

STRESS-RATIO-DEPENDENT MATERIAL PARAMETERS FOR IMPROVED NUMERICAL SIMULATIONS OF TEXTILE MEMBRANE STRUCTURES

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Abstract. *A recently proposed orthotropic hyperelastic material model for the geometrically nonlinear simulation of woven textile membranes is extended to become the basis of a new approach for structural simulations of improved accuracy. The model is polyconvex and benefits from anisotropic metric tensors in the construction of structural tensors. The proposed nonlinear model is a competitive replacement for the commonly used, however, oversimplified linear elastic formulation. Solving an optimization problem, the nonlinear model parameters are initially identified and fixed for general groups of fabrics, e.g., glass-PTFE or PES-PVC. For each particular material type, the remaining linear material parameters will be adjusted for the given set of experimental data.*

To improve the accuracy of the model response in structural simulations we propose a new framework to identify and modify material parameters based on the stress ratio locally varying within the structure. To this end, in the first step, the material parameters are separately adjusted to rather classical biaxial tensile tests with load ratios e.g., 1:0.5, 1:1, etc, and further rather uncommon load ratios 1:0.25, 1:0.125, etc. In the second step, an iterative simulation procedure will be applied, where, according to the resulting, locally distributed stress ratio, the corresponding parameter value will be updated at the integration points in the discretized structural problem. For this purpose, suitable interpolation schemes will be applied to identify the parameters at load ratios in between the experimentally considered ones. The iterative scheme will be repeated until the overall parameter change on all points becomes negligible. The efficacy of the proposed method will be assessed by comparing results using the new approach with the results obtained from fixed parameters.

1 INTRODUCTION

In general, woven textile membranes are composed of a network of interlaced yarns, warp and fill, which are covered by a coating material. The two main groups of technical fabrics extensively used in engineering applications are the PES-PVC and the glass-PTFE fabrics. To characterize the material behavior of textile membranes the material response of a cruciform fabric is studied under application of biaxial tensile loading in the warp- and fill direction [1]. The commonly applied biaxial tensile tests are the ones with the load ratios of 1:0.5, 1:1 and 0.5:1 (warp:fill), where e.g., in 1:0.5, 1 indicates that the maximum load is applied in the warp direction and the fill direction is pulled by half of the maximum load. It is also recommended to study the fabric behavior additionally under uniaxial tensile tests [2]. Nonetheless, the obtained data of lateral strains are usually neglected in the identification of elastic constants, see [1, 2, 3], and thus, uniaxial tests are not considered in this work. However, the material response of the fabric will be investigated instead using some uncommon load ratios, i.e. the 1:0.125, 1:0.25 and 1:0.75 as well as the 0.125:1, 0.25:1 and 0.75:1 ones. Such load ratios are more likely to appear in real engineering structures and the relatively realistic large-scale experiments, see [4].

In this work, a new framework will be outlined which may improve the overall accuracy of the model response in the simulation of woven fabrics. It is well known that due to the complex behavior of woven fabrics, fitting all load ratios together requires complex material formulations [5]. The recently proposed orthotropic nonlinear model, as a replacement for the simple linear elastic formulation, was able to improve the general representation of woven fabric behavior by more than 40% [6]. However, using the new framework, significant improvement in the structural simulation of technical fabrics can be expected. The basic idea is to separately fit each stress ratio to obtain so-called stress-ratio-dependent material parameters. By employing an interpolation system the material parameters will be identified and modified according to the stress ratios appearing locally in the structural problem using the previously identified ones. Subsequently, an iterative simulation procedure can be employed, where, based on the resulting locally distributed stress ratio, the corresponding parameter value will be updated at the integration points in the discretized structural problem. To this end, appropriate interpolation schemes can be utilized to modify the parameters at load ratios which were not considered in the experiments. The iterative scheme shall be repeated until the overall parameter change on all integration points becomes negligible. The effectiveness of the proposed framework will be evaluated by comparing results using the new approach with the results obtained from fixed parameters.

2 MATERIAL MODEL AND PARAMETER ADJUSTMENT

2.1 Nonlinear polyconvex orthotropic model

The orthotropic nonlinear material model was primarily proposed for the geometrically nonlinear simulation of glass-PTFE textile membranes, see [7]. Nevertheless, the adequacy

of the model in the representation of mechanical response of PES-PVC fabrics was also demonstrated in [8, 6]. Owing to its polyconvex energy terms, the model is guaranteed to be materially stable and robust in numerical simulations [9, 10, 11]. The advantages of including the polyconvex orthotropic term was showcased and compared with other competitive nonlinear models, see [12, 13, 7]. In [4], the model was validated by means of the simulation of a recently designed large-scale pressure chamber test on a woven fabric.

Taking into account the elements of a woven coated fabric, the heterogeneous body can be idealized as a continuum membrane with two orthogonal tensile reinforced directions; i.e. warp- and fill directions. For hyperelastic modeling, the existence of a strain energy function ψ is postulated and therefore, the model may be formulated as an additive decomposition of separate energy terms. In the model it is assumed that each independent nonlinear term ψ_i represents a specific response. The intensity of the desired response may simply be regulated via only one linear coefficient α_i available per energy term; i.e. $\alpha_i\psi_i$. This is the decisive feature of the proposed nonlinear formulation, since at the end, only 3 material parameters α_i remain which are to be adjusted for each individual fabric, i.e. $\psi(\boldsymbol{\alpha}, \mathbf{C}, \boldsymbol{\gamma}) = \sum_{i=1}^3 \alpha_i\psi_i(\mathbf{C}, \boldsymbol{\gamma})$. The included nonlinear term $\psi_i(\mathbf{C}, \boldsymbol{\gamma})$ is a function of the right Cauchy-Green deformation tensor $\mathbf{C} = \mathbf{F}^T\mathbf{F}$, with \mathbf{F} being the deformation gradient, and $\boldsymbol{\gamma}$ are the nonlinear model parameters. The internal model parameters $\boldsymbol{\gamma}$ control the nonlinearity of each deformation mode. For each group of fabrics; i.e. glass-PTFE or PES-PVC, the model parameters can be initially identified and remain fixed, see e.g. [4, 8, 6], following the basic idea in a biomechanics context proposed by [14]. For the fixed $\boldsymbol{\gamma}$, the orthotropic model for glass-PTFE fabrics becomes linear in the material parameters α_i while remaining nonlinear in the deformation modes $\psi_i(\mathbf{C})$ and is built as the following

$$\psi(\boldsymbol{\alpha}, \mathbf{C}) = \alpha^{\text{int}}\psi^{\text{int}}(\mathbf{C}) + \alpha_w^{\text{ti}}\psi_w^{\text{ti}}(\mathbf{C}) + \alpha_f^{\text{ti}}\psi_f^{\text{ti}}(\mathbf{C}) + \epsilon\psi^{\text{vol}}. \quad (1)$$

In the above, the orthotropic term is denoted as ψ^{int} mainly controls the crosswise inter-yarns interactions. The two transversely isotropic terms act as tensile reinforcement in the fill- and warp directions which are denoted as ψ_f^{ti} and ψ_w^{ti} . In addition, one volumetric term is included to assure a nearly incompressible material response and thus, the coefficient is chosen as $\epsilon = 1\text{E}6 \text{ kN/m}$.

In the formulation of the implemented interaction term, the notion of anisotropic metric tensor \mathbf{G} is used as the so-called structural tensor which may solve the typical issues of the multiplicative non-polyconvex ones, see [12, 15, 16]. In general, a full component metric tensor enhances the description of generic classes of anisotropy and thereby, a great solution which enables various couplings and interactions between principal material directions [10]. The anisotropic metric tensors \mathbf{G} are basically the more generalized form of the classical structural tensors $\mathbf{M}_i = \hat{\mathbf{a}}_i \otimes \hat{\mathbf{a}}_i$, see [17]. In the classical form, also used in the transversely isotropic terms ψ_i^{ti} , $\hat{\mathbf{a}}_i$ is a unit vector showing the yarn direction, where the properties $|\hat{\mathbf{a}}_i| = 1$, $\text{tr}[\mathbf{M}_i] = 1$ and $i = \text{w/f}$ hold. The metric tensor is formulated as $\mathbf{G} := \mathbf{H}\mathbf{H}^T$ with tensor \mathbf{H} being a linear tangent map of a fictitious

Cartesian basis \hat{e}_i into the material principal directions; i.e. $\mathbf{H} : \hat{e}_i \rightarrow \bar{\mathbf{a}}_i$ and $\bar{\mathbf{a}}_i = \mathbf{H}\hat{e}_i$ where $i \in \{1, 2, 3\}$. If the material basis coincides with the global Cartesian basis, the orthotropic metric tensor becomes $\mathbf{G} = \text{diag}[a^2, b^2, c^2]$ with 3 non-zero components a, b, c . These coefficients are assumed to be the length of material basis in the warp, fill and thickness directions, thereby, they are able to scale a material property in the associated direction. For textile membranes, the thickness component can be set to zero $c = 0$. The metric tensor components belong to the model parameters and thus, a and b are respectively denoted as γ_1 and γ_2 . The final form of the metric tensor is obtained as $\mathbf{G} = \text{diag}[\gamma_1^2, \gamma_2^2, 0]$. The individual nonlinear term/mode are given as

$$\psi^{\text{int}} = \frac{1}{(\gamma_3 + 1)g^{\gamma_3}} \left[J_4^{\gamma_3+1} + J_5^{\gamma_3+1} - \ln I_3^{(\gamma_3+1)g^{\gamma_3}} - 2g^{\gamma_3+1} \right], \quad (2)$$

$$\psi_{\text{w}}^{\text{ti}} = \langle \bar{J}_{4\text{w}} - 1 \rangle^{\gamma_4}, \quad \psi_{\text{f}}^{\text{ti}} = \langle \bar{J}_{4\text{f}} - 1 \rangle^{\gamma_5}, \quad \psi^{\text{vol}} = (I_3^2 + I_3^{-2} - 2).$$

The principal invariants of tensor \mathbf{C} are $I_1 = \text{tr}[\mathbf{C}]$, $I_2 = \text{tr}[\text{Cof}[\mathbf{C}]]$ and $I_3 = \det[\mathbf{C}]$. The mixed invariants in the transverse isotropic terms are defined as $J_{4\text{w}} = \text{tr}[\mathbf{C}\mathbf{M}_{\text{w}}]$ and $J_{4\text{f}} = \text{tr}[\mathbf{C}\mathbf{M}_{\text{f}}]$. Acting only as tensile reinforcements, the Macaulay brackets are used to cancel out the transverse isotropic terms under compression; i.e. $\langle (\bullet) \rangle = [|\bullet| + (\bullet)]/2$. The bar above the quantities implies the volume-preserving part of in invariant, e.g. $\bar{J}_{4i} = J_{4i}/I_3^{1/3}$. The invariants in interaction term are defined as $J_4 = \text{tr}[\mathbf{C}\mathbf{G}]$ and $J_5 = \text{tr}[\text{Cof}[\mathbf{C}]\mathbf{G}]$ using the metric tensor. The trace of metric tensor is denoted as $g = \text{tr}[\mathbf{G}] = \gamma_1^2 + \gamma_2^2$.

2.2 Material parameter adjustment, stress-ratio-dependent parameters

The basis of the parameter adjustment is the stress-strain data of the first load cycle (LC) of 9 biaxial tensile tests. The tests are applied on 9 separate virgin glass-PTFE fabrics of the same material and same production. It is known to the authors that the material response of woven fabrics in the first load cycles is elasto-plastic and highly nonlinear [18]. However, for the current work, the first LC data was taken into account to demonstrate only the efficacy of the proposed method. Furthermore, the typical uniaxial tensile tests were dropped from the applied experiments due to the following reasons. (a) Such loading case rarely happens and actually should not appear in any realistic structural behavior. (b) The challenging strong lateral contraction data is usually canceled in the parameter adjustment procedure, see [1, 2, 3]. Thus, the uniaxial tests were replaced by some uncommon, however, more realistic biaxial cases; i.e. 1:0.125, 1:0.25, 0.125:1 and 0.25:1. For the purpose of parameter interpolation in the next section, two other stress ratios were also added to the experiments; i.e. 1:0.75 and 0.75:1. These new experiments, to a great extent, provide decent information on the evolution of stiffness parameters and the structural response of tensile fabrics on every practical stress ratios. As shown in [8], the model parameters γ_i are identified initially as a part of the nonlinear model and kept fixed. To this end, by fitting to all 9 load ratios at each individual stress-strain point, the deviation of the material parameters α_i from their average value $\bar{\alpha}_i$ becomes minimized, see [14]. Once this deviation becomes minimized, the model parameters are identified.

Table 1: The identified model parameters for glass-PTFE fabric using first LC data.

model parameters	$\gamma_1[-]$	$\gamma_2[-]$	$\gamma_3[-]$	$\gamma_4[-]$	$\gamma_5[-]$
glass-PTFE	1.25	0.85	7	3	5

The application of this method has the following advantages for the nonlinear model. (a) The total number of remaining material parameters to be identified is reduced to maximum 3 parameters, which is very similar to the linear elastic formulation, however it is more accurate [4]. (b) As it will be shown, the fixed model parameters will be used for identification of all stress-ratio-dependent material parameters. (c) Following the method in [14], a unique identification of material parameters can become feasible since the remaining parameters appear only linearly in the formulation and the least-square error functional in terms of the stress deviation becomes convex in the material properties α_i . In addition, as the material parameters mainly act as scaling factors on each deformation mode ψ_i , thus, a fairly small sensitivity on the model response will be naturally included. The obtained values of model parameters are listed in Table 1. Despite the mentioned advantages, the last step of unique identification of material parameters, point (c), will not be employed here since the results obtained by this method will not guarantee the actual stress ratio applied during each load driven biaxial tests. Therefore, to obtain the desired stress-ratio-dependent material parameters a proper least-square error functional in terms of the strain deviation is introduced as

$$\mathcal{L}_\varepsilon(\boldsymbol{\alpha}) = \sum_k \mathcal{L}_{\varepsilon,k}(\boldsymbol{\alpha}) \quad \text{with} \quad \mathcal{L}_{\varepsilon,k}(\boldsymbol{\alpha}) = \sqrt{\frac{1}{n_{\text{mp}}} \sum_i^{n_{\text{mp}}} \left(\frac{(\bullet)_i^{\text{comp}} - (\bullet)_i^{\text{exp}}}{\max(\bullet)_k^{\text{exp}}} \right)^2}. \quad (3)$$

In the above, $\mathcal{L}_\varepsilon(\boldsymbol{\alpha})$ denotes the overall error of relative strain differences summed over all measure points of all experiments. The $\mathcal{L}_{\varepsilon,k}(\boldsymbol{\alpha})$ is the strain deviation of each stress ratio k . The quantities in (\bullet) are the computed and measured engineering strains in the warp and fill directions. The number of total measure points is denoted as n_{mp} . In general, it is assumed that the material behavior of woven coated fabrics is incompressible, therefore, during optimization the volume change is preserved by enforcing $\det(\mathbf{F}) = 1$. Herein, the components of deformation gradient in the thickness direction becomes a function of the other two in-plane coefficients $\lambda_3 = 1/\lambda_1/\lambda_2$. Moreover, the penalty term $\psi^{\text{vol}} = p(I_3 - 1)$ is added to the strain energy function. The quantity p is interpreted as a pressure-like Lagrange multiplier [19, 20]. Now, by minimizing the least-square error functional, i.e. $\tilde{\boldsymbol{\alpha}} = \text{argmin}(\mathcal{L}_\varepsilon(\boldsymbol{\alpha}))$, the 3 linear material parameters will be identified in two separate procedures. In the first optimization procedure, the parameters are adjusted fitting all 9

Table 2: Comparison of obtained errors using two optimization procedures.

Total error	$\mathcal{L}_\varepsilon(\boldsymbol{\alpha})$	$\mathcal{L}_\varepsilon^*(\boldsymbol{\alpha})$	Improvement [%]
Nonlinear model	3.35	1.99	40[%]

Table 3: Stress-ratio-dependent material parameters obtained for the nonlinear model.

	0.125:1	0.25:1	0.5:1	0.75:1	1:1	1:0.75	1:0.5	1:0.25	1:0.125	all
α^{int} kN/m	8.4	8.5	9.9	12.6	13.7	8.9	8.0	7.6	7.4	12
α_w^{ti} kN/m	0.0	0.0	0.0	0.0	0.0	25000	17506	4394	4390	10527
α_f^{ti} kN/m	707	756	1052	1201	1522	4105	10974	1e+6	100	1205

experiments together; i.e. directly minimizing \mathcal{L}_ε . In the second procedure, the material parameters are identified fitting each load ratio separately; i.e. minimizing separately each $\mathcal{L}_{\varepsilon,k}$ term. Therefore, the obtained parameters are called the stress-ratio-dependent material parameters and the total error by summing all terms is denoted as $\mathcal{L}_\varepsilon^*$. The stress-ratio-dependent parameters are listed in Table 3. To better understand the impact of separately fitting each load ratio in the overall representation of experimental data, the obtained errors by two optimization procedures are reported in Table 2. The relative difference listed in the last column of Table 2 proves that fitting each stress ratio separately will improve the general model response by almost 40[%]. Figure 1 shows the obtained model response using the stress-ratio-dependent parameters for the nonlinear orthotropic model.

3 ITERATIVE SCHEME IN IMPROVED NUMERICAL SIMULATION

In the following, we present a convenient iterative scheme which facilitates the implementation of stress-ratio-dependent parameters in the structural simulation.

3.1 Interpolation scheme based on stress ratio

The basic idea is to iteratively update and assign stress-ratio-dependent material parameters to the locally varying stress ratio at the integration points within the structure during numerical simulation. Therefore, an interpolation system should be designed which is able to practically identify/estimate the parameters for the stress ratios which were not included in the experiments. Taking into account the variation of material parameters in Table 3, a simple power function can be a reasonable choice:

$$f_{\alpha_i}(x) = ax^b + c \tag{4}$$

In the above function, x is assumed to be the computed stress ratio and $f_{\alpha_i}(x)$ is the interpolated value of parameter α_i at ratio x . The constants a, b, c are identified separately for each parameter at different intervals of x . Figure 2 shows the adequacy of the interpolation function in fitting of α^{int} and α_f^{ti} at different stress ratios. It should be noted that the stress ratios are replaced by their computed values in Figure 2, e.g. 1:0.125 corresponds to 8 and 0.5:1 becomes 0.5. (The stress ratios are computed as the ratio of nominal stress in the warp to the stress in the fill directions P_w/P_f .) Note that the response of the interpolation function is in agreement with the restrictions of the nonlinear model, e.g. if the ratio goes beyond 8, due to contraction in the fill direction the α_f^{ti} tends

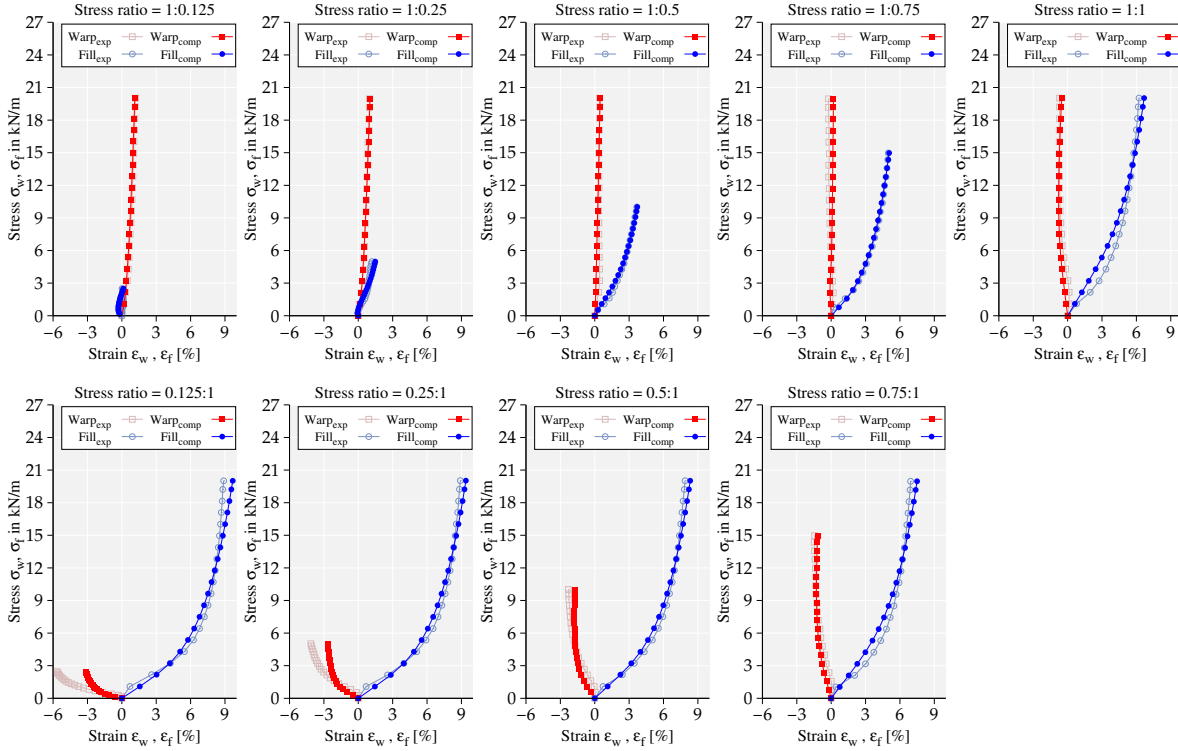


Figure 1: Representation of experiments using the stress-ratio-dependent material parameters.

to zero. Thus, for the values out of the range of the applied stress ratios the function can still fairly estimate the parameters. As it is apparent from Figure 2, for each parameter at least 2 independent interpolation functions with different constants were used to fit all ratios.

3.2 Impact of iterative parameter modification on the structural response

In this section, the impact of the stress-ratio-dependent material parameters in the simulation of a simple boundary value problem (BVP) is investigated. The boundary value problem is a flat membrane which is fixed on all edges in all displacement directions. The membrane is square with the length of 1400mm and thickness of 1mm. The surface is loaded with pressure load of 7kN/m². The BVP was chosen so that it resembles the boundary condition of the large-scale experiment in [4]. The applied pressure was opted such that the maximum generated stress within the surface would be in accordance with the maximum applied load in the biaxial tests. Hence, it is expected that the locally distributed stress ratios at any point will not notably exceed the upper or lower bounds of the experimentally considered stress ratios. For the structural simulations, the nonlinear material model was implemented as a user material subroutine (UMAT) into the commercial software Abaqus. The surface is discretized using quadrilateral shell elements S4R of Abaqus with 1225 elements. Note that by exerting an initial mesh convergence study the proper number of elements were chosen such that the results would not be

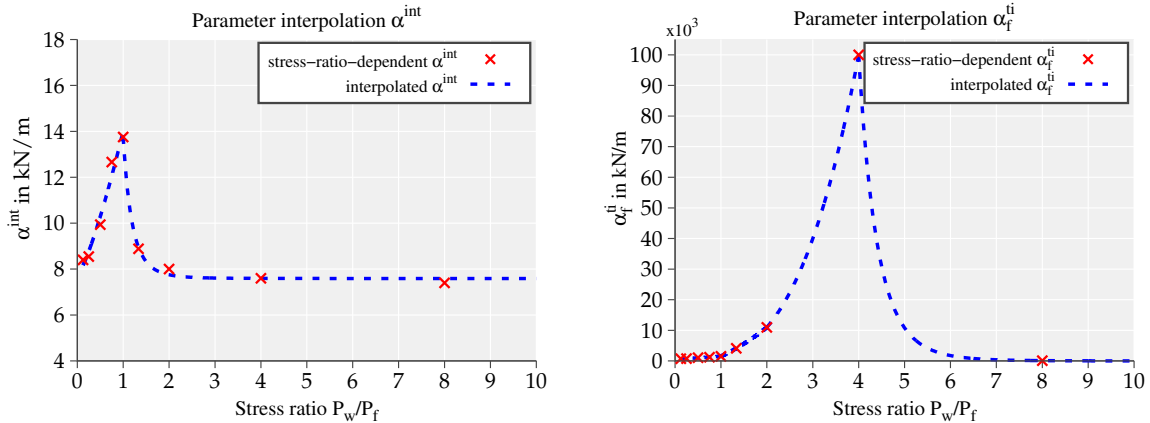


Figure 2: Interpolation over stress-ratio-dependent material parameters.

affected by the number of elements.

In the first step, using the material parameters fitted to all stress ratios, the BVP is simulated, i.e. in the simulation material parameters are fixed/identical on all integration points within the discretized structure (level 0). The obtained displacement distribution using the fixed parameters is plotted in Figure 3a. At the end of the simulation, the stress ratio in the principal material directions P_w/P_f at each integration point is computed and saved. Subsequently, the stored stress ratio will be inserted into the interpolation functions to compute/identify the stress-ratio-dependent parameters for the next simulation. The computed stress ratio distribution of the fixed parameters is plotted in Figure 3b. In the next step, the same BVP is re-executed with the major difference being that the material parameters are the interpolated ones based on the computed stress ratio of the previous step. That also means that the value of each material parameter may vary locally at each integration point over the discretized surface. Note that since the modification in the parameters was carried out through the interpolation scheme, no sudden change/jump in the model response would be expected. After finishing level 1 simulation, again the stress ratios are computed on each integration point. Subsequently, the stress-ratio-dependent parameters are updated for the next simulation similarly to the previous procedure. The parameter update will be repeated until the overall parameter change on all points becomes negligible. This can be achieved by formulating a proper error function to compute the L^2 -norm of all parameter change at each integration point

$$\tilde{\mathcal{L}} = \sum_{i=1}^3 \tilde{\mathcal{L}}_{\alpha_i} = \tilde{\mathcal{L}}_{\alpha^{\text{int}}} + \tilde{\mathcal{L}}_{\alpha_w^{\text{ti}}} + \tilde{\mathcal{L}}_{\alpha_f^{\text{ti}}}, \quad (5)$$

$$\tilde{\mathcal{L}}_{\alpha_i} = \left(\sum_{j=1}^{n_{\text{el}}} (\alpha_{i,j}^{n+1} - \alpha_{i,j}^n)^2 \cdot (dA)_j \right) \cdot \frac{1}{\sum_{j=1}^{n_{\text{el}}} (dA)_j} \cdot \frac{1}{\max(\alpha_i^n)^2}.$$

The L^2 -norm of each parameter change at the integration points of all elements is denoted as $\tilde{\mathcal{L}}_{\alpha_i}$. The dA is the area element. The indices i, j and n show the material parameter,

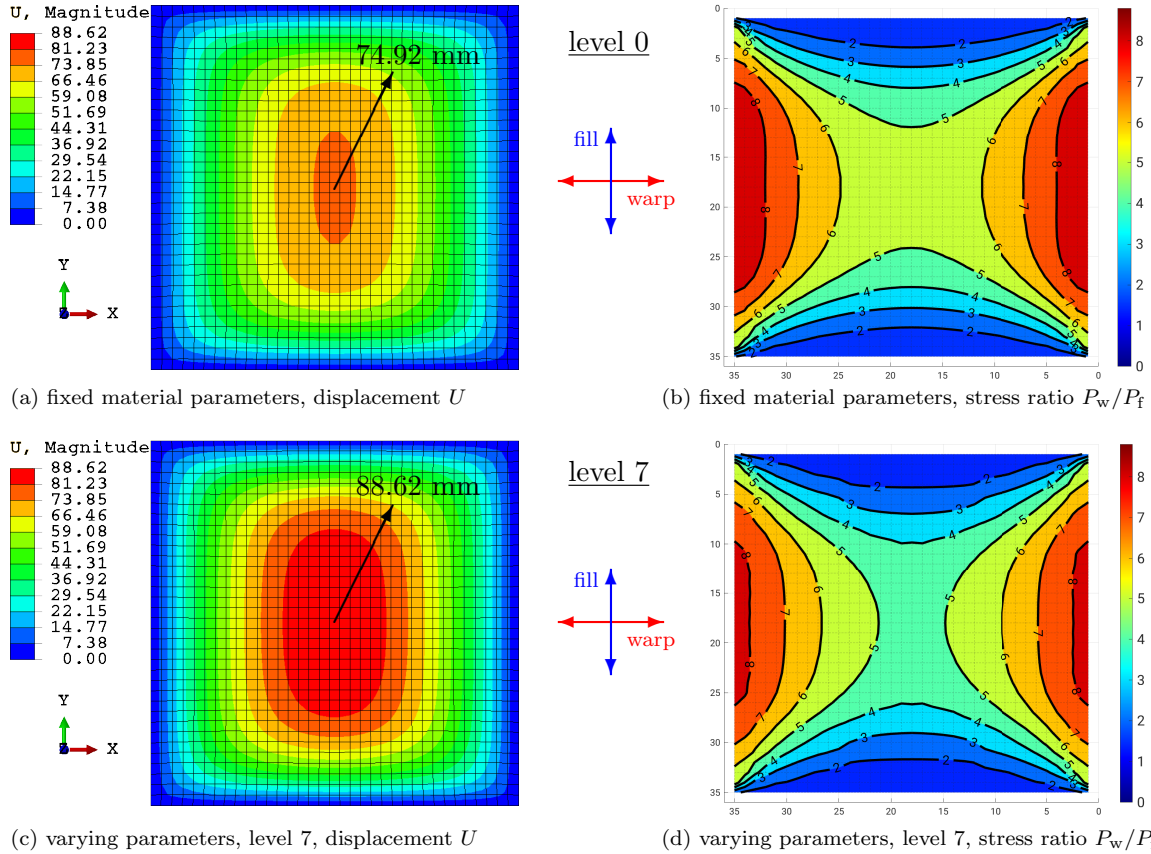


Figure 3: Mechanical response of the nonlinear model. (a,b) Displacement and stress ratio distribution obtained for the fixed material parameters.(c,d) Displacement and stress ratio distribution obtained for the varying stress-ratio-dependent material parameters, level 7.

element index and the level of iterative updates, respectively. To establish the effectiveness of the iterative scheme, the L^2 -norm of parameter change during 7 levels of updates is illustrated in Figure 4a. The displacement change of all nodes is also computed using a similar error function in Eq.(5) and the result is shown in Figure 4a. Although, after 7 iterations the L^2 -norm of parameters change becomes basically zero, however, even after the first level of update, the total structural response becomes almost constant. Thus, the update of parameters after level 1 in principle only marginally changes the global material response. The convergence in the maximum displacement appearing at the middle element of membrane is illustrated in Figure 4b. The maximum displacement using the fixed parameters, i.e. level 0, is about 75mm which alters by 18% after one level of update and remains almost constants during the next iterations. The same trend was also observed for the stress quantities, i.e. almost 17% decrease in the maximum in-plane stresses. Furthermore, the attained results of displacement and stress ratio distribution after 7 levels of update are illustrated in Figures 3c and 3d. Taking into account the obtained results in Figures 3 and 4, although the qualitative response does not vary much

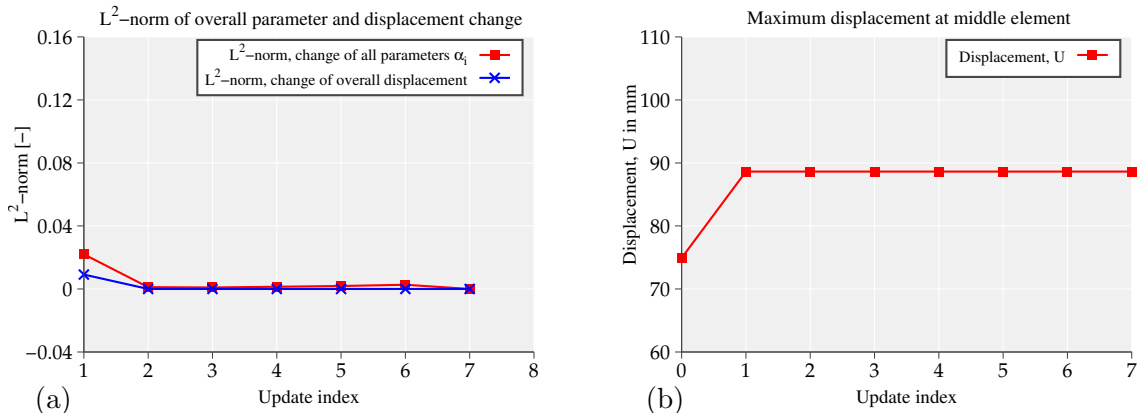


Figure 4: Convergence study of structural response by updating material parameters.

in the surface, however, the computed differences indicate remarkable improvement in the overall material response by almost 18%. Note that the stress-ratio-dependent material parameters were 40% more accurate in representation of the tensile behavior of the tested fabric. It is worth to mention that the numerical time during these simulations was not at all influenced by using varying parameters in the structural problem.

4 CONCLUSION

In this work, we established a new method which effectively improved the model response of the nonlinear material model in structural simulation of woven fabrics. To this end, a new framework was proposed to identify and modify the material parameters according to the locally resulting stress ratios within the structure. The material parameters were separately identified for the experimentally considered stress ratios, the so-called stress-ratio-dependent parameters. Using a suitable interpolation scheme, the material parameters were identified even at the stress ratios which were not considered in the controlled standard tensile tests. Using the iterative simulation approach, the corresponding parameter values were updated at the integration points in the discretized structural problem. That is, the utilized parameter would have been the most suitable value for the appearing stress ratio. The iterative scheme was repeated until the overall parameter change on all integration points became negligible.

Using the stress-ratio-dependent material parameters, the overall representation of tensile experiments was 40% improved. In the structural simulation, the iterative scheme, even only after one parameter update, could improve the overall mechanical response by almost 18%. During the next updates, the global material response remained almost unchanged indicating a highly efficient convergence behavior of the stress-ratio iteration. Although within the structure the parameter values vary at every integration point, this change in neighboring points did not significantly influence the numerical cost of simulation. In the future studies, the new iterative scheme shall be assessed by considering a more complex geometry.

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