_1$ and $\mathbf{x}_2, \mathbf{v}_2$, an encounter may be computed by evaluating the time increment Δt at which the distance between the two is minimized, i.e.

$$[\mathbf{x}_1 + \Delta t\mathbf{v}_1 - (\mathbf{x}_2 + \Delta t\mathbf{v}_2)]^2 \rightarrow min. \tag{9}$$

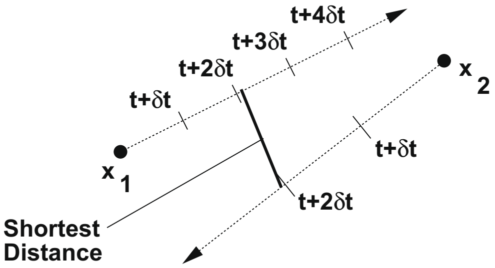

Fig. 1. Encounter of two pedestrians.

This results in

$$[(\mathbf{x}_1 - \mathbf{x}_2) + \Delta t_m(\mathbf{v}_1 - \mathbf{v}_2)] \cdot (\mathbf{v}_1 - \mathbf{v}_2) = 0, \tag{10}$$

or

$$\Delta t_m = -\frac{(\mathbf{x}_1 - \mathbf{x}_2) \cdot (\mathbf{v}_1 - \mathbf{v}_2)}{(\mathbf{v}_1 - \mathbf{v}_2) \cdot (\mathbf{v}_1 - \mathbf{v}_2)}. \tag{11}$$

The minimum distance is given by

$$\delta_{min} = |(\mathbf{x}_1 - \mathbf{x}_2) + \Delta t_m(\mathbf{v}_1 - \mathbf{v}_2)|. \tag{12}$$

Obviously, the force will only become active if $\Delta t > 0$. The general nature of this force is such that it will decrease with distance. It is clear that it should diminish with distance. How exactly this decrease function looks like is unknown at the present time (it may even be random). We have used a function based on the normalized distance between the pedestrians

$$\rho = \frac{|\mathbf{x}_1 - \mathbf{x}_2|}{r_1}, \tag{13}$$

where r_1 is the characteristic radius of pedestrian 1, which is of the form:

$$f = f_{max} \frac{1}{1 + \rho^p}, \quad p = 2. \tag{14}$$

The advantage of this form is that a well defined maximum force can be identified (f_{max}). The polynomial power p is, of course, a guess (as it turns out, it delivers surprisingly good results, see below). This simple repulsion force may be refined further by considering the directions along and normal to the current velocity vector $\mathbf{v}_1$, denoted by $\mathbf{t}$ and $\mathbf{n}$, respectively. The normalized distance ρ may be into tangential and normal components:

$$\rho_t = \frac{|\mathbf{t} \cdot (\mathbf{x}_1 - \mathbf{x}_2)|}{r_1}, \quad \rho_n = \frac{|\mathbf{n} \cdot (\mathbf{x}_1 - \mathbf{x}_2)|}{r_1}. \tag{15}$$

This leads to tangential and normal forces of the form:

$$\mathbf{f} = -f_{max} \frac{1}{1 + \rho_t^2} \mathbf{t} - f_{max} \frac{1}{1 + \rho_n^2} \mathbf{n}. \tag{16}$$

The value of f_{max} can be related to the relaxation time, i.e. is not too difficult to obtain. We have used $f_{max}/m_p = O(2)\alpha[\mathrm{m/s}^2]$.

2.2.2. Near-range force

Once a pedestrian has moved close enough to obstacles or other pedestrians, its velocity will be decreased markedly. Unlike the long-range forces, these forces act in the direction of the normalized difference vector

$$\mathbf{s} = \frac{\mathbf{x}_2 - \mathbf{x}_1}{|\mathbf{x}_2 - \mathbf{x}_1|}. \tag{17}$$

We have used, as before, a force of the form

$$\mathbf{f} = -f_{max} \frac{1}{1 + \rho^p} \mathbf{s}, \quad p = 2, \tag{18}$$

although the exact nature of this force is unknown at the present time. The value of f_{max} can again related to the relaxation time. We have used $f_{max}/m_p = O(4)\alpha[\mathrm{m/s}^2]$.

2.2.3. Contact force

Once a pedestrian is so close to other pedestrians that contact occurs, the repulsion forces will increase markedly. The contact force acts in the direction of the normalized difference vector $\mathbf{s}$ given by Eq. (17), but unlike the previous forces it is symmetric, i.e. the same for both pedestrians. Defining:

$$\rho_{12} = \frac{|\mathbf{x}_1 - \mathbf{x}_2|}{r_1 + r_2}, \tag{19a}$$

$$\alpha_{12} = max(\alpha_1, \alpha_2), \tag{19b}$$

where $r_1, r_2, \alpha_1, \alpha_2$ are the characteristic radii and accelerations of pedestrians 1, 2, the force is given by:

- $\rho_{12} \leqslant 1$:

$$\mathbf{f}_{12} = -f_{max}\frac{1}{1+\rho^p}\mathbf{s}, \quad p = 2, \tag{20a}$$

- $\rho_{12} \geqslant 1$:

$$\mathbf{f}_{12} = -2f_{max}\frac{1}{1+\rho^p}\mathbf{s}, \quad p = 2. \tag{20b}$$

As before, the value of f_{max} can be related to the relaxation time. We have used $f_{max}/m_p = O(8)\alpha_{12}[\mathrm{m/s^2}]$. If a difference in tangential velocity (i.e. normal to $\mathbf{s}$) exists between the pedestrians, contact forces will lead to friction forces. The assumption made is of the Coulomb type, i.e. the friction force is given by the magnitude of $\mathbf{f}_{12}$ multiplied by a coefficient:

$$\mathbf{f}^t_{12} = c_f|\mathbf{f}_{12}|\mathrm{sign}(1, (\mathbf{v}_1 - \mathbf{v}_2)\cdot\mathbf{t})\mathbf{t}, \tag{21}$$

where $\mathbf{s}\cdot\mathbf{t} = 0$ and $c_f = O(0.2)$.

2.3. Kinematic constraints: motion inhibition

The ability to perform the next step in order to achieve a certain desired velocity may be hampered by the presence of other pedestrians. Therefore, a 'motion inhibition' model has to be incorporated for situations where the pedestrian density is high.

Given the characteristic radiuses r_1, r_2 of pedestrians 1, 2, the minimum safe distance between them may be written as

$$\begin{aligned} d_s &= \xi r_1 + \xi r_2 \quad \text{if}: \mathbf{v}_1\cdot\mathbf{v}_2 < 0, \\ d_s &= \xi r_1 + (1-\xi) r_2 \quad \text{if}: \mathbf{v}_1\cdot\mathbf{v}_2 > 0. \end{aligned} \tag{22}$$

The factor ξ, which lies between $0 < \xi < 1$, measures the influence of distance required for the forward step, and $1-\xi$ the distance required behind a pedestrian. With the positions and velocities of the two pedestrians, as well as the timestep Δt, the new distance d_n between them can be computed. This new distance is then used to evaluate by how much the current velocity must be reduced in order to maintain a minimum distance:

$$c_r = min\left(1, max\left(0, \frac{d_n - d_s}{d_s}\right)\right). \tag{23}$$

If we split the velocity in the directions along the difference vector $\mathbf{x}_2 - \mathbf{x}_1$ and normal to it:

$$\mathbf{v}_1 = v_t\mathbf{t} + v_n\mathbf{n}, \tag{24}$$

the reduced (or possible) velocity will be

$$\mathbf{v}_1 = c_r v_t\mathbf{t} + v_n\mathbf{n}. \tag{25}$$

As the reduction of one velocity may influence the reduction factors of neighbouring pedestrians, the reduced velocities have to be obtained in several passes.

2.4. Obstacle forces

2.4.1. Wall force

A stream of pedestrians will be enclosed by the walls in or around the structures in which it occurs. We have assumed that this force acts in the direction of the gradient of the distance-to-wall function $d_w(\mathbf{x})$. This function, which measures the closest distance to a wall from any given location $\mathbf{x}$, is assumed known. In practice, it is evaluated in a pre-processing step. As before, we have used, for lack of any empirical data, a function of the form:

$$\mathbf{f} = -f_{max}\frac{1}{1+\left(\frac{d_w}{r}\right)^p}\cdot\nabla d_w, \quad p = 2, \tag{26}$$

with $f_{max}/m_p = O(16)\alpha[\mathrm{m/s^2}]$ (note that $|\nabla d_w| = 1$). Depending on the cultural setting, some pedestrians may avoid closeness to walls, while others prefer to walk rapidly close to walls. In order to model this, we only apply the force given by Eq. (26) if the pedestrian is closer than a given distance to wall. This 'threshold distance' is varied randomly among pedestrians in order to achieve a higher degree of realism in the simulations.

2.4.2. Motion inhibition

The ability to perform the next step in order to achieve a certain desired velocity may be hampered by the presence of a wall. Therefore, a 'motion inhibition' model has to be incorporated for situations where the pedestrian is headed for a collision with a wall.

Given the characteristic radius r of the pedestrian, the velocity normal to the wall is set to zero if:

– The pedestrian is headed towards the wall; and
– $d_w < r$.

2.5. Summary of forces

Note that all the forces are related to the characteristic acceleration of the individual α, and are structured in such a way that:

$$\text{will-force} < \text{avoidance-force} < \text{contact-force} < \text{wall-force}.$$

It was found that this combination of force magnitudes gives a rather faithful reproduction of pedestrian flows.

3. Data structures

In order to achieve a real-time modeling for pedestrian simulations with more than 10^4 pedestrians, it is of paramount importance to devise proper data structures that minimize CPU and memory requirements. In fact, it may be argued that after the proper behavioral phenomena have been identified, the realization of real-time modeling hinges on these data structures.

For the pedestrian motion simulation, proper data structures must be designed in order to access rapidly the following information:

(a) Data Carried by the Individual:
  – Personal Data (height, width, fitness, nationality, familiarity with surroundings, objectives, etc.).
  – Current Physical and Mental State.
  – Current Motion Data (location, velocity, etc.).

(b) Geographic Data:
  – Surroundings (e.g. obstacles, signs, guidance personnel, etc.).
  – Terrain Data.
  – Climate Data.
  – Distance to Walls.

(c) Neighbour Data:
  – Nearest Neighbours.
  – Nearest+1 Neighbours, etc.

Data from the first two groups will affect the will forces, as well as the forces due to proximity of walls. The data from the third group is used for the collision avoidance forces.

3.1. Data carried by the individual

This data, which includes, among others, items such as the pedestrians height, width, fitness, nationality, familiarity with surroundings, objectives, current physical and mental state, as well as the current motion data (location, velocity, etc.), is stored in arrays attached directly to the pedestrians. In this way, this data becomes easily accessible at any time. In order to save storage, the individual's height, width and fitness are taken from a data-base, i.e. through a table look-up. In this table, the data belonging to a limited number of representative groups is stored, and the individual is tagged as belonging to one of these groups. In order to enhance realism, a random variation number for personal data is also attached to each individual. The savings in storage accrued by this indirect storage assignment are considerable: if we have N_d personal data items, conventional storage would require $O(N_d * N_p)$ items, whereas indirect data assignment only requires $O(2 * N_p + N_d * N_g)$ items. This implies that even for moderate realism, with O(10) data items per person, indirect storage provides savings of O(5).

Besides the physical, emotional and ethnical data, the prediction of an individual's motion requires the current coordinates (x, y, z) and velocities (v_x, v_y). These are stored at the pedestrian level, requiring $O(5 * N_p)$ storage locations. Furthermore, an internal, time-like parameter is required in order to evaluate the current goals of an individual.

Typical pedestrians will follow a path that requires them to reach a certain place at a certain time, with loiter intervals in between. A table of paths is devised before the simulation, and each pedestrian is assigned a path according to some statistical distribution. For museums, basilicas, or other places of interest the number of possible paths can be considerable. In some cases, it may be better to input destinations and loiter time, and obtain the possible paths through a combinatorial

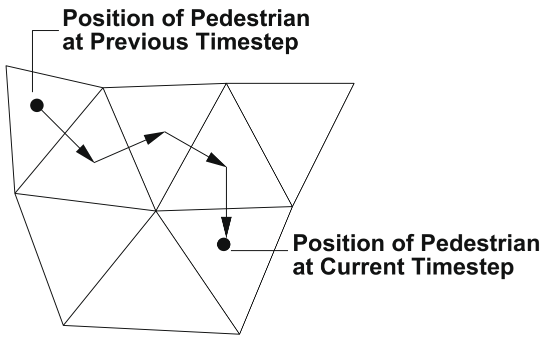

Fig. 2. Nearest neighbour tracking procedure.

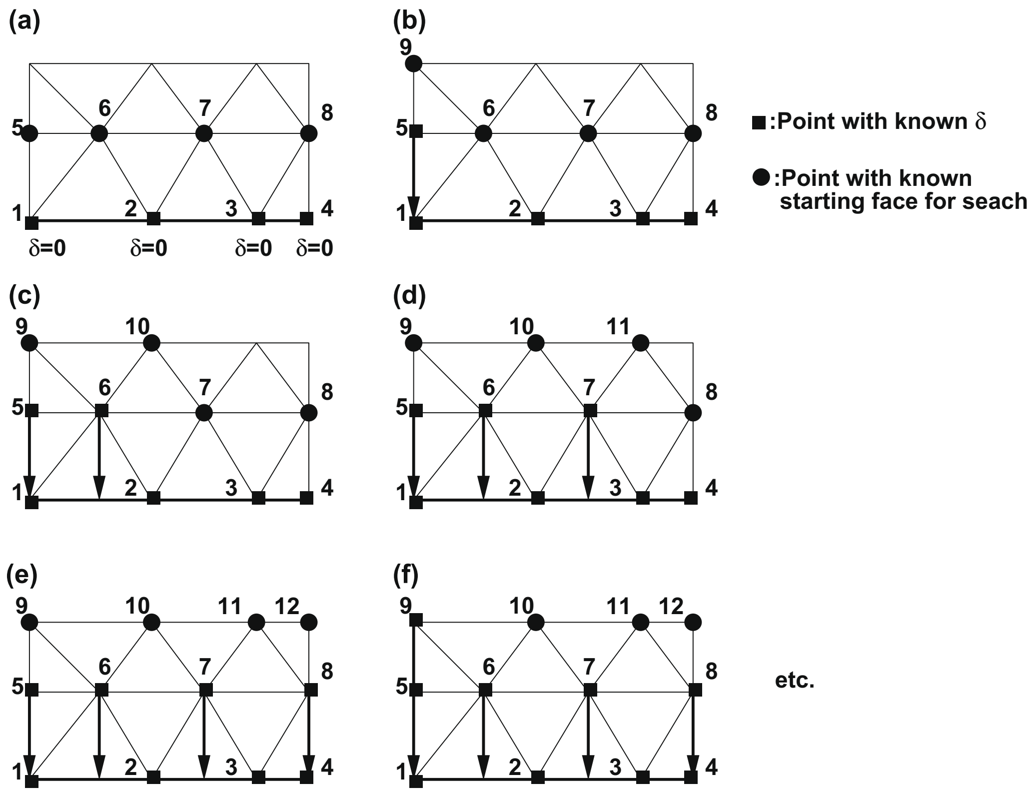

Fig. 3. Obtaining the closest distance to walls.

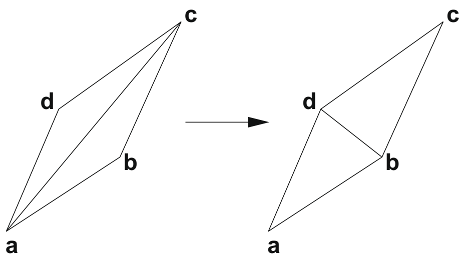

Fig. 4. Diagonal swap using min–max criterion.

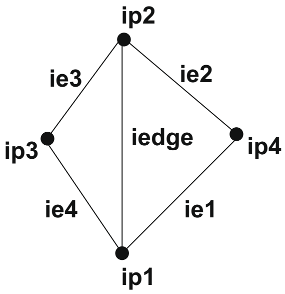

Fig. 5. Edge-based nearest neighbour data.

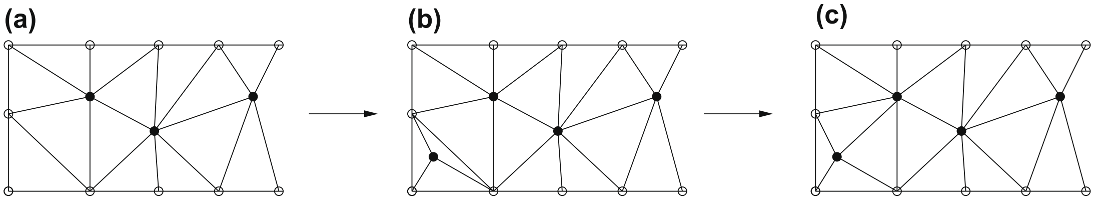

Fig. 6. Introduction of pedestrians into domain.

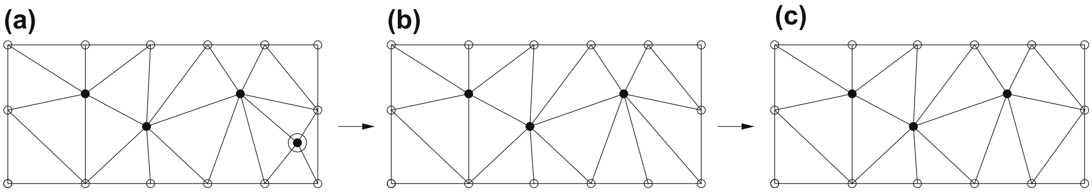

Fig. 7. Removal of pedestrians from domain.

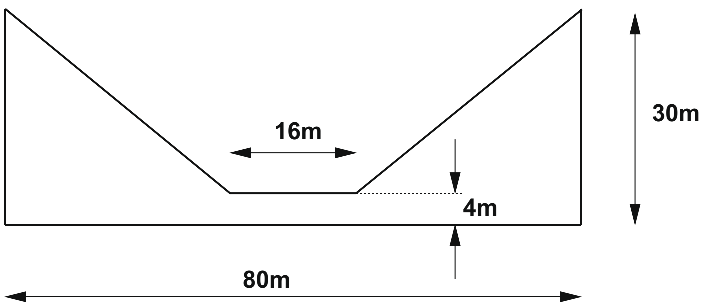

Fig. 8a. Passage geometry.

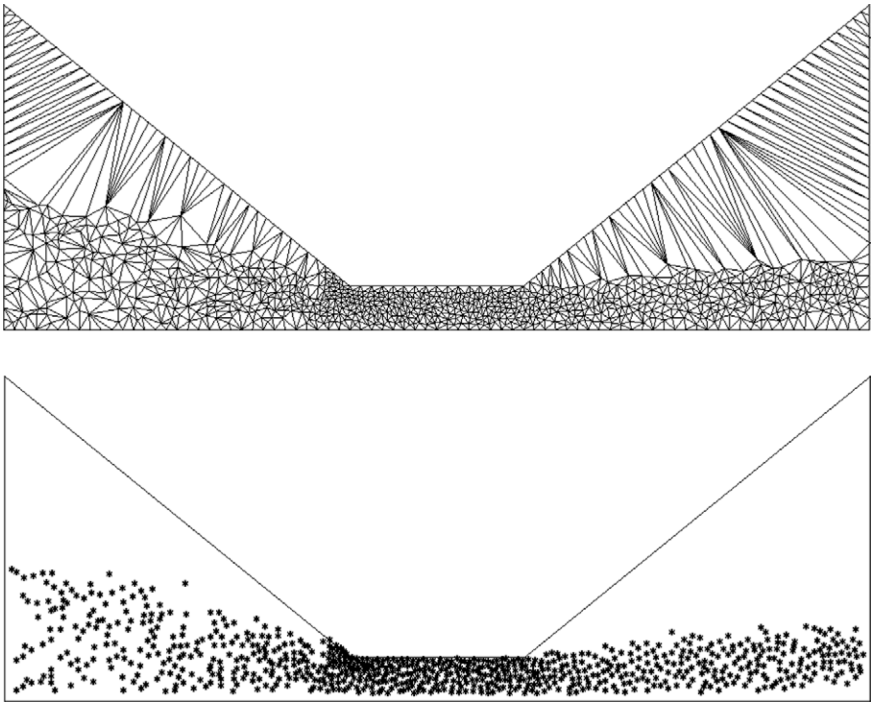

Fig. 8b. Passage: result.

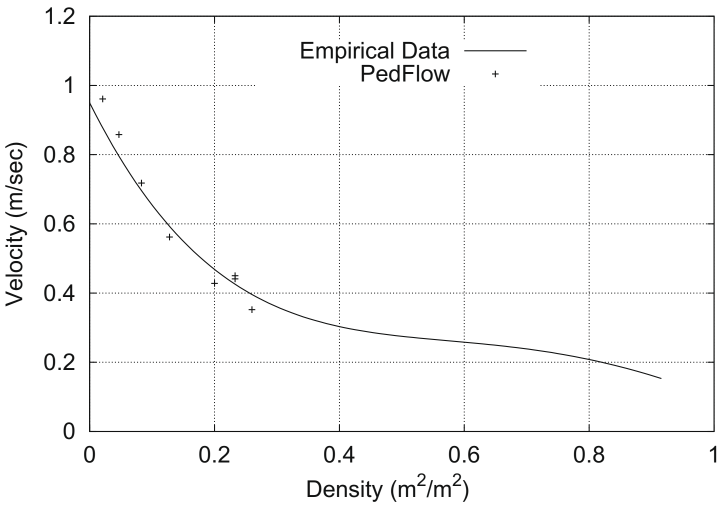

Fig. 8c. Velocity vs.density.

assignment of the stations, with due care for conflicting paths (zig-zag, reverse, etc.). For evacuation simulations, this does not have to be done, as pedestrians will move in the direction of the closest perceived exit.

3.2. Geographic data

Under geographic data we consider, among others, items such as terrain data (inclination, soil/water, escalators, obstacles, etc.), climate data (temperature, humidity, sun/rain, visibility), signs, the location and accessability of guidance personnel, as well as doors, entrances and emergency exits. This data is stored in a so-called background grid consisting of triangular elements. This background grid is used to define the geometry of the problem, and is generated automatically using the advancing front method [18].

All geometrical and climatological data is attached to this grid. This implies that the amount of data stored and used for the pedestrian movement depends only on the level of detail stored in this mesh, and is proportional to the number of elements in it. For obvious reasons, the size of this mesh (number of elements and points) should be limited to the necessary and available amount of information required for the simulation. At every instance, a pedestrian will be located in one of the

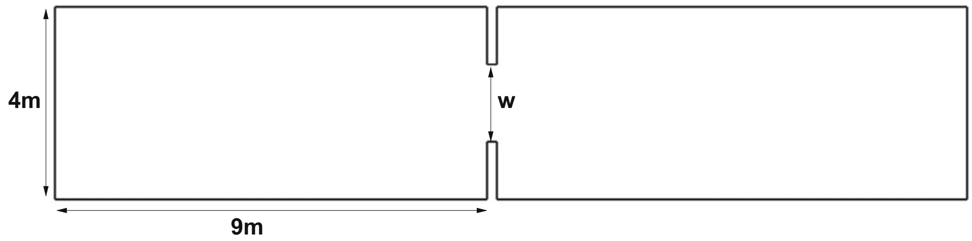

Fig. 9a. Geometry of 'exit through a gap' experiment.

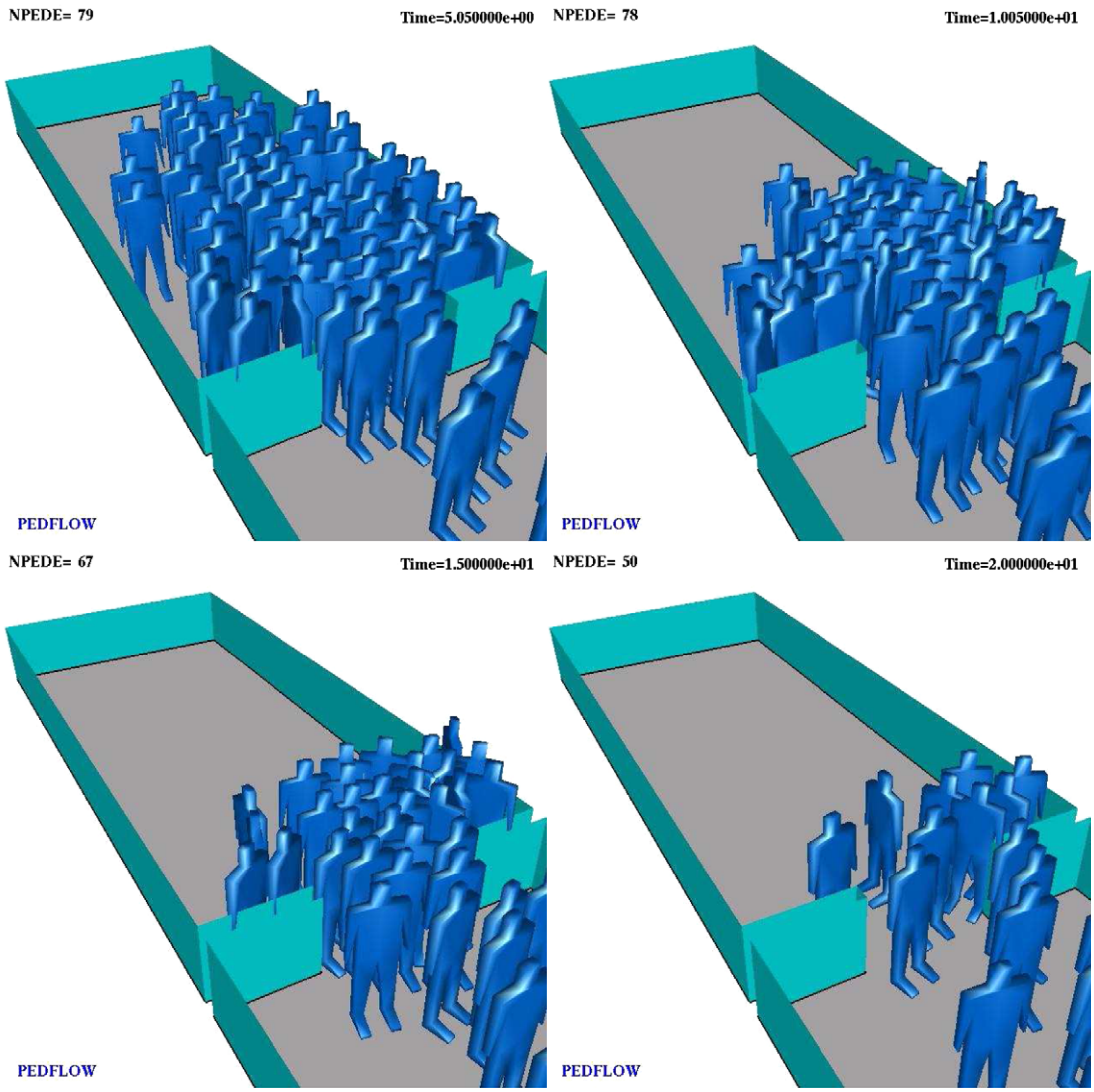

Fig. 9b. Snapshots during exit (gap size: 160 cm).

elements of the background grid. This 'host element' is updated continuously with the nearest neighbour tracking procedure shown schematically in Fig. 2. The present implementation uses a vectorized nearest neighbour tracking procedure [19]. Given the host element, the geographic data, stored at the nodes of the background grid, is interpolated linearly to the pedestrian.

The closest distance to a wall δ_w for any given point is evaluated via a fast nearest neighbour/heap list technique [18]. Starting from the nodes on the walls, the points of the background grid are traversed by evaluating the distance for the neighbours of the closest point to a wall found. After evaluating the δ_w for a point, the closest face on the boundary is stored as a starting search value for the neighbouring points for which δ_w is still unknown. In order to proceed from the walls to the interior in an orderly fashion, the new points are stored in a heap list, with the point having the smallest value for δ_w at the top. The procedure, which yields an operational complexity of $O(N \log N)$, is shown diagrammatically in Fig. 3.

3.3. Neighbour data

By far the most time-consuming portion of pedestrian simulations is the evaluation of the interaction between nearest neighbours. These nearest neighbours of every pedestrian must be identified and accounted for at every timestep. A possible way of solving this task efficiently is via optimal spatial data structures, such as bins or octrees [20–24]. In the present case, a different approach, based on the Delauney triangulation, was followed. Assuming an arbitrary cloud of points, a Voronoi tesselation, or its dual, the Delauney triangulation, uniquely defines the nearest neighbours of a point. The Delauney criterion states that the circumscircle of any triangle does not contain any other point. In two dimensions, this is equivalent to minimizing the maximum angle for any combination of triangles adjacent to an edge (see Fig. 4).

For dynamically moving points, the Delauney or min–max criterion will be violated in parts of the mesh. Every so often (e.g. after every timestep), the mesh must be modified in order to restore it. For 2-D grids like the ones contemplated here,

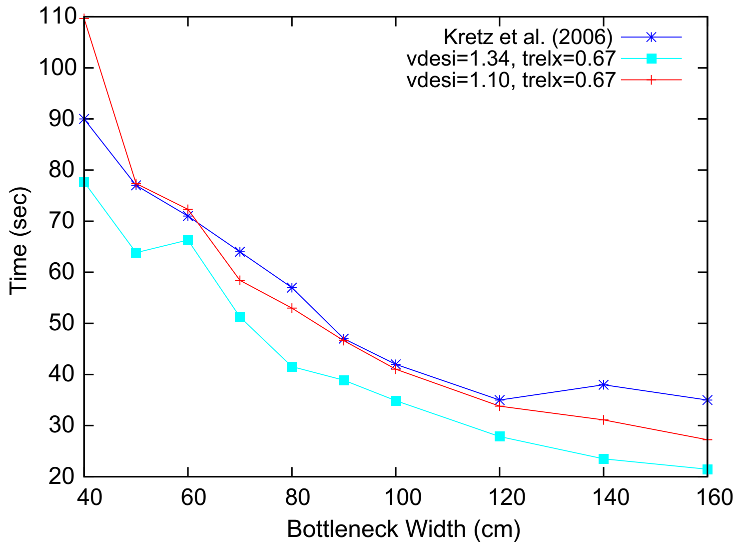

Fig. 9c. Effect of desired velocity, comparison to experiment.

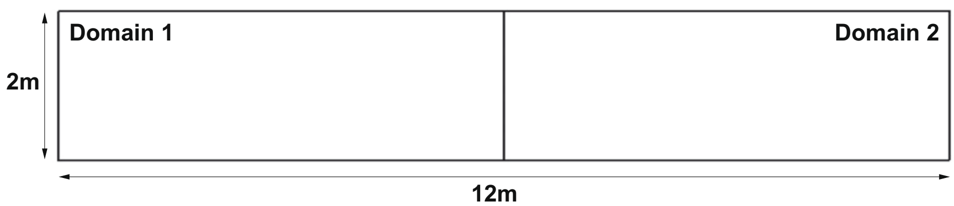

Fig. 10a. Crossing in a passage: problem definition.

this is best achieved by flipping diagonals until the Delauney or min–max criterion has been restored. In this way, no boundary recovery is required, and the procedure is guaranteed to yield a valid mesh unless pedestrians stray outside the domain. This last situation can be easily detected, as it would produce elements with negative areas.

An edge-based data structure that is well suited for this purpose stores the two points of the edge, the neighbours on either side of the edge, as well as the four edges that enclose the edge (see Fig. 5). For boundary edges, some of these items will be missing, making it easy to identify them.

In order to expedite the search, a first pass is performed over all edges, storing those that require a modification. After flipping these diagonals, the neighbouring diagonals are stored for further inspection. The procedure is then repeated until no edges remain in the list. In order to identify possible collisions or crossings, a loop over the **edges** of the present pedestrian 'mesh' is performed. With the data stored for each edge `iedge` as shown in Fig. 5, the nearest neighbours of points `ip1`, `ip2`, as well as the next layer of neighbours for points `ip3`, `ip4` can be obtained immediately and processed further.

The addition and removal of pedestrians (points) from the Delauney mesh is illustrated in Figs. 6 and 7. At boundaries where pedestrians are streaming into the domain new pedestrians are added into the elements bordering the domain. Diagonal flipping is then carried out until a Delaunay mesh is obtained. Pedestrian removal is carried via edge collapse. Once the pedestrian and the associated edges have been removed, diagonal flipping is again invoked until a Delaunay mesh is obtained.

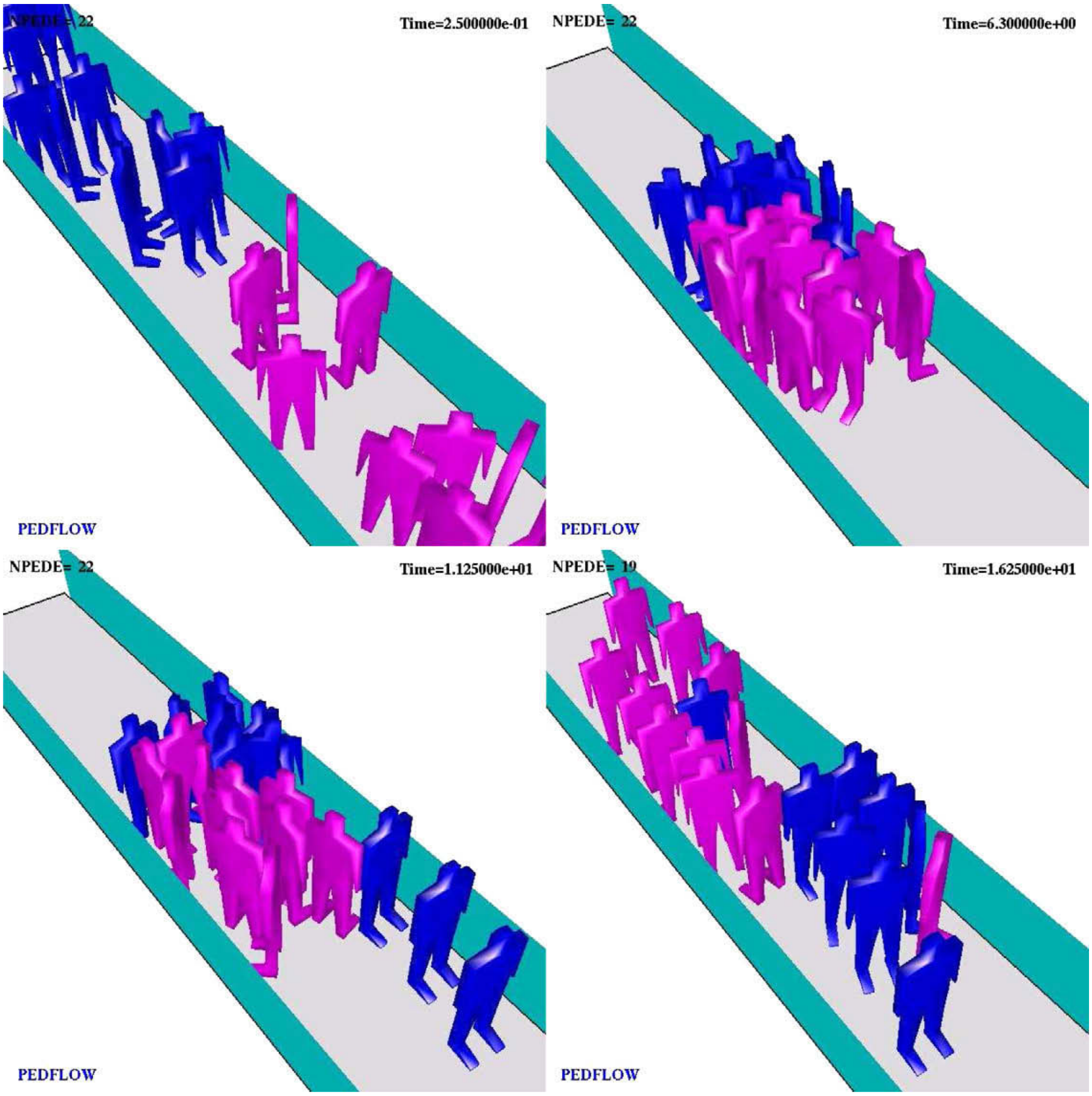

Fig. 10b–e. Snapshots during crossing in a passage: 2×11 pedestrians.

4. Examples

4.1. Passage

Pedestrian motion in passages is one of the few cases where reliable empirical data exists. In order to assess the validity of the proposed pedestrian motion model, a typical 'passage flow' was selected. The geometry of the problem is shown in Fig. 8a. Pedestrians enter the domain from the left and exit to the right. For this case, each pedestrian has the goal of reaching first the entrance of the passage, then to traverse it to the other end, and finally to exit on the right. A typical snapshot during one of the simulations is shown in Fig. 8b.

The pedestrian parameters chosen were as follows:

- Desired velocity: 1 ± 0.02 m/s.
- Relaxation time: 0.50 ± 0.1 s.
- Pedestrian radius: 0.2 ± 0.02 m.

The problem was run repeatedly with an increasing number of pedestrians entering the domain per time unit. The resulting pedestrian density (i.e. pedestrian surface vs. floor surface) was measured in the passage, as well as the average velocity. The results obtained are compared to published data [25] in Fig. 8c. As one can see, the proposed model provides surprisingly accurate results.

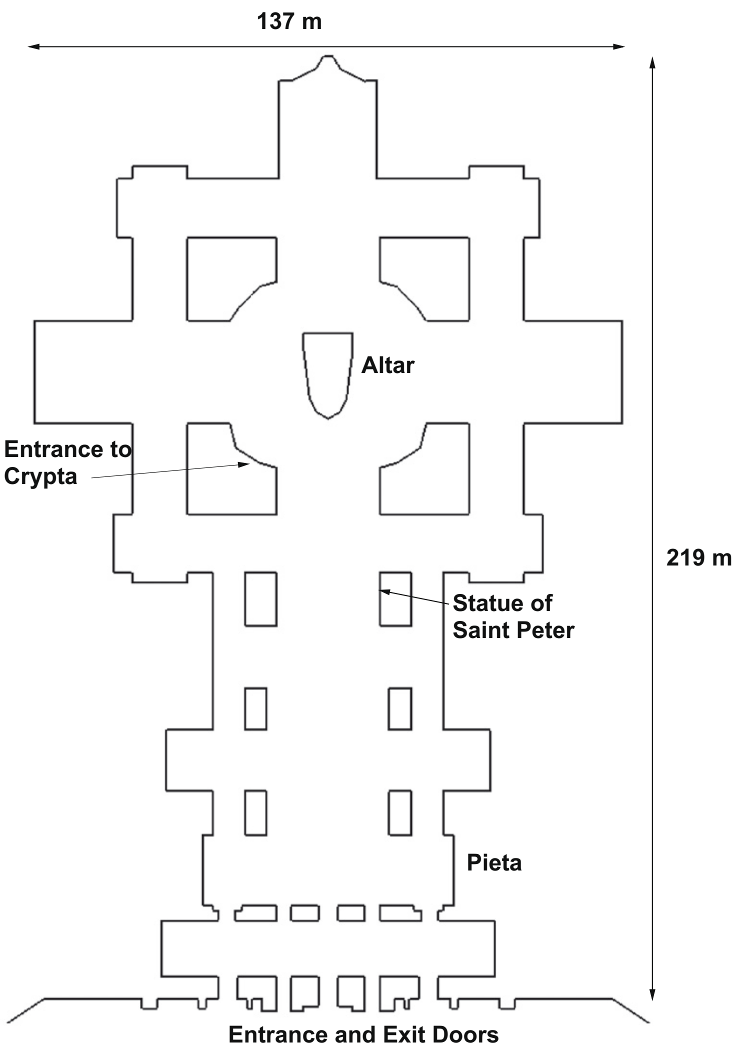

Fig. 11a. Saint Peter's Basilica: problem definition.

4.2. Exit through a gap

This case is used to reproduce the experimental results of Kretz et al. [25]. The geometry of the problem is shown in Fig. 9a. Eighty pedestrians are placed in the room on the left, and are expected to leave through the gap. A typical instance in a run is shown in Fig. 9b. The results of a parametric study varying gap size as well as desired velocities and relaxation times is shown in Fig. 9c, and compared to experimental results [25].

4.3. Crossing in a passage

This case, taken from [12], considers a random initial distribution of pedestrians in two domains as shown in Figs. 10a and b. The pedestrians in each domain are then asked to proceed to the exit of the other domain. This leads to crossing and 'passage forming'. The pedestrian parameters chosen were as follows:

- Desired velocity: 1.15 ± 0.10 m/s.
- Relaxation time: 0.50 ± 0.05 s.
- Pedestrian radius: 0.2 ± 0.05 m.

Fig. 10b–e show the pedestrians at different times. After an initial encounter phase (Fig. 10c) lanes form and the pedestrians can cross each other.

4.4. Saint Peter's basilica

The final example shows a simulation for Saint Peter's Basilica in the Vatican. The planform used is shown in Fig. 11a, together with some of the points of interest. Ten different paths through the Basilica were used as input. These took into consideration typical prayer and observation times at different locations. Pilgrim and tourist groups were modeled as well. For these cases, one pedestrian was designated as the leader; at any given instance, the other members of the group then had

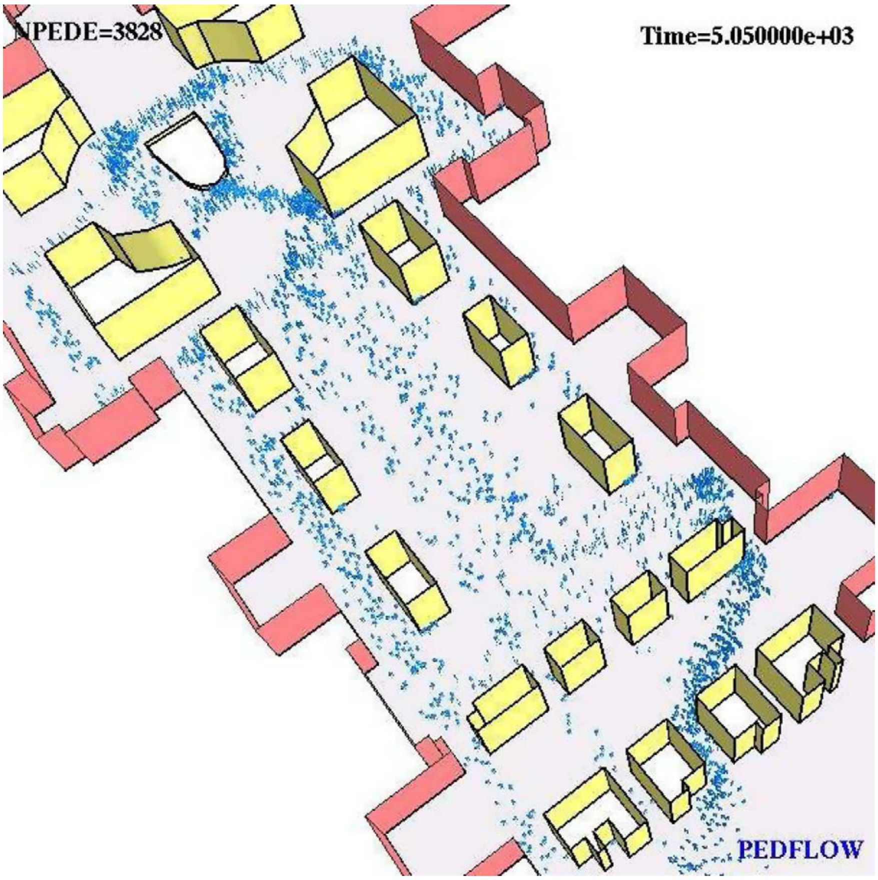

Fig. 11b. Overall view at time $t = 5050$ s.

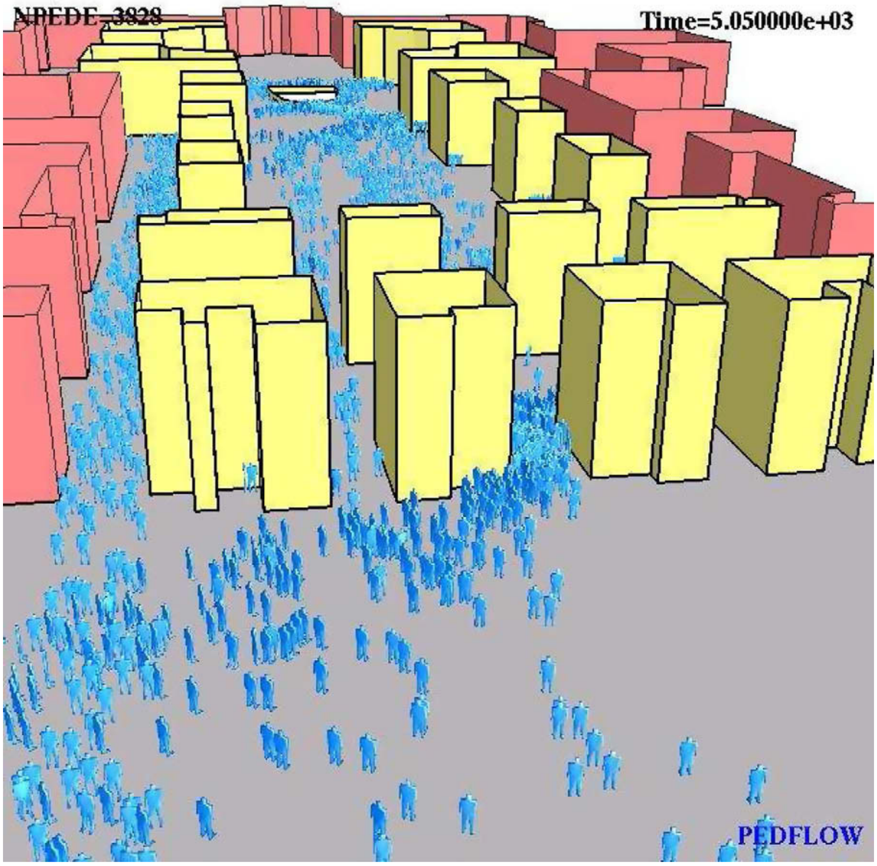

Fig. 11c. View from entrance at time $t = 5050$ s.

the goal of reaching the vicinity of the leader. Fig. 11b–d show snapshots of one of the simulations with approximately 3600 pedestrians.

5. Conclusions

A model for the simulation of pedestrian flows and crowd dynamics has been developed. The model is based on a series of forces, such as:

- Will forces (the desire to reach a place at a certain time).
- Pedestrian collision avoidance forces.
- Obstacle/wall avoidance forces.
- Pedestrian contact forces; and
- Obstacle/wall contact forces.

It is assumed that for any given pedestrian these forces are the result of local (nearest neighbour) situations, and are not influenced by pedestrians that are far away. The near-neighbour search problem is solved by a Delaunay triangulation that is updated at every timestep. An efficient edge data structure has been developed that is sufficiently fast so that this part requires less than 15% of the overall simulation time.

In order to allow for general geometries a so-called background triangulation is used to carry all geographic information. At any given time the location of any given pedestrian is updated on this mesh. This task is solved efficiently using a near-neighbour interpolation algorithm.

The results obtained to date show that:

- Crowd dynamics, such as lane forming, overtaking, avoidance of obstacles and panic behaviour can be accurately modeled.
- The model allows for real time simulation of up to 5000 pedestrians on a laptop PC.

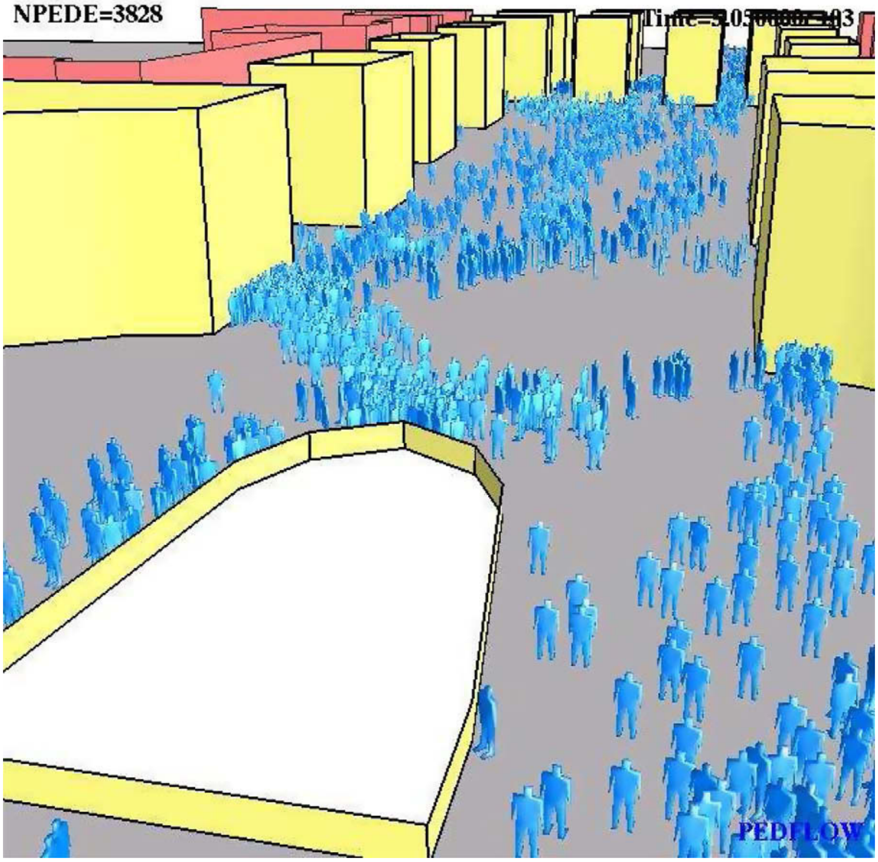

Fig. 11d. View from apsis at time $t = 5050$ s.

– The model allows realistic, accurate and timely computation of pedestrian flows with over 100,000 individuals on parallel, shared-memory machines.

Future work will center on the following topics:

– Further increases in realism for pedestrian behaviour modeling, such as:
  – Queuing at certain locations.
  – Aleatoric stopping; and

– Detailed comparison of results with video footage obtained from large-scale pilgrimage events.

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