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Comparison of Classical and Weakly-singular Representations of Free-surface Potential Flows

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Within the potential-flow framework, a flow in a domain is determined in terms of the flow at the boundary surface of the flow domain by means of a classical boundary-integral representation, which defines the flow potential ϕ in terms of a Green function G and its gradient ∇G. An alternative flow representation is the weakly-singular representation (which defines ϕ in terms of G and a vector Green function $\mathbf{G}$ associated to G via the relation $\nabla \times \mathbf{G} = \nabla G$) given by the authors for diffraction-radiation by a ship advancing in regular waves. The alternative mathematical representations of far-field waves associated with the classical and weakly-singular potential representations are compared here in the special case of steady flows.

INTRODUCTION

Wave diffraction-radiation by a ship advancing in time-harmonic waves at the free surface of a large body of water, within the framework of a 3-dimensional potential-flow frequency-domain analysis, is one of the most classical and important core issues in ship hydrodynamics. Indeed, 3-D wave diffraction-radiation with forward speed is relevant to hydrodynamic hull-form design and optimization (notably of fast and unconventional vessels and at early stages), viscous ship hydrodynamics (via coupling with RANSE near-field calculations), and time-domain simulations of ship motions in large waves (for which added-mass and wave-damping coefficients are useful elements). Accordingly, the problem has been extensively considered in the literature.

In the special case of wave diffraction-radiation without forward speed (by an offshore structure), robust and practical panel methods have been developed and are routinely used to solve the canonical wave diffraction-radiation problems that yield added-mass and wave-damping coefficients, and wave exciting forces and moments. These potential-flow methods are based on the numerical solution of a boundary-integral equation obtained using a Green function that satisfies the linear free-surface boundary condition for diffraction-radiation of regular waves without forward speed.

Application of this classical approach, often identified as the free-surface Green function method, to wave diffraction-radiation by ships (i.e. with forward speed) has also led to useful methods (Diebold, 2003; Boin et al., 2000, 2002; Chen et al., 2000; Guilbaud et al., 2000; Fang, 2000; Wang et al., 1999; Du et al., 1999, 2000; Iwashita and Ito, 1998; Iwashita, 1997), although not of a comparable degree of practicality, as forward speed introduces major difficulties.

A basic difficulty is that the Green function that satisfies the linear free-surface boundary condition for diffraction-radiation of time-harmonic waves (frequency ω) with forward speed $\mathcal{U}$ is considerably more complicated than the Green functions corresponding to the special cases $\mathcal{U}=0$ or $\omega=0$, which can be evaluated relatively simply and efficiently, at least in deep water (e.g. Ponizy et al., 1994). Several free-surface Green functions, based on alternative mathematical representations, have been proposed and used in the literature on wave diffraction-radiation with forward speed. These alternative free-surface Green functions, and related singularity distributions over flat rectangular or triangular panels, have been considered in numerous studies, and relatively efficient numerical-evaluation methods have been developed (Maury, 2000; Chen, 1999; Boin et al., 1999; Brument and Delhommeau, 1997; Ba and Guilbaud, 1995; Iwashita and Ohkusu, 1992; Bougis and Coudray, 1991; Jankowski, 1990; Hoff, 1990; Wu and Eatock Taylor, 1987; Guevel and Bougis, 1982; Inglis and Price, 1982; Kobayashi, 1981; Bessho, 1977; and Wehausen and Laitone, 1960). Nevertheless, Green functions that satisfy the free-surface boundary condition for wave diffraction-radiation with forward speed are relatively complicated building blocks.

Another basic difficulty is related to the property that the boundary-integral representation of the velocity potential for time-harmonic (or steady) free-surface flows about a ship advancing with speed $\mathcal{U}$ in waves (or in calm water) involves a surface integral over the ship hull and a line integral around the ship waterline. This line integral (not present if $\mathcal{U}=0$) has important effects, notably on irregular frequencies (Ba et al., 2001). Further, the contributions of the waterline integral and of the hull-surface integral largely cancel out (Noblesse et al., 1990; Noblesse and Yang, 1995; and present study), which can result in a loss of accuracy.

These 2 basic difficulties of a frequency-domain analysis of wave diffraction-radiation with forward speed are examined in Noblesse and Yang (2004a and b), which provide, respectively, an alternative free-surface Green function and an alternative boundary-integral representation of the potential. The usual free-surface Green function for wave diffraction-radiation with forward speed and the simpler Green function (which satisfies the free-surface boundary condition accurately in the far field and approximately in the near field) given in Noblesse and Yang (2004a) are considered and compared in Yang et al. (2004) for the particular case of steady flows. The classical boundary-integral representation of the potential and the alternative weakly-singular potential representation given in Noblesse and Yang (2004b) are considered and compared here, also for the particular case of steady flows.

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