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DEVELOPMENT OF A FLUID-STRUCTURE INTERACTION METHOD WITH FREE SURFACE USING IGA

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Abstract. *The purpose of this study is to develop a fluid-structure interaction analysis method using IGA. For fluid analysis, the numerical method based on the VOF method is employed. The stabilized finite element method with IGA is applied as the spatial discretization method and the Crank-Nicolson method as the temporal discretization method. Several numerical examples are presented to demonstrate the promise and potential of the present method to solve the solid-fluid interaction problems with free surface(WCCM-APCOM 2022).*

1 INTRODUCTION

IGA (Isogeometric Analysis)[1, 2] is a method that has been actively researched and applied in recent years in numerical analysis in the fields of medicine, automobiles and precision machinery. IGA uses the Spline function, which is used to represent CAD (Computer Aided Design) geometry, as the basis function, allowing analysis meshes to be created directly from the geometry model drawn in CAD. This reduces the mesh creation process, and curves can be represented without shape errors.

For example of the application of IGA to civil engineering field, the coupled fluid-structure analysis in a region with curved geometry, such as the effect of water sloshing in a spherical tank on a tank is applied. In this report, as a fundamental study, IGA with NURBS function is applied to a free surface flow problem and the results are compared with those obtained by the finite element method using tetrahedral first-order elements.

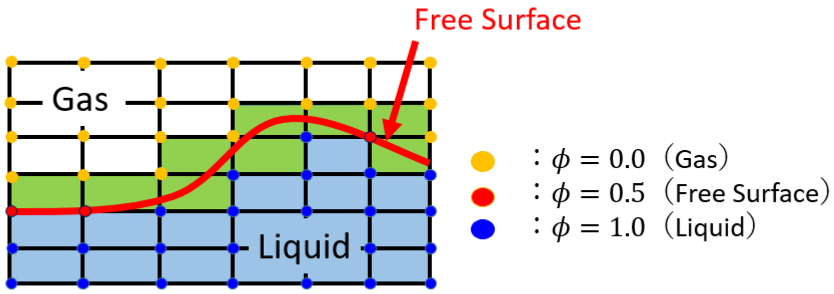

Figure 1: Definition of the VOF function

2 NUMERICAL ANALYSIS METHODS

2.1 Calculation of density and viscosity coefficient

The VOF method expresses the free surface position by a scalar function ϕ called the VOF function. The VOF function ϕ takes the values 1.0 for liquid, 0.5 for on the free surface, and 0.0 for gas at each node (see Figure -1). The density and viscosity coefficient of gas and liquid are expressed using the VOF function ϕ as follows.

$$\rho = \rho_l \phi + \rho_g (1 - \phi) \tag{1}$$

$$\mu = \mu_l \phi + \mu_g (1 - \phi) \tag{2}$$

ρ and μ are the density and viscosity coefficient in each element, and ρ_l, ρ_g, μ_l, and μ_g are the density of liquid, density of gas, viscosity coefficient of liquid and viscosity coefficient of gas.

2.2 Calculation of flow velocity and pressure

In free surface flow analysis using the VOF method, the following the Navier-Stokes equations and the Continuity equations are used to obtain the flow velocity and pressure at each node.

$$\rho \left(\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} \right) + \frac{\partial p}{\partial x_i} - \mu \frac{\partial}{\partial x_j} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) = 0 \quad \text{in} \quad \Omega \tag{3}$$

$$\frac{\partial u_i}{\partial x_i} = 0 \quad \text{in} \quad \Omega \tag{4}$$

Ω is the analysis domain bounded by the boundary Γ, u_i is the flow velocity and p is the pressure.

For the governing equations (3) and (4), when the stabilized finite element method based on the SUPG/PSPG method is used to discretize the spatial direction, the following weak form is obtained by applying the weighted residual method.

$$\int_\Omega w_i \rho \left(\frac{\partial u_i}{\partial t} + \bar{u}_j \frac{\partial u_i}{\partial x_j} \right) d\Omega - \int_\Omega \frac{\partial w_i}{\partial x_j} p d\Omega + \int_\Omega \mu \frac{\partial w_i}{\partial x_j} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_i}{\partial x_i} \right) d\Omega + \int_\Omega q \frac{\partial u_i}{\partial x_i} d\Omega$$
$$+ \sum_{e=1}^{n_{el}} \int_{\Omega_e} \left(\tau_S \bar{u}_k \frac{\partial w_i}{\partial x_k} + \tau_P \frac{\partial q_i}{\partial x_k} \right) \left(\frac{\partial u_i}{\partial t} + \bar{u}_j \frac{\partial u_i}{\partial x_j} + \frac{\partial p}{\partial x_j} \right) d\Omega = \int_{\Gamma_h} w_i h_i d\Gamma \tag{5}$$

Where w_i and q are the weight functions of the Galerkin terms for the Navier-Stokes equations and the Continuity equation, Ω_e represents the domain of each element when the entire analysis domain is divided into M elements, and Γ_h represents the h denotes the boundary given the Neumann boundary condition. In addition, τ_S and τ_P are the stabilization parameters for the SUPG and PUPG terms.

$$\tau_S = \tau_P = \left(\left(\frac{2}{\Delta t} \right)^2 + \left(\frac{2||\bar{u}_i^e||}{h_e} \right)^2 + \left(\frac{4}{Re h_e{}^2} \right)^2 \right)^{-\frac{1}{2}} \tag{6}$$

Δt is the micro time increment, $\bar{u}_i^e$ is the element advection velocity, h_e is the element length, and $||\bar{u}_i^e||$ is the norm of the element advection velocity, each expressed as follows.

$$h_e = 2 \left(\sum_{\alpha=1}^{n_{en}} \left| \left(\frac{u_i^e}{||u_i^e||} \right) \left(\frac{\partial N_\alpha^e}{\partial x_i} \right) \right| \right)^{-1} \tag{7}$$

$$||\bar{u}_i^e|| = \sqrt{\sum_{i=1}^{n_d} (\bar{u}_i^e)^2} \tag{8}$$

N_{en} is the number of nodes of the element, N_α^e is the shape function, and the NURBS function is used in this study. Also, N_D denotes the number of dimensions.

For the governing equations (3) and (4), stabilized finite element method based on the SUPG/PSPG method is used to discretize in the spatial direction, and the Crank-Nicolson method with quadratic accuracy is applied to discretize in the temporal direction for the flow velocity, while the flow velocity and pressure in the continuous equation are treated implicitly. The advection velocity $\bar{\mathbf{u}}_i$ is approximated explicitly and linearized by the second-order accurate Adams-Bashforth method shown in Eq.（9）.

$$\bar{\mathbf{u}}_i = \frac{3}{2}\mathbf{u}_i^n - \frac{1}{2}\mathbf{u}_i^{n-1} \tag{9}$$

The finite element equation shown in equation (10) is obtained from the above.

$$\begin{aligned} &(\mathbf{M}+\mathbf{M}_S)\frac{\mathbf{u}_i^{n+1}-\mathbf{u}_i^n}{\Delta t} + (\mathbf{A}+\mathbf{A}_S)\frac{1}{2}\left(\mathbf{u}_i^{n+1}+\mathbf{u}_i^n\right) - (\mathbf{G}_i - \mathbf{G}_{Si})\,\mathbf{p}^{n+1} + \mathbf{D}_{ij}\frac{1}{2}\left(\mathbf{u}_i^{n+1}+\mathbf{u}_i^n\right) \\ &+ \mathbf{S}_C\mathbf{u}_i^{n+1} + \mathbf{C}_j\mathbf{u}_i^{n+1} + \mathbf{M}_{Pj}\frac{\mathbf{u}_j^{n+1}-\mathbf{u}_j^n}{\Delta t} + \mathbf{A}_{Pj}\frac{1}{2}\left(\mathbf{u}_i^{n+1}+\mathbf{u}_i^n\right) + \mathbf{G}_P\mathbf{p}^{n+1} = 0 \end{aligned} \tag{10}$$

The **M**, **A**, **G**, **D**, **C**, and **S** are the coefficient matrices for the time derivative, advection, pressure, viscosity, continuous, and shock trapping terms. The subscripts S, P, and C are the matrices due to the SUPG, PSPG, and shock trapping terms. The GPBi-CG method, which is an iterative solution method, is used to solve the finite element equations.

2.3 Calculation of free surface location

The advection equation (11) is used as the governing equation for the VOF function that describes the position of the free surface.

$$\frac{\partial \phi}{\partial t} + u_i \cdot \left(\frac{\partial \phi}{\partial x_i} \right) = 0 \quad \text{in} \quad \Omega \tag{11}$$

Where u is the advection velocity and the value obtained by solving Eq. (10).

The following finite element equations are obtained by discretizing equation (11) by applying a stabilized finite element method based on the SUPG method in the spatial direction and the Crank-Nicolson method in the temporal direction.

$$\frac{1}{\Delta t}\left\{\mathbf{M}+\mathbf{M}_S\right\}\phi^{n+1}+\frac{1}{2}\left\{\mathbf{A}+\mathbf{A}_S\right\}\phi^{n+1}+\mathbf{S_C}\phi^{n+1}=\frac{1}{\Delta t}\left\{\mathbf{M}+\mathbf{M}_S\right\}\phi^{n}-\frac{1}{2}\left\{\mathbf{A}+\mathbf{A}_S\right\}\phi^{n} \tag{12}$$

Where $\mathbf{M}$, $\mathbf{A}$, and $\mathbf{S}$ are the coefficient matrices of the time derivative, advection, and shock trapping terms, and the subscripts S and C are the matrices due to the SUPG and shock trapping terms. The GPBi-CG method, which is an iterative solution method, is used to solve the finite element equations.

2.4 Calculation of fluid force

Applying the weighted residual method based on the Galerkin method to the governing equations (3) and (4) in the flow field, and applying partial integration to the pressure and viscosity terms, the following weak form is obtained.

$$\begin{aligned}&\int_{\Omega^0} w_i\left(\frac{\partial u_i}{\partial t}+\bar{u}_j\frac{\partial u_i}{\partial x_j}-f_i\right)d\Omega-\int_{\Omega^0}\frac{\partial w_i}{\partial x_j}p d\Omega+\int_{\Omega^0}q\frac{\partial u_i}{\partial x_i}d\Omega\\&+\int_{\Omega^0}\frac{\partial w_i}{\partial x_j}\mu\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_i}{\partial x_i}\right)d\Omega=\int_{\Gamma_{in}}w_i\left(-p\delta_{ij}+\mu\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_i}{\partial x_i}\right)\right)n_j d\Gamma\end{aligned} \tag{13}$$

Where Ω^0,Γ_{in} denotes the area around the structure and the structure boundary. The integral term on the right side of equation (13) is the fluid force (drag force) acting on the structure. Substituting the velocity and pressure obtained by solving equations (10) into equation (13), the fluid force acting on the structure is obtained.

2.5 NURBS

In this study, the NURBS function is used as the Spline function for the shape function because it can represent various shapes with a small number of elements, depending on the weights assigned to the control points. The NURBS function in a two-dimensional domain is represented by a two-way B-spline basis function, weights assigned to control points, and position vectors of control points. B-Spline basis function is defined by the Cox de Boor asymptotic formula in Eq. (14).
For $p=0$:

$$\begin{aligned}N_{i,0}(\xi)&=1 &&\text{if}\quad \xi_{\mathrm{i}}\le\xi\le\xi_{\mathrm{i+1}}\\N_{i,0}(\xi)&=0 &&\text{otherwise}\end{aligned}$$

For $p=1,2,3\cdots$:

$$N_{i,p}(\xi)=\frac{\xi-\xi_i}{\xi_{i+p}-\xi_i}N_{i,p-1}(\xi)+\frac{\xi_{i+p+1}-\xi}{\xi_{i+p+1}-\xi_{i+1}}N_{i+1,p-1}(\xi) \tag{14}$$

Where N is the B-Spline basis function in the ξ direction, i is the control point number, p is the order of the B-Spline basis function, and ξ_i is the knot, which is the coordinate in parameter space, given by a uniformly increasing number sequence called the knot vector as shown below.

$$\Xi = (\xi_1, \xi_2, ...\xi_{n+p+1}) \tag{15}$$

The knot vector is a sequence of numbers obtained from the CAD drawn shape model and is a parameter that defines the B-Spline basis functions and the elements in IGA.

Using the B-Spline basis functions expressed in equation (14), the basis functions $R_{i,j}^{p,q}(\xi,\eta)$ and the NURBS surface $S(\xi,\eta)$ are expressed as in Eq. (17) are expressed as in the following equation.

$$R_{i,j}^{p,q}(\xi,\eta) = \frac{N_{i,p}(\xi)M_{j,q}(\eta)w_{i,j}}{\sum_{\hat{i}=1}^{n}\sum_{\hat{j}=1}^{m} N_{\hat{i},p}(\xi)M_{\hat{j},q}(\eta)w_{\hat{i},\hat{j}}} \tag{16}$$

$$S(\xi,\eta) = \sum_{i=1}^{n}\sum_{j=1}^{m} R_{i,j}^{p,q}(\xi,\eta)B_{i,j} \tag{17}$$

Where M is the B-spline basis function in the η direction, j is the control point number of the B-spline basis function in the η direction, q is the order of the B-spline basis function in the η direction, $w_{i,j}$ is the weight assigned to the control point, which is the coordinate in physical space, and $B_{i,j}$ is the location vector of the control point.

Since the Spline function is used for the shape function in IGA, the flow velocity, pressure and and the weight function at each element are expressed as in Eq.(18), (19) and (20) using the NURBS function shown in Eq.(16).

$$u_i^e(\xi,\eta) = \sum_{I=1}^{n_{en}} R_{e_I}(\xi,\eta)u_{i,I} \tag{18}$$

$$p^e(\xi,\eta) = \sum_{I=1}^{n_{en}} R_{e_I}(\xi,\eta)p_I \tag{19}$$

$$w_i^e(\xi,\eta) = \sum_{I=1}^{n_{en}} R_{e_I}(\xi,\eta)w_{i,I} \tag{20}$$

Where n_{en} is the number of control points constituting the element. The Spline function is not a function of the physical space, but a function of the parameter space of knots, and the numerical computation is performed in the parameter space. Therefore, it is necessary to perform variable transformation in the physical space $\Omega_e(x,y)$ and the parameter space $\Omega_e(\xi,\eta)$. In addition, since the variable transformation makes analytical integration difficult, a variable transformation is performed from the parameter space $\Omega_e(\xi,\eta)$ to the parent element $\hat{\Omega}_e(\hat{\xi},\hat{\eta})$, and then numerical integration is performed using the Legendre-Gauss integral formula shown in Eq.(16).

$$\int_{-1}^{1}\int_{-1}^{1} F(\hat{\xi},\hat{\eta})d\hat{\xi}d\hat{\eta} = \sum_{i=1}^{ngp}\sum_{j=1}^{ngp} F(\hat{\xi}_i,\hat{\eta}_j)w_i w_j \tag{21}$$

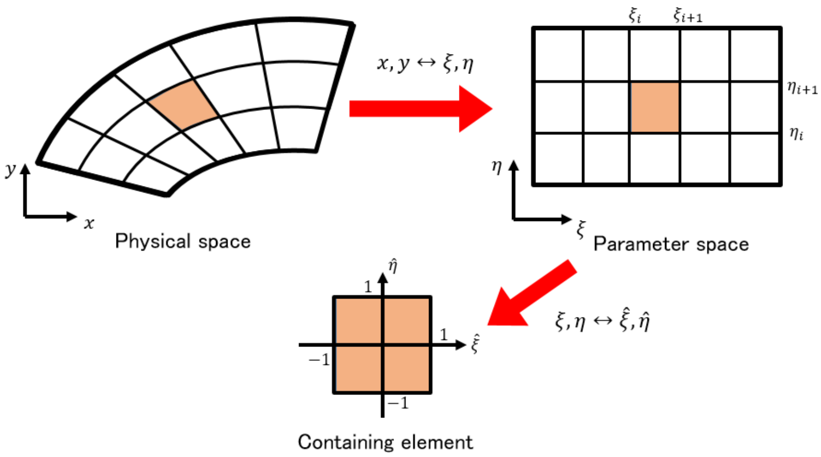

Figure 2: Variable transformation

Where ngp is the number of integration points, $\bar{\xi}_i, \bar{\eta}_j$ are the coordinates of integration points in the parent element, and w_i, w_j are the weights of integration points. Thus, in IGA, it is necessary to perform the variable transformation twice (see Figure -2). Here, the variable transformation from the parameter space to the parent element is performed by the following equation.

$$\hat{\xi} = -\frac{\xi_{i+1} + \xi_i}{\xi_{i+1} - \xi_i} + \frac{2}{\xi_{i+1} - \xi_i}\xi \tag{22}$$

$$\hat{\eta} = -\frac{\eta_{i+1} + \eta_i}{\eta_{i+1} - \eta_i} + \frac{2}{\eta_{i+1} - \eta_i}\eta \tag{23}$$

3 NUMERICAL ANALYSIS EXAMPLE

In this study, a sloshing problem in an area with curved shape is taken up and analyzed by IGA. The validity of the program will be verified by comparing it with the results of conventional finite element analysis, and the results will be compared with the experimental results.

4 CONCLUSIONS

In this report, an analytical method for free surface flow problems using IGA with NURBS functions is described. The conclusions obtained from the comparison with the conventional finite element method through application examples will be presented in the presentation.

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