



Modelling unsaturated soil behaviour during normal consolidation and at critical state

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ABSTRACT

An analysis of results from published laboratory tests on Jossigny silt and Barcelona clayey silt is presented to confirm the existence of a unique capillary bonding function linking the quotient between unsaturated and saturated void ratio, at the same mean average skeleton stress, to a single capillary bonding scalar variable. The analysis confirms that the same capillary bonding function applies to both normally consolidated and critical stress states.

The above two experimental sets with the addition of further published data for Speswhite Kaolin are also used to study the relationship between unsaturated critical shear strength, mean average skeleton stress and capillary bonding variable. The results of such analysis are assessed in the light of a similar modelling framework proposed in the literature.

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1. Introduction

Early research on the fundamental behaviour of unsaturated soil (e.g. [3]) attempted to relate the mechanical behaviour of this material to a single effective stress σ'_{hk} of the form:

$$\sigma'_{hk} = \sigma_{hk} - \delta_{hk}[u_a - \chi(u_a - u_w)] \quad (1)$$

where σ_{hk} is the total stress, u_a is the air pressure, u_w is the liquid pressure, δ_{hk} is Kronecker's delta and χ is a soil parameter depending on degree of saturation and ranging from zero, when the soil is dry, to unity, when the soil is saturated. Eq. (1) is probably the simplest and most intuitive way of generalizing the principle of effective stress postulated by Terzaghi [19] for saturated soils to the unsaturated domain. Indeed Eq. (1) reduces to the definition of the saturated effective stress when the degree of saturation, and hence the parameter χ , becomes equal to unity. The debate on the existence of a single effective stress variable in unsaturated soils, as it is the case for saturated soils, has traditionally been linked to the analysis of two distinct aspects of behaviour, i.e. deformation and shear strength.

In [9], Jennings and Burland described the difficulties of modelling important characteristic of the volumetric behaviour of unsaturated soils by using only one effective stress as a constitutive variable. In [4], Bishop and Blight observed that the parameter χ in Eq. (1) is linked to the degree of saturation via a relationship that

depends not only on the type of soil but also on the particular stress path being followed. This is a significant limitation to the formulation of a general constitutive model for unsaturated soils in terms of a single effective stress. This was indeed recognized by Bishop and Blight [4], who however concluded that the effective stress defined by Eq. (1) "... can be used with much less difficulty in terms of shear strength ... than in terms of volume change ...".

Eq. (1) provides an approximate measure of the average stress acting on the unsaturated soil skeleton by weighing the pore pressures exerted by both liquid and gas [8,13]. The use of Eq. (1) as a single effective stress variable is however difficult because the mechanical response of an unsaturated soil is governed not only by the change of the average stress acting on the solid skeleton but also by the change of the soil "structure" associated to the inter-particle capillary bonding generated by water menisci. A stress-strain model formulated in terms of a single effective stress variable, such as that given by Eq. (1), is potentially capable of capturing the former effect but not the latter one.

A number of constitutive models for unsaturated soils have been proposed during recent years where the bonding exerted by water menisci at inter-particle contacts has been accounted for by introducing a scalar constitutive variable in addition to a tensorial stress variable. The form of stress variable is not unique and various alternatives of Eq. (1) have been proposed where χ is a function of degree of saturation or suction or both, e.g. Nuth and Laloui [15], Karube and Kawai [11], Loret and Khalili [14]. The most commonly used stress variable is the net stress, e.g. Alonso et al. [1], given by Eq. (1) where χ is equal to zero. The additional scalar

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variable is usually suction e.g. [1] or degree of saturation e.g. [18] or a combination of these two e.g. [7].

Consistently with the above physical argument, Gallipoli et al. [7] presented a constitutive model formulated in terms of one tensorial variable and one scalar variable. This model is capable of predicting elasto-plastic volumetric changes in an unsaturated soil that is either normally consolidated or at critical state.

The tensorial variable is the “average skeleton stress” σ''_{hk} defined as the stress given by Eq. (1) with the parameter χ equal to the degree of saturation S_r [10]:

$$\sigma''_{hk} = \sigma_{hk} - \delta_{hk}[u_a - S_r(u_a - u_w)] \quad (2)$$

The scalar variable is the capillary bonding ξ defined in terms of degree of saturation S_r and suction s :

$$\xi = f(s)(1 - S_r) \quad (3)$$

Eq. (3) is based on the postulate that capillary bonding depends on two factors, namely the number of inter-particle water menisci per unit volume of solid and the degree of inter-particle pull generated by the suction within each water meniscus. Under the simplifying assumptions introduced by Gallipoli et al. [7], the former factor is uniquely related to the variable $(1 - S_r)$ whereas the latter factor is described by the variable $f(s)$, which is a function of suction based on the simplified capillary model of Fisher [6]. The capillary bonding ξ defined by Eq. (3) combines both the above factors in a single measure defined as the product of the corresponding variables $(1 - S_r)$ and $f(s)$.

Gallipoli et al. [7] showed that, during plastic loading, the void ratio of an unsaturated soil, e can be calculated as a proportion of the void ratio of the same soil in saturated conditions, e_s at the same mean average skeleton stress and subjected to the same type of loading (e.g. normally consolidated or at critical state). The factor of proportionality $\frac{e}{e_s}$, which is greater than unity, is a unique function of the current value of the capillary bonding ξ :

$$\frac{e}{e_s} = g(\xi) \quad (4)$$

The function of Eq. (4), which for simplicity we will name “capillary bonding function” in the following part of this paper, also appears to be the same for soil samples that are normally consolidated or at critical state.

A single capillary bonding function is therefore necessary to extend the application of saturated soil models to the domain of unsaturated soils and this only requires a limited number of additional model parameters. Another appealing feature of the proposal by Gallipoli et al. [7] is the unified interpretation of irreversible volumetric strains in terms of a single yield curve regardless whether irreversible strains are caused by loading, wetting or drying (see also [23]). The ability to capture irreversible strains by activating the same yield locus during drying as well as during loading or wetting is a significant improvement over previous constitutive frameworks such as, for example, the “Barcelona Basic Model” of Alonso et al. [1].

The first part of this paper provides additional validation of the assumptions introduced by Gallipoli et al. [7] for modelling volumetric strains against experimental data from laboratory tests on Jossigny silt performed by Cui [5] and Barcelona clayey silt performed by Barrera [2]. In the second part of the paper the effect of partial saturation on soil strength at critical state is analysed in the light of the previous two data sets as well as additional results from tests on Speswhite Kaolin performed by Sivakumar [17] and Wheeler and Sivakumar [24]. In particular, the dependency of the unsaturated critical state strength on the capillary bonding ξ is explored and compared with the dependency of volumetric strains on the same variable.

2. Validation of modelling assumptions by Gallipoli et al. [7]

Gallipoli et al. [7] assumed that during plastic loading of an unsaturated soil element the quotient between the current void ratio e and the void ratio e_s in saturated conditions, at corresponding values of the average skeleton stress, is a unique function of the capillary bonding ξ . This modelling assumption has been validated by Gallipoli et al. [7] for normally consolidated and critical stress states by using data from laboratory tests performed on three unsaturated soils, i.e. a compacted Speswhite Kaolin clay [17] and [24], a compacted mixture of Bentonite and Kaolin clays [16] and a compacted Kiunyu gravel [20]. Gallipoli et al. [7] also postulated that the capillary bonding function is a property of the soil and, based on the same set of data, they demonstrated the uniqueness of such function for both normally consolidated and critical states.

In this paper, the modelling assumption by Gallipoli et al. [7] will be further validated against results from laboratory tests on two other types of unsaturated soil samples, i.e. Jossigny silt samples [5] prepared by one-dimensional static compaction to a dry density of 1.67 g/cm³ and a water content of 18% (which correspond approximately to the Proctor optimum) and Barcelona clayey silt samples [2] prepared by isotropic static compression to 600 kPa at a water content of 11% (this results in a dry density of 1.65 g/cm³, which corresponds to dry of optimum on the isotropic static compaction curve at 600 kPa). These two unsaturated silts have intermediate grain size between the clays and gravels considered in the original paper by Gallipoli et al. [7] and therefore provide a useful complement to the validation of their modelling ideas.

2.1. Validation against data for Jossigny silt [5]

Cui [5] carried out a series of triaxial tests on Jossigny silt at constant suction levels of 0 (saturated), 200, 400, 800, and 1500 kPa. A number of samples were sheared to critical state at each value of suction, after consolidation to variable levels of mean net stress.

The data by Cui [5] are here used to validate the model of Gallipoli et al. [7] by defining the capillary bonding function with reference to critical states (it is here assumed that all shearing stages performed by Cui [5] take soil samples to critical state). Such capillary bonding function is subsequently used to predict soil behaviour at normally consolidated states.

For the definition of the capillary bonding function, it is first necessary to identify the reference saturated critical state line in terms of void ratio and mean average skeleton stress. Given that Cui [5] presents only two saturated shearing tests at a suction of 0 kPa, the results from two additional shearing tests at suction of 200 kPa have been considered in this work to improve estimation of the saturated critical state line. Such two additional tests achieve, at the end of shearing, values of the degree of saturation of 92% and 98%, respectively and are therefore considered to be representative of the saturated soil behaviour. In summary, the saturated critical state line is calculated on the basis of four experimental points (i.e. two at a suction of 0 kPa and two at a suction of 200 kPa) as

$$e_s = \Gamma - \psi \cdot \ln p'' \quad (5)$$

where p'' is the mean average skeleton stress (coinciding with Terzaghi's mean effective stress for a saturated soil), e_s is the saturated void ratio and the values of the parameters Γ and ψ are equal to 1.13 and 0.0932, respectively. Fig. 1 shows such critical state line together with the four critical state points used for its definition. Fig. 1 also shows additional points representing the unsaturated critical states achieved during shearing stages performed by Cui [5] at various values of suction.

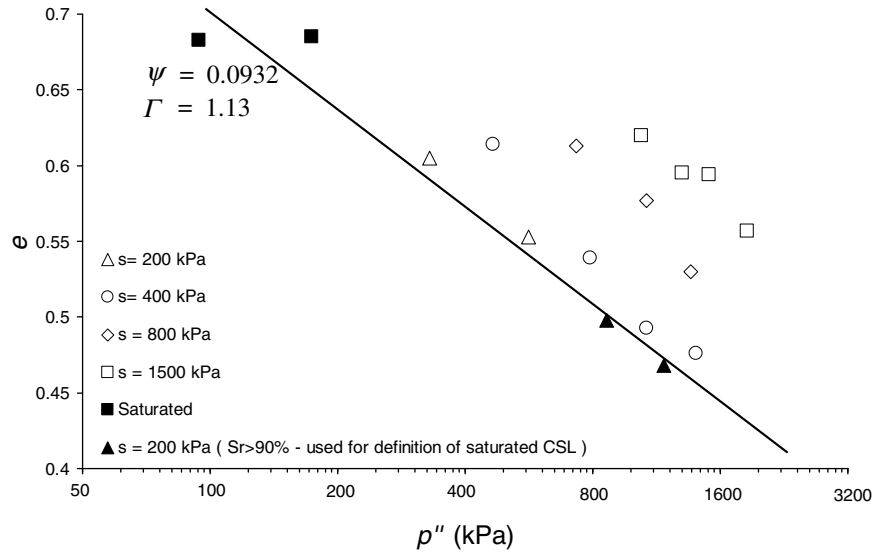


Fig. 1. Critical state void ratios at different suctions and at saturation plotted against mean average skeleton stress for compacted Jossigny silt (data by Cui [5]) together with interpolated saturated critical state line.

Fig. 2 plots the quotients $\frac{e}{e_s}$ between unsaturated and saturated critical state void ratios (where the saturated void ratio is calculated according to Eq. (5) at the same average skeleton stress as the unsaturated void ratio) against the corresponding values of the capillary bonding ξ . The points in Fig. 2 are fitted by a particular form of the capillary bonding function of Eq. (4) given by the following exponential relationship as proposed in Gallipoli et al. [7]:

$$\frac{e}{e_s} = 1 - a \cdot (1 - \exp(b \cdot \xi)) \quad (6)$$

with the values of the material parameters a and b are equal to 0.0173 and 6.53, respectively.

The good agreement shown by Fig. 2 between experimental data at different suctions and the exponential relationship of Eq. (6) provides further proof of the existence of a capillary bonding function as postulated by Gallipoli et al. [7]. Note that the circled points in Fig. 2 refer to those tests where the deviator stress passes through a peak during shearing and strain localization is likely to occur during the post-peak phase. For these tests it is questionable

whether the degree of saturation and void ratio measured at the end of shearing are representative of a homogenous sample. The corresponding points in Fig. 1 have therefore been neglected when calculating the best-fit line.

Cui [5] also presents four isotropic tests at constant suctions of 200, 400, 800, and 1500 kPa involving loading up to a mean net stress of 600 kPa, i.e. well beyond the yield mean net stress corresponding to these four levels of suction. Nevertheless, the plastic part of the stress path followed in these tests can not be used to undertake a similar analysis as previously done for critical states, because the saturated normal compression line is unknown and water content values are only reported at the end of each test. Based on the assumption that Eq. (6) is unique for critical and normally consolidated stress states, it is however possible to estimate the saturated void ratio corresponding to the mean average skeleton stress at the end of each of these four tests. The results of such analysis are summarized in Fig. 3 where the measured void ratios at the end of the four tests are shown together with the corresponding estimated saturated void ratios.

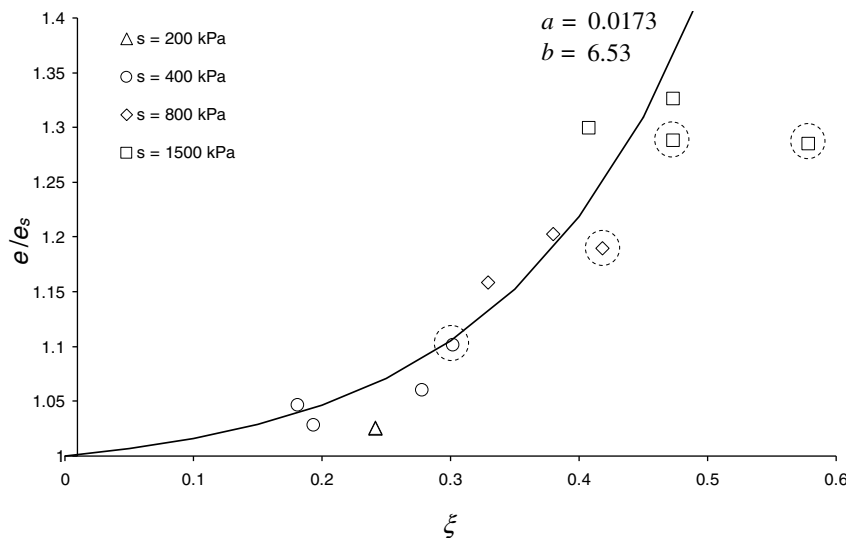


Fig. 2. Quotient between unsaturated and saturated critical state void ratios at the same mean average skeleton stress plotted against capillary bonding for compacted Jossigny silt (data by Cui [5]) together with interpolated capillary bonding function.

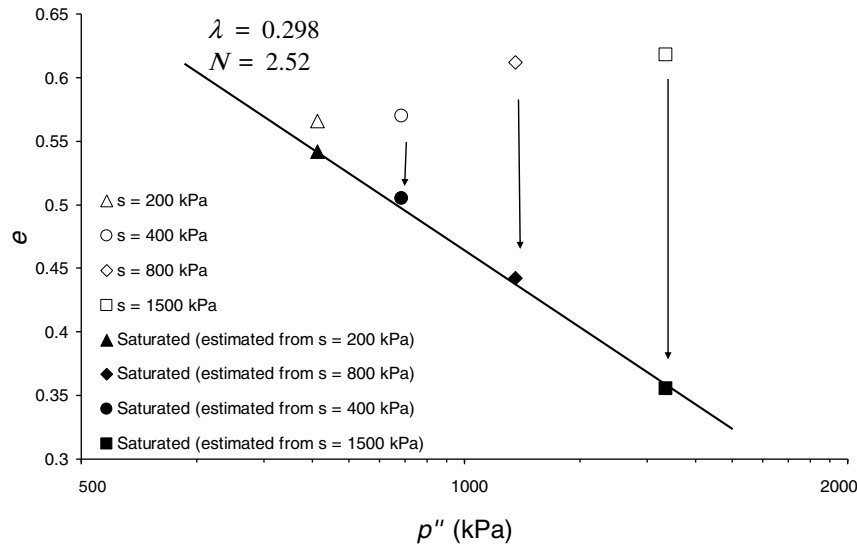


Fig. 3. Normally consolidated values of unsaturated void ratio plotted against mean average skeleton stress for compacted Jossigny silt (data by Cui [5]) together with corresponding estimates of normally consolidated saturated values of void ratio and interpolated saturated normal compression line.

Inspection of Fig. 3 indicates that, as expected, the estimated saturated points fit a linear relationship in the semi-logarithmic plane $e - \ln p''$ and the slope λ of such line is equal to 0.298. This value is also consistent with data from oedometric tests on saturated Jossigny silt reported by Vicol [22], who observed that the slope of the virgin line in the semi-logarithmic plane $e - \ln \sigma_v''$ ranges between 0.25 and 0.28 with an average value of 0.27 (σ_v'' is the vertical average skeleton stress coinciding with the vertical effective stress for a saturated soil). These results provide further support to the assumption by Gallipoli et al. [7] that the relationship given by Eq. (6) is unique for both normally consolidated and critical stress states. It is worth noting here that the estimated slope of 0.298 for the saturated normal compression line is considerably different from the measured slope of 0.0932 for the saturated critical state line, which apparently contradicts experimental evidence suggesting that these two lines are parallel in the semi-logarithmic plane $e - \ln p''$. This discrepancy might be partly explained by the observation that, despite having assumed that critical state is attained in all triaxial tests performed by Cui [5], a considerable tendency to change in volume is still evident at the end of three out of four shearing stages used for the definition of the saturated critical state line in Fig. 1. Equally, a tendency to reduce in volume is still appreciable at the end of most constant suction shearing stages performed on unsaturated soil samples reported in Fig. 1. Notwithstanding such uncertainties about the achievement of critical state, the resulting capillary bonding function is still consistent with the expected trend and capable of predicting a realistic value of the saturated normal compression slope.

2.2. Validation against data for Barcelona clayey silt [2]

Barrera [2] carried a series of isotropic and triaxial shearing tests on Barcelona clayey silt under controlled suction and net stress. The isotropic tests involve rather complex paths with a combination of both loading/unloading at constant suction and wetting/drying at constant mean net stress. These also include one isotropic loading test under saturated conditions, which takes the soil well within the plastic region. This saturated test has been used in the present work to define the following equation of the saturated normal compression line to be used as a reference for the definition of the capillary bonding function:

$$e_s = N - \lambda \cdot \ln p'' \quad (7)$$

where the values of the parameters N and λ are equal to 0.934 and 0.0723, respectively.

Fig. 4 shows this saturated normal compression line together with the experimental plastic points used for its definition. These points are identified in Fig. 4 by the label SAT-1, which is the same label used in Barrera [2] to identify this test (note that the same test labels as in Barrera [2] are used in all relevant figures of this paper to facilitate cross-referencing with the original set of data). Fig. 4 also shows additional data points that correspond to plastic behaviour of unsaturated soil samples subjected to isotropic stress states. Some of these points lie on isotropic loading paths at constant suction whereas other points belong to wetting paths at constant mean net stress (a point on a wetting path is assumed to represent a plastic state if the sample experiences compressive behaviour at that particular point). These data points are used in the present analysis to define the capillary bonding function for normally consolidated states regardless of whether they belong to a loading path or a wetting path. Due to the uniqueness of the normal consolidation surface in the void ratio-mean net stress-suction space (as first postulated by Alonso et al. [1]), these points are all representative of normally consolidated behaviour.

Fig. 5 plots the quotients $\frac{e}{e_s}$ between unsaturated and saturated void ratios (where the saturated void ratio is calculated from the normal compression line of Eq. (7) at the same average skeleton stresses as the unsaturated void ratio) against the capillary bonding ξ . Again, the points in Fig. 5 follow a clear trend that is interpolated by the capillary bonding function, which this time appears to follow a rather linear pattern. This is different from the case of Jossigny silt [5] shown in Fig. 2 and from the cases of Speswhite Kaolin clay [17] and [24], Bentonite/Kaolin clay [16] and Kiunyu gravel [20] shown in Gallipoli et al. [7], which all seem to suggest a more or less pronounced exponential increase of the ratio $\frac{e}{e_s}$ with capillary bonding ξ . The reason of this difference is likely to be found in the different soil fabrics and, in particular, in the distinct pore size distribution, which controls the number of water menisci that exist per unit volume of solid fraction at each given value of degree of saturation.

The data points in Fig. 5 are well fitted by Eq. (6), where the value of the material parameters a and b are equal to 1.11 and 0.492, respectively.

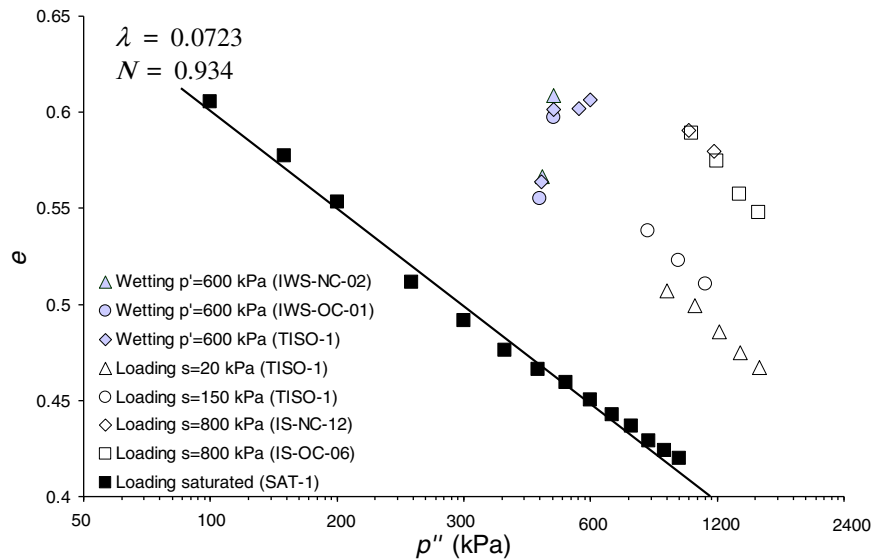


Fig. 4. Normally consolidated void ratios at different suctions and at saturation plotted against mean average skeleton stress for compacted Barcelona clayey silt (data by Barrera [2]) together with interpolated saturated normal compression line. The test labels within brackets are the same as those in Barrera [2].

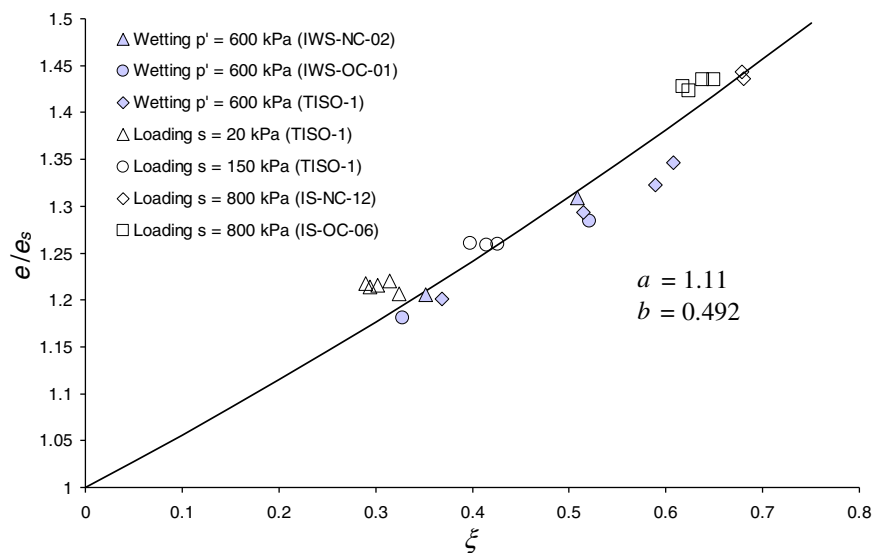


Fig. 5. Quotient between unsaturated and saturated normally consolidated void ratios at the same mean average skeleton stress plotted against capillary bonding for compacted Barcelona clayey silt (data by Barrera [2]) together with interpolated capillary bonding function. The test labels within brackets are the same as those in Barrera [2].

Barrera [2] also performed six triaxial shear stages, five at a suction of 800 kPa and one at a suction of 20 kPa. Some tests present rather complex consolidation paths prior to shearing, which involve both loading/unloading at constant suction and wetting/drying at constant mean net stress. In addition, Barrera [2] presents data from four saturated shearing tests but only two of these tests include information on volumetric strains and are reported here. All saturated and unsaturated shearing tests are assumed to load samples up to critical state.

Similarly to the logic used for the analysis of Jossigny silt, we assume that the capillary bonding function for normally consolidated conditions remains unchanged at critical states and can therefore be used to estimate the saturated critical state void ratios in correspondence of the mean average skeleton stresses attained at the end of the unsaturated shearing tests. The results of such analysis are shown in Fig. 6 where the unsaturated critical state void ratios from the six shearing tests are shown together with the corre-

sponding estimates of saturated void ratios. The estimated saturated points fit the linear relationship of Eq. (5) in the semi-logarithmic plane $e - \ln p''$ with the values of the parameters Γ and ψ equal to 0.867 and 0.0611, respectively. Fig. 6 also shows that the critical state points from the two saturated shearing tests are consistent with the estimated saturated critical state line. This result further corroborates the assumption made by Gallipoli et al. [7] of the existence of a unique capillary bonding function at normally consolidated and critical states.

3. Unsaturated critical shear strength

In parallel with the investigation of the strain behaviour, previous research on unsaturated soils has also explored the existence of a relationship between the critical state shear strength and a single stress variable, as for saturated soils. One of the earliest contributions in this respect is due to Bishop and Blight [4], who

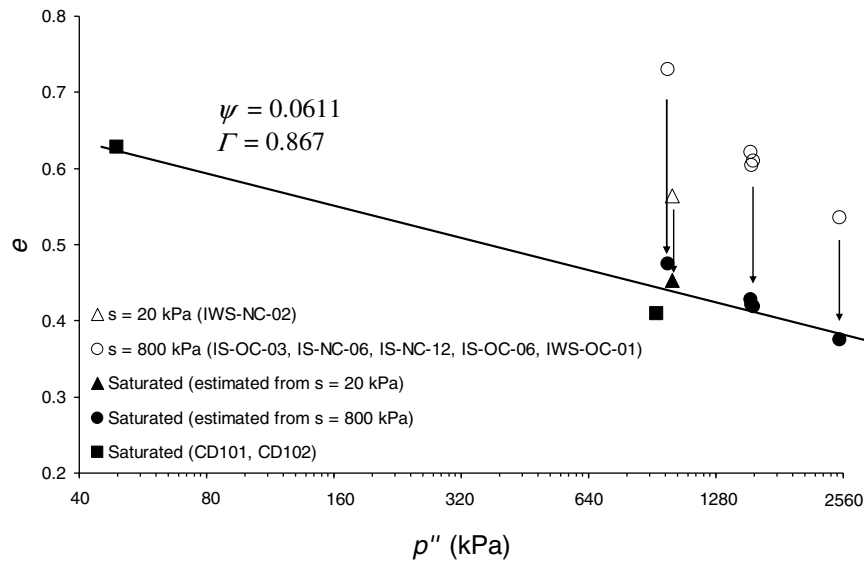


Fig. 6. Critical state values of unsaturated void ratio plotted against mean average skeleton stress for compacted Barcelona clayey silt (data by Barrera [2]) together with corresponding critical state estimates of saturated void ratio and interpolated saturated critical state line. The test labels within brackets are the same as those in Barrera [2].

concluded that, unlike volumetric behaviour, the critical shear strength of an unsaturated soil may be reasonably related to a single stress given by Eq. (1) where χ is a material parameter depending on the degree of saturation. More recently, Khalili and Khabbaz [12] suggested that the critical shear strength is uniquely related to the stress of Eq. (1) where χ is a power function of the ratio between the current suction and the air-entry suction of the soil.

In this paper, the dependency of the critical shear strength on the average skeleton stress alone (i.e. the stress variable of Eq. (2), where the χ parameter of Eq. (1) is replaced by degree of saturation) is initially explored. Subsequently, the additional dependency of the critical shear strength on capillary bonding is investigated to account for the potential influence of the unsaturated soil fabric at critical state. The motivation for such study stems from the observation, made in the previous section, that different values of void ratio, and therefore different material fabrics, can exist at critical state corresponding to the same mean average skeleton stress depending on the level of capillary bonding. This in

turn poses the question whether such different fabrics also influence the critical deviator stress that the soil is capable of sustaining at a given mean average skeleton stress. The capillary bonding ξ , which is uniquely related to the ratio $\frac{e}{e_s}$, can be taken as a measure of how “open” the unsaturated soil fabric is. Hence, any dependency of critical shear strength on the capillary bonding ξ may be considered as a sign of the influence of the unsaturated soil fabric on the critical shear strength.

3.1. Unsaturated critical shear strength of Jossigny silt [5]

In Fig. 7 the deviator stress at critical state is plotted against the corresponding mean average skeleton stress from the shearing tests performed on saturated and unsaturated samples (at constant suctions of 200, 400, 800, and 1500 kPa) of compacted Jossigny silt [5]. Fig. 7 also shows a best-fit critical state line passing through the origin and having a slope M equal to 0.84. The circled points refer to tests where the deviator stress passes through a peak before

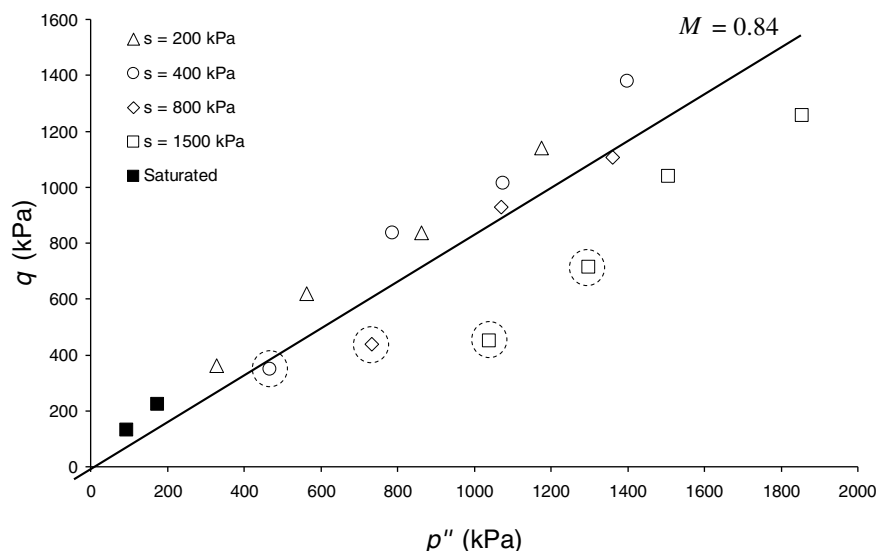


Fig. 7. Critical state values of deviator stress plotted against mean average skeleton stress for compacted Jossigny silt (data by Cui [5]) for saturated and unsaturated samples at different suctions together with interpolated critical state line.

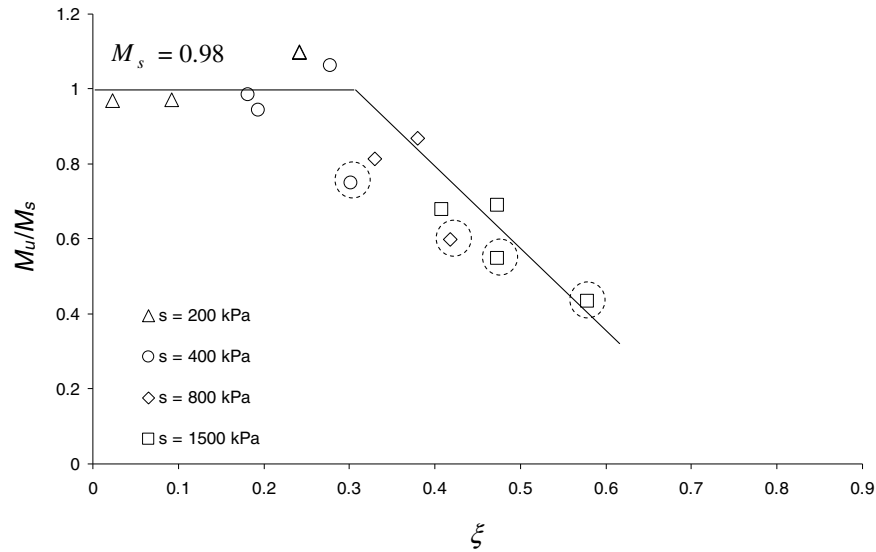


Fig. 8. Normalized critical state ratios plotted against capillary bonding for compacted Jossigny silt (data by Cui [5]) at different suctions.

achieving critical state and they have not been considered in the calculation of the best-fit line for the reason explained in Section 2.1.

Inspection of Fig. 7 indicates that the data present a moderate scatter at low suctions with a tendency to deviate substantially from the linear fit at the highest suction of 1500 kPa. The considerable dispersion of the data may suggest that the ratio M_u between deviator stress and mean average skeleton stress at critical state is influenced, for unsaturated soils, by the fabric corresponding to different levels of capillary bonding. In order to investigate further this possibility, the critical stress ratio M_u normalized with respect to the slope of the saturated critical state line M_s (whose value is here estimated equal to 0.98), is plotted in Fig. 8 against the corresponding value of the capillary bonding variable. The normalization of the unsaturated critical stress ratio M_u with respect to the slope of the saturated critical state line M_s facilitates direct comparison among the different data sets presented in this paper.

Note that any error in the measurement of stresses at critical state is reflected in a corresponding error in the calculation of the stress ratio. The significance of the latter error increases mark-

edly as the stress magnitude decreases and, for this reason, the two saturated points corresponding to the lowest stresses in Fig. 7 have not been included in the plot of Fig. 8.

A clear trend can be identified in Fig. 8 whereby the normalized critical stress ratio stays approximately equal to unity when the capillary bonding ξ remains close to zero, i.e. when the degree of saturation and suction remain close to unity and zero, respectively. However, if the capillary bonding ξ increases beyond a given threshold (and the corresponding unsaturated soil fabric becomes gradually more open), the normalized stress ratio drops below unity. This confirms that the critical strength of compacted Jossigny silt depends not only on the mean average skeleton stress but also on the capillary bonding ξ and, hence, on the “openness” of the soil fabric.

3.2. Unsaturated critical shear strength of Speswhite Kaolin [17] and [24]

Fig. 9 shows the deviator stress plotted against the corresponding mean average skeleton stress measured at critical state during

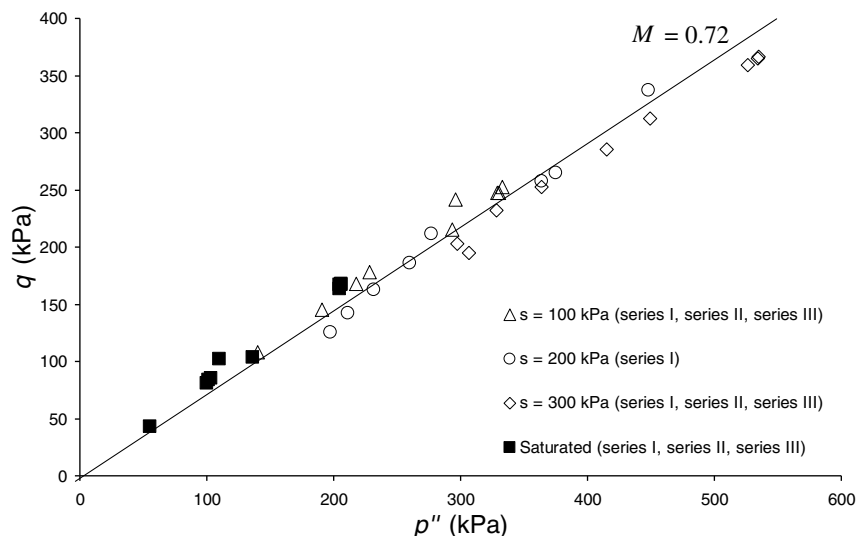


Fig. 9. Critical state values of deviator stress plotted against mean average skeleton stress for compacted Speswhite Kaolin (data by Sivakumar [17] and Wheeler and Sivakumar [24]) for saturated and unsaturated samples together with interpolated critical state line.

shearing tests on saturated and unsaturated samples (at constant suctions of 100, 200, and 300 kPa) of compacted Speswhite Kaolin performed by Sivakumar [17] and Wheeler and Sivakumar [24]. The different experimental series in Fig. 9 are associated to different procedures adopted for the compaction of Speswhite Kaolin samples at the same water content, as explained in Wheeler and Sivakumar [24]. Fig. 9 also shows a best-fit critical state line passing through the origin (having a slope M equal to 0.72), which indicates that the data follow a relatively clear linear trend. Although the points in Fig. 9 present some degree of scatter, the dispersion is significantly less than that observed in Fig. 7 for Jossigny silt.

The potential dependency of the critical strength on the unsaturated soil fabric generated by capillary bonding is considered in Fig. 10, where the critical stress ratio M_u of individual samples is normalized with respect to the slope of the saturated critical state line M_s (whose value is here estimated equal to 0.80) and plotted against the corresponding value of the capillary bonding ξ . Note that Fig. 10 does not include the five saturated points correspond-

ing to the lowest stresses in Fig. 9 because the normalized critical state ratios for these points are likely to be affected by relatively large errors as explained in Section 3.1.

Inspection of Fig. 10 shows a decrease of the normalized critical stress ratio with increasing values of the capillary bonding ξ . This confirms the trend in behaviour already observed in the previous section, although the rate of decrease appears to be considerably less for Speswhite Kaolin in comparison to Jossigny silt.

3.3. Unsaturated critical shear strength of Barcelona clayey silt [2]

Fig. 11 shows the deviator stress plotted against the corresponding mean average skeleton stress measured at critical state from the shearing tests performed by Barrera [2] on saturated and unsaturated samples (at constant suctions of 20 kPa and 800 kPa) of compacted Barcelona clayey silt.

Inspection of Fig. 11 indicates that, throughout the experimental stress range, the data for Barcelona clayey silt present consider-

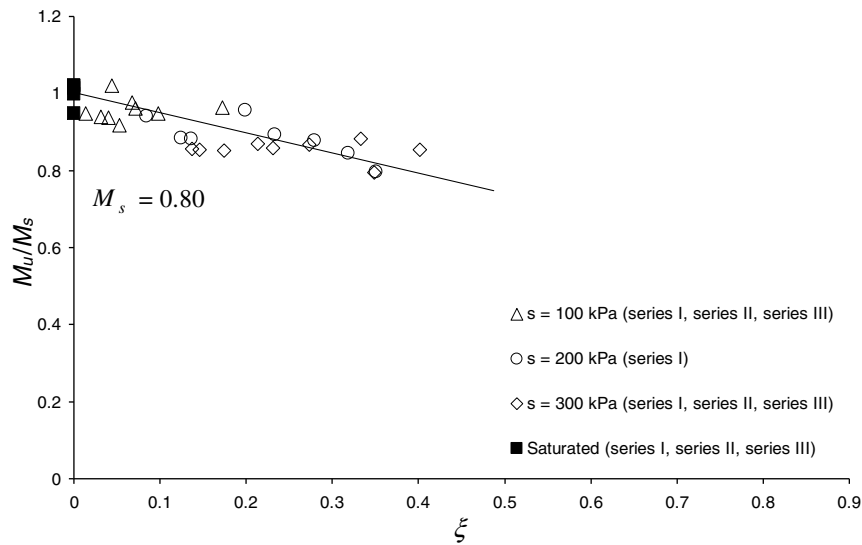


Fig. 10. Normalized critical state ratios plotted against capillary bonding for compacted Speswhite Kaolin (data by Sivakumar [17] and Wheeler and Sivakumar [24]) at different suctions.

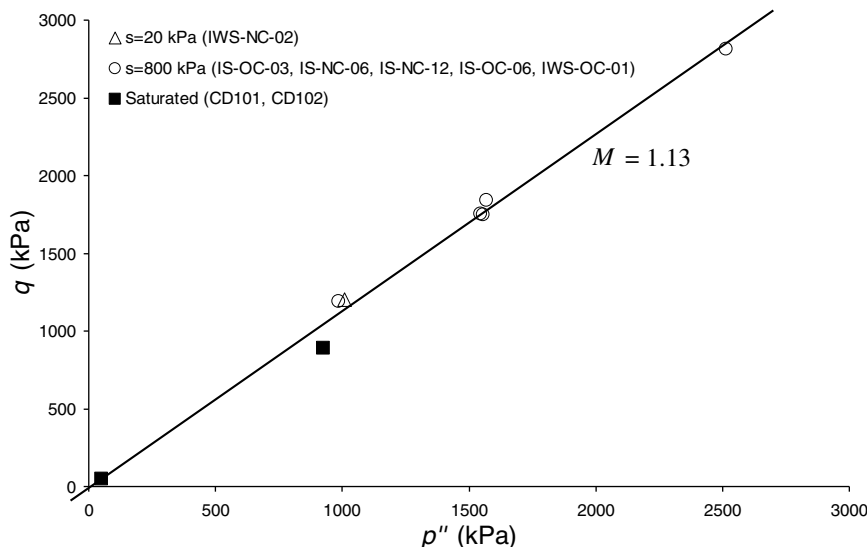


Fig. 11. Critical state values of deviator stress plotted against mean average skeleton stress for compacted Barcelona clayey silt (data by Barrera [2]) for saturated and unsaturated samples at different suctions together with interpolated critical state line. The test labels within brackets are the same as those in Barrera [2].

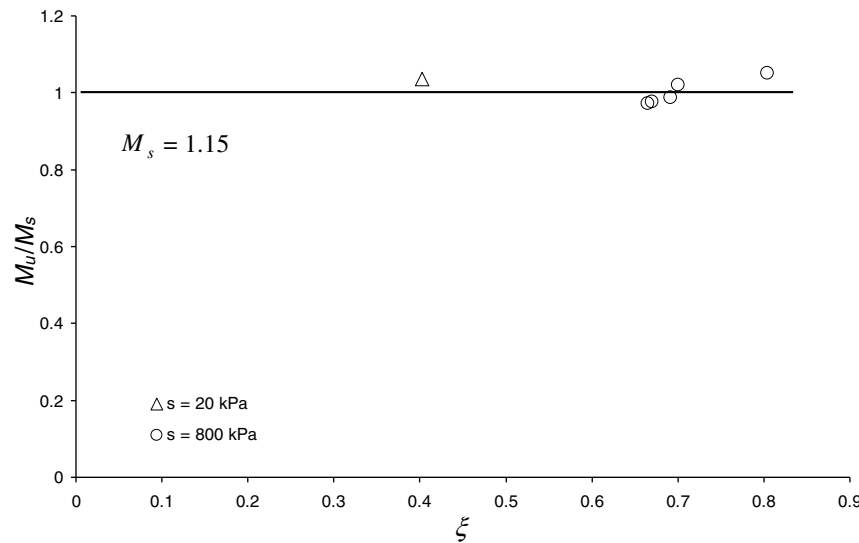


Fig. 12. Normalized critical state ratios plotted against capillary bonding for compacted Barcelona clayey silt (data by Barrera [2]) at different suctions. The test labels within brackets are the same as those in Barrera [2].

ably less scatter in comparison to the data for Jossigny silt and Speswhite Kaolin shown in Figs. 7 and 9, respectively, although the number of data points available for Barcelona clayey silt is significantly less than for the other two materials. The data in Fig. 11 are also well interpolated by a linear relationship passing through the origin and having a slope M equal to 1.13. Note that one data point at suction of 800 kPa refers to a sample subjected, prior to shearing, to a wetting/drying cycle involving a variation of suction from 800 kPa to 10 kPa and back to 800 kPa under a constant isotropic net stress state. Yet this data point fits the same linear trend as for the other points and the hydraulic cycle prior to shearing does not seem to produce any anomaly in the relation between critical strength and mean average skeleton stress.

The points shown in Fig. 11 are used to calculate the corresponding values of the critical stress ratio M_u normalized with respect to the slope of the saturated critical state line M_s (assumed here to be equal to 1.15), which are plotted in Fig. 12 against the capillary bonding ξ . Similarly to the previous data sets, the normalized critical stress ratios shown in Fig. 12 do not include the two saturated tests at the lowest stresses reported in Fig. 11.

As expected from the limited scatter in Fig. 11, the normalized critical state ratio stays approximately equal to unity over the whole experimental range. This seems to suggest that, for this particular soil, the unsaturated fabric at critical state has little influence on the corresponding strength. Such behaviour is different from that observed for Jossigny silt and Speswhite Kaolin in Sections 3.1 and 3.2, respectively. It is worth noting that the analysis of the volumetric behaviour in Section 2.1 also shows an atypical form for the capillary bonding function, which appears to be closer to a straight line rather than to the characteristic exponential law observed for all other materials. At this stage it is only possible to speculate that these distinctive aspects of behaviour in terms of both critical strength and volumetric deformation originate from the same cause and might be related to the particular evolution of capillary bonding during changes of degree of saturation.

4. Conclusions

Two sets of data from laboratory tests on compacted Jossigny silt and compacted Barcelona clayey silt performed by Cui [5] and Barrera [2], respectively are analysed in the light of the consti-

tutive model proposed by Gallipoli et al. [7]. The results for each soil confirm the existence of a unique function, valid for both normally consolidated and critical stress states, linking the ratio between unsaturated and saturated void ratio at the same mean average skeleton stress to the capillary bonding ξ . For the tests on Barcelona clayey silt, the capillary bonding function is able to capture volumetric changes of the soil under normally consolidated conditions regardless of whether these volumetric changes are generated during a wetting or loading path.

The analysis of Jossigny silt and compacted Barcelona clayey silt presented in this paper provides a useful complement to the validation of the model by Gallipoli et al. [7] because these two soils have an intermediate grading between the clays and gravel of the experimental programmes considered in Gallipoli et al. [7].

The second part of the paper analyses the relationship between unsaturated critical shear strength, mean average skeleton stress and capillary bonding ξ based on the previous two sets of experimental data, with the addition of a further data set from laboratory tests on compacted Speswhite Kaolin performed by Sivakumar [17] and Wheeler and Sivakumar, [24]. The analysis indicates that, for Jossigny silt and Speswhite Kaolin, the normalized critical stress ratio (i.e. the ratio between deviator stress and mean average skeleton stress at critical state, normalized with respect to the slope of the saturated critical state line) appears to decrease when the capillary bonding ξ increases and such trend is more marked for the former data set with respect to the latter one. This corresponds to the observation that, as the capillary bonding ξ increases, progressively larger void ratios are attained at critical state under any given level of mean average skeleton stress. Therefore, for larger values of the capillary bonding ξ , a more “open” critical state fabric exists and this is likely to have an effect on soil strength.

This behaviour, however, is not apparent from the analysis of Barcelona clayey silt, which shows no variation of the normalized critical stress ratio with the capillary bonding ξ . In this case, unlike Jossigny silt and Speswhite Kaolin, the critical strength is little affected by the unsaturated fabric and is therefore suitably modelled by assuming a dependency on the average skeleton stress alone. The Barcelona clayey silt also shows an atypically linear capillary bonding function in contrast with the characteristic exponential trend of the capillary bonding functions of all other materials. Both such distinctive aspects of behaviour may be related to the partic-

ular evolution of capillary bonding during changes of degree of saturation in this material.

Finally, it is worth noting that the description of the critical shear strength presented in this paper can be recast according to the formulation presented in Toll [20] and Toll and Ong [21]. These authors used two independent coefficients M_a and M_b , depending on degree of saturation, to relate the variation of deviator stress at critical state with mean net stress and suction, respectively. It is easily shown that the dependency of deviator stress at critical state on mean average skeleton stress and capillary bonding proposed in this paper results in the calculation of the two coefficients M_a and M_b as

$$M_a = M_s \frac{M_u}{M_s}(\xi) \quad (8)$$

$$M_b = S_r M_s \frac{M_u}{M_s}(\xi) \quad (9)$$

where S_r is the degree of saturation and $\frac{M_u}{M_s}(\xi)$ is the function relating the normalized critical stress ratio to the capillary bonding ξ as shown in Figs. 8, 10 and 12 for Jossigny silt, Speswhite Kaolin and Barcelona clayey silt, respectively.

For Jossigny silt and Speswhite Kaolin, both coefficients M_a and M_b decrease for decreasing values of degree of saturation. This can be easily understood from Eqs. (8) and (9) taking into account the particular form of the function $\frac{M_u}{M_s}(\xi)$ for these two materials (see Figs. 8 and 10) and the fact that the capillary bonding ξ increases for decreasing value of degree of saturation.

On the other hand, for Barcelona clayey silt, the coefficient M_a remains constant and equal to M_s over the entire experimental range whereas the coefficient M_b decreases for decreasing values of degree of saturation. This is a consequence of the fact that, for this material, the normalized critical state ratio in Eqs. (8) and (9) remains constant and equal to unity (see Fig. 12).

Also note that, consistently with the formulation presented in Toll [20] and Toll and Ong [21], the coefficient M_b is equal or smaller than the coefficient M_a for any value of degree of saturation because of the multiplying factor S_r (smaller or equal than unity) present in Eq. (9).

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