

A Reduced Order Modeling approach for optimal allocation of Distributed Generation in power distribution systems

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Abstract—This paper presents an “offline-online” strategy for optimal allocation and sizing of Distributed Generation. In traditional optimization approaches, each function evaluation requires the solution of a power flow problem, which makes global optimality a computationally challenging goal. In the proposed strategy the power flow solver is invoked only once and a parametric solution is constructed with a monolithic solver. Despite the fact that the parametrized power flow equations result in a high-dimensional problem, the proposed algorithm is specifically designed to circumvent the curse of dimensionality. This is achieved through the application of Model Reduction, in particular the Proper Generalized Decomposition combined with a nonlinear solver. Numerical examples are carried out for showing the validity of the proposed method.

Index Terms—Distributed power generation, Model Reduction, Proper Generalized Decomposition, Optimal DG allocation.

I. INTRODUCTION

The optimal allocation of a distributed generator (DG) is a key feature to ensure the introduction of distributed generation in electrical power systems to be successful.

This consideration explains the intense research activity of the last decades aimed to the identification of optimization techniques in power grid analysis. In particular, [1] defines the optimal distributed generation placement problem (ODGP) claiming that it provides the best locations and sizes of DGs to optimize electrical distribution network operation and planning. When ODGP is solved, the objective function can be single or multi-objective. On one hand, the main single-objective functions are: minimization of energy losses, minimization of system average interruption duration index (SAIDI), minimization of cost, etc. On the other hand, in [2], ODGP multi-objective formulations are classified as multi-objective function with weights, where the multi-objective formulation is transformed into a single-objective function using the weighted sum of individual objectives; goal multi-objective index, where the multi-objective formulation is transformed into a single-objective function using the goal programming

method and multi-objective formulation considering more than one often contrasting objectives.

As also in this paper, several studies have been focused on single-objective optimization, which in most cases corresponds to the minimization of the total power loss of the system. An example can be found in [3] where an implementation method of Tabu Search to find how much distribution loss can be reduced if DGs are optimally allocated is discussed. Authors in [4] and [5] proposed a genetic algorithm and an analytical method in order to minimize the total real power losses in the system. In [6] an analytical expression to calculate the optimal size and an effective methodology to identify the corresponding optimum location for DG placement for minimizing the total power losses in primary distribution systems is also presented. More recently, new methodologies have been proposed based on the optimal allocation of DG units in the distribution system so as to minimize annual energy loss [7]–[9]. Most optimization techniques require repeated evaluation of the objective function, each demanding the solution of a different power flow problem in order to compute the associated losses. When the number of parameters to optimize is high and global optimization is sought, this approach can become costly from the computational viewpoint.

The aim of this work is to illustrate a new strategy for optimal placement and sizing of DGs in power distribution systems based on a “offline-online” approach that separates the solution of the associated parametric power flow problem and the optimization into distinct steps. The novelty of this methodology is the fact that diverse parameters of the power flow problem are now considered as variables rather than input quantities. This perspective leads us to solve the power flow problem in a high-dimensional parametric space where the position of the DG, its generated power and the power demanded by the loads are regarded as additional coordinates. We address to this problem as the Parametric Power Flow (PPF).

PPF naturally arises as a high dimensional problem. In order to reduce the complexity of this problem we rely on Reduced

Order Modelling (ROM). The ROM approach has been already successfully applied to a number of applications in power engineering, namely: optimization, uncertainty quantification and real time control [10]–[15].

Although in this paper we only take into account a single objective function, the concepts that are presented can be easily extended to multi-objective optimization. After solving the PPF problem once and for all, any particular objective function can be defined and the optimization step can be performed efficiently since it just requires inexpensive function evaluations and no longer power flow solutions.

The layout of the paper is organized as follows: an overview of the Proper Generalized Decomposition (PGD) method is given in section II, while section III illustrates the Parametric Power Flow problem and the iterative scheme including PGD for solving it. In section IV, numerical examples are presented for a problem in two and eight dimensions. Finally, the conclusions are discussed in section V.

II. PROPER GENERALIZED DECOMPOSITION

The drawback of high dimensional problems is that the solution may become complicated due to the exponential increase of the degrees of freedom. Indeed, in D dimensions if each parameter assumes n possible states, the extensive exploration of the parametric space is associated to a volume of information that scales with n^D . This particular feature is known as the curse of dimensionality. In order to overcome this problem some alternatives could be taken into account, for instance the application of a ROM method.

The particular way in which the PGD method reduces the dimensional complexity is by approximating a multivariate function by the sum of products of one-dimensional functions:

$$V(s_1, s_2, \dots, s_n) \approx \sum_m^M \mathcal{V}_1^m(s_1) \mathcal{V}_2^m(s_2) \dots \mathcal{V}_n^m(s_n). \quad (1)$$

This form, called the separated variables representation, is computed using a greedy enrichment procedure, in which a single term per iteration is introduced in the summation. Each new term is determined using a fixed point algorithm in which each function is updated individually, using an alternating minimization approach.

The advantage of this approach is that the solution can be described by $n \cdot D \cdot M$ degrees of freedom instead of n^D and that the algorithm has linear complexity with respect to D . Due to the important reduction in the complexity (from exponential to linear) the PGD approach is drastically alleviating the curse of dimensionality [16]–[19].

This strategy has been successfully applied to parametric problems in computational mechanics. For detailed information look at the references [20]–[22].

In general, the application of PGD to linear problems is straightforward, nevertheless the extension to nonlinear problems could be really arduous depending on the particular problem to be solved. In this work, the Z bus method [23] is applied to the power flow equations. This, can be seen

as a particularization of the method of Alternating Search Directions (ASDM) [24]. This method is chosen because, unlike other classical methods such as Newton-Raphson, it is relatively easy to combine with PGD, since the nonlinear iteration is broken down into two linear steps that can be solved with PGD. For more details refer to [25].

III. THE PARAMETRIC POWER FLOW PROBLEM

The governing equations for the PPF problem and the corresponding separated variables forms are introduced in this section.

A. Parametrized power flow equations

When we consider the Parametric Power Flow problem, the voltage V , the power source S and the current I are no longer vectors of nodal values but vector valued functions of the problems parameters. Arranging all the parameters in a vector

$$\xi = [s_1, s_2, \dots, s_n]^T \quad (2)$$

the parametrized Kirchhoff's current law writes as:

$$\mathbf{Y}V(\xi) = I_0 + I(\xi), \quad (3)$$

where \mathbf{Y} is the admittance matrix, V is the vector of unknown voltage evaluated at each node, I_0 is the vector containing the constant current originated by the contribution of the slack node and I the vector of currents due to a power source S . All these vectors belong to \mathbb{C}^d while the matrix $\in \mathbb{C}^{d \times d}$ if d is the number of degrees of freedom. Currents, voltages and powers are nonlinearly related through power balance equations, which can be written in vector form as:

$$S(\xi) = V(\xi) \odot I(\xi)^*, \quad (4)$$

with I^* being the complex conjugate of I and the symbol \odot denoting the Hadamard product of vectors.

By incorporating (4) into (3), the following nonlinear system is obtained

$$\mathbf{Y}V(\xi) = I_0 + S(\xi)^* \oslash V(\xi)^*, \quad (5)$$

where the symbol \oslash denotes the component-wise quotient between vectors.

B. Separated variables representation

In the remainder of this paper the following parameters are considered:

- The DG position q
- The nominal generated power r
- The time t , since the power flow analysis will be performed with time-varying loads over the period of a year.

Once the datum S is expressed as a separated variables functions depending on q , r and t

$$S(r, t) = \sum_h^H \check{Q}^h \check{\mathcal{R}}^h(r) \check{T}^h(t), \quad (6)$$

the objective of the PGD is to find the unknown voltage V in a separated variables representation consisting of the sum of products of one-dimensional functions

$$V(r, t) \approx V_{PGD} = \sum_m^M Q^m \mathcal{R}^m(r) \mathcal{T}^m(t), \quad (7)$$

where H, M are the number of terms in the summations, $\hat{\mathcal{R}}^h(r)$, $\hat{\mathcal{T}}^h(t)$, $\mathcal{R}^m(r)$ and $\mathcal{T}^m(t)$ are parametric modes. The modes express the functional dependency from the corresponding parameters q, r, t and are recombined through the vectors coefficients \hat{Q}^h and Q^m .

Once the solution V_{PGD} is obtained, this can be used to generate a separated variables representation for the losses used for optimization:

$$L(q, r, t) = \sum_p^P \hat{Q}^p(q) \hat{\mathcal{R}}^p(r) \hat{\mathcal{T}}^p(t), \quad (8)$$

where P is the number of terms in the sum, $\hat{Q}^p(q)$, $\hat{\mathcal{R}}^p(r)$ and $\hat{\mathcal{T}}^p(t)$ are parametric modes depending on the corresponding parameter q, r and t .

C. Algorithm overview

Representing the variables in (5) in separated variables form, the Z bus method at iteration step γ reads as:

$$\mathbf{Y} V_{PGD}^{\gamma+1} = I_0 + S^* \odot V_{PGD}^{\gamma*}. \quad (9)$$

This algorithm can be broken down into the following two steps:

- 1) Given V_{PGD}^{γ} , the first step is to compute the injected currents $I^{\gamma+1}$ in a separated variables form, evaluating the quotient

$$I^{\gamma+1} = S^* \odot V_{PGD}^{\gamma*}. \quad (10)$$

Since the numerator S and the denominator V_{PGD}^{γ} are separated variables functions, the evaluation of the quotient is not straightforward, therefore PGD is applied to the associated linear problem

$$V_{PGD}^{\gamma*} \odot I^{\gamma+1} = S^*, \quad (11)$$

in order to find a separated representation of $I^{\gamma+1}$. The PGD algorithm is stopped once the residual of equation (11) is smaller than a given tolerance parameter ϵ_g . Each new mode in the summation is computed to a given precision ϵ_{fp} .

- 2) The second step consists of solving the linear system

$$V^{\gamma+1} = \mathbf{Y}^{-1}(I^{\gamma+1} + I_0) \quad (12)$$

which does not present any additional difficulty because the matrix \mathbf{Y} does not depend on the DG positioning or the power. For this reason, the factorization of the matrix can be stored and reused. Consequently, the modes for the voltage $V^{\gamma+1}$ can be straightforwardly computed by applying backward and forward substitution to the modes of $I^{\gamma+1}$.

The strategy presented is based on a particular iterative scheme and the necessity of an initial guess for the voltage comes up. This is taken as the voltage circulating in the grid under no loads or generation:

$$V_0 = \mathbf{Y}^{-1} I_0. \quad (13)$$

The nonlinear algorithm is terminated once the difference between the solution at two successive iterations is smaller than a tolerance parameter ϵ_n .

IV. NUMERICAL APPLICATIONS

In this section we show the proposed method for a test system. The network's diagram is shown in Fig. 1. The model, taken from [26], is a three-phase system with different topologies and load characteristics including a simplified representation of the high-voltage system. Some of the main characteristics of the substation transformer are given below:

- High-voltage rating: 230 kV
- Low-voltage rating: 4.16 kV
- Rated power of substation transformer: 10000 kVA.

The distributed generator is connected to the system through a step-up interconnection transformer allowing the switch from high-voltage to low-voltage. The authors of this work presented an application of parallel computing for allocating distributed generation for optimum reduction of energy losses using Monte Carlo method.

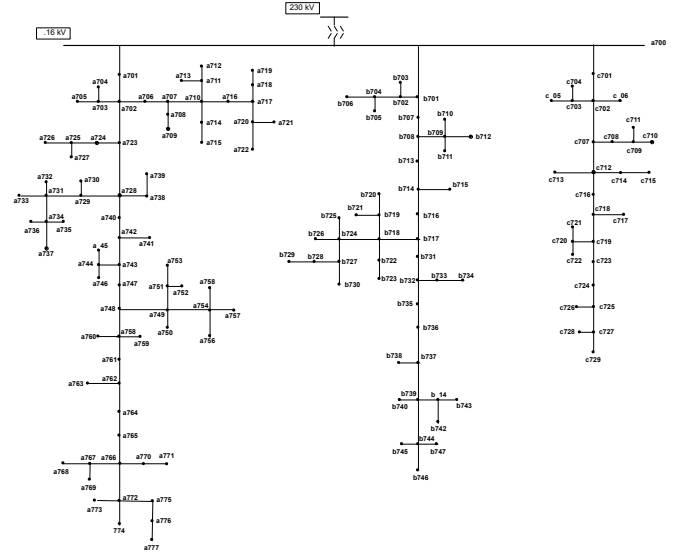


Fig. 1: Diagram of the test system

A. Optimal positioning of a DG unit with a fixed loads

We show as a first example the resolution of the Parametric Power Flow problem when two additional parameters being the position q of the new DG and the generated active power r are considered as variables. We seek the values of q and r that minimize the system losses. The problem consists in finding

the separated form of the voltage solution at each node of the grid

$$V(r) \approx \sum_m^M Q^m \mathcal{R}^m(r), \quad (14)$$

$\forall q = 1, \dots, 82$ corresponding to the bus position in the two first branches in the network, and $\forall r$ in the set of possible values of active power that the DG can provide, that is in the partition of the interval $[0, r_{max}]$ when the increment is $N_r = 100$ (number of samples) and $r_{max} = 8 \cdot 10^5 \text{ W}$.

The approach “offline-online” mentioned in section III-C is followed. The tolerances are $\epsilon_{fp} = 10^{-10}$, $\epsilon_g = 10^{-8}$ and $\epsilon_n = 10^{-6}$ and a solution could be found with 3 modes in just 6 nonlinear iterations as can be seen in Fig. 2, showing the convergence diagram of the method. In Fig. 3 the norm 2 of the 3 terms representing the solution are shown. These values quantify the contribution of each term of the summation in the separated representation of the solution, in this example the solution could be represented with sufficient precision just by the first 2 terms. It is worth mentioning that, if N_q is the number of candidate positions where the DG can be positioned and N_d is the number of degrees of freedom for the voltage nodes, the volume of information necessary to represent the whole parametric space is proportional to $N_q \times N_r \times N_d \approx 6.3 \cdot 10^7$, whereas the same information can be approximated with great accuracy by $3(N_q + N_r + N_d) \approx 2.8 \cdot 10^3$ degrees of freedom with a reduction rate of about $2.25 \cdot 10^4$.

The normalized real part of the functions of the parameter r are shown in Fig. 4. Finally, as a post-process the evaluation of the system losses is carried out evaluating (8). The fully two dimensional representation of the losses is presented in Fig. 5 where we can observe that the minimum value of the losses are concentrated around the second branch. Optimization can be carried out with ease using any algorithm, since the objective function and its gradients are now explicitly available. In practice, the minimum loss is $1.93 \cdot 10^3 \text{ W}$ when the DG is set at the position *lb736* with power $4.6 \cdot 10^5 \text{ W}$.

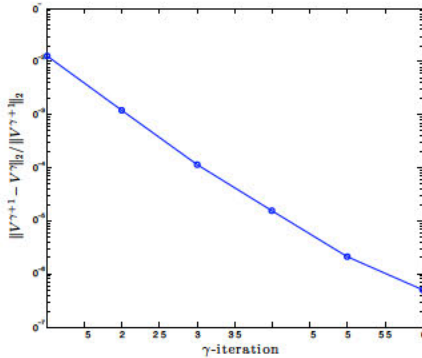


Fig. 2: Nonlinear PGD convergence diagram

B. Optimal positioning of three DG units with time varying loads

The goal in the last example is to minimize the power losses over a year when three DGs, one per branch, are set

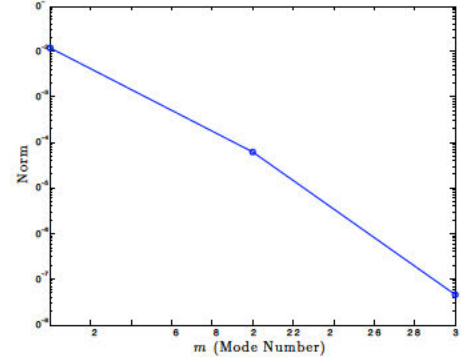


Fig. 3: Norm of the individual terms in the separated representation of V

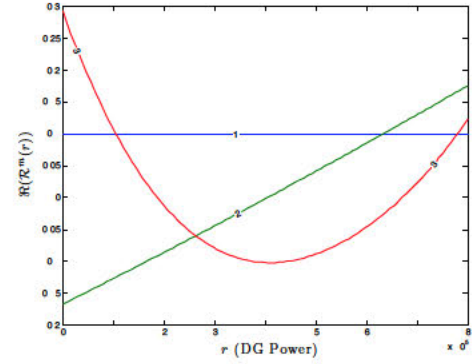


Fig. 4: Functions of the DG output active power r for V

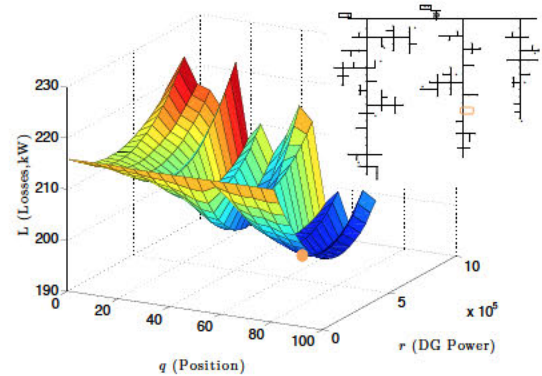


Fig. 5: Reconstructed System Losses

in the network. The parameters associated to the power flow problems are the position of the DGs q_1, q_2 and q_3 , the active power of the DGs r_1, r_2 and r_3 and the time t . Considering also the nodal position where the voltage is computed, that is the physical coordinate of the system, the parametric problem is eight-dimensional. The representation of the solution does not always have to deal with one-dimension separated modes. Occasionally, considering functions that depend on more than one parameter is also efficient. In particular, although the problem is eight-dimensional the decomposition chosen is 4D-3D-1D as can be seen in the representation of the input S ,

$$\begin{aligned}
S(q_1, q_2, q_3, r_1, r_2, r_3, t) = & \mathcal{Q}^1(q_1)\mathcal{R}^1(r_1)\mathcal{T}^1(t) \\
& + \mathcal{Q}^2(q_2)\mathcal{R}^2(r_2)\mathcal{T}^2(t) + \mathcal{Q}^3(q_3)\mathcal{R}^3(r_3)\mathcal{T}^3(t) \\
& + \sum_{h=4}^{27} \check{Q}^h\check{R}^h(r_1, r_2, r_3)\check{T}^h(t).
\end{aligned} \quad (15)$$

For the sake of simplicity, the parameters q_1, q_2, q_3 varying from the positions 1 to 19 at each branch. Similarly, due to the characteristics of the first two branches, the variation of r_1 and r_2 is the same with $r_{max} = 4 \cdot 10^5 \text{W}$, while the maximum value for r_3 is $22 \cdot 10^5 \text{W}$ and N_r is also equals to 100. Expression (15), requires the load and generation profiles during a year. For this particular system, 24 load curves $\check{T}^h(t)$ were generated using the software HOMER described in [27]. The time parameter t is varying from 1 to $8760h$ with a time step of $1h$.

A very accurate solution can be represented with only 18 PGD modes in 6 iterations as can be seen in the convergence diagram of the nonlinear algorithm Fig. 6. In this case the tolerances are $\epsilon_{fp} = 10^{-8}$, $\epsilon_g = 10^{-7}$ and $\epsilon_n = 10^{-5}$. If N_t is the number of hours in a year, the whole parametric space is represented by a considerable amount of information $N_q \cdot N_r \cdot N_d \cdot N_t \approx 1.14 \cdot 10^{10}$, however the same information can be described by just $18(N_q + N_r + N_d + N_t) \approx 1.73 \cdot 10^5$ degrees of freedom, with a reduction of about 10^5 .

In order to be able to visualize the shape of the normalized functions representing the DG power, the position and the time, the first ten modes are shown in Fig. 8, 9 and 10. The objective function, that is the average of power losses during a year, can be computed by post-processing the voltage solution, see Fig. 7. The optimal position of the DGs are $la724$, $lb726$ and $lc726$ and the values of the power are $r_1 = 1.21 \cdot 10^5 \text{W}$ and $r_2 = 6.66 \cdot 10^5 \text{W}$.

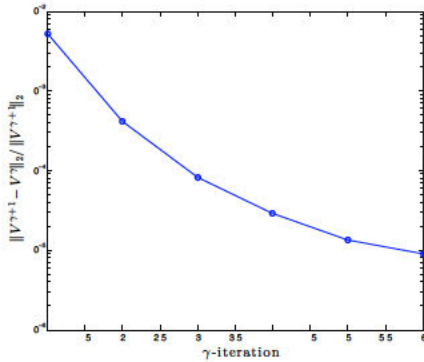


Fig. 6: Nonlinear PGD convergence diagram

V. CONCLUSIONS

This work presented the results obtained when applying a non-linear solver combined with the Proper Generalized Decomposition to the Parametric Power Flow problem for optimal location and sizing of distributed generation.

The “offline-online” approach shown allows to compute once and for all the solution in the “offline” phase while

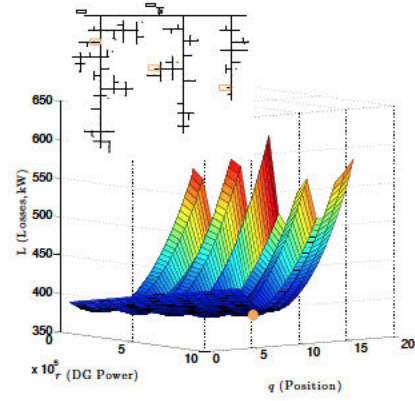


Fig. 7: Reconstructed System Losses

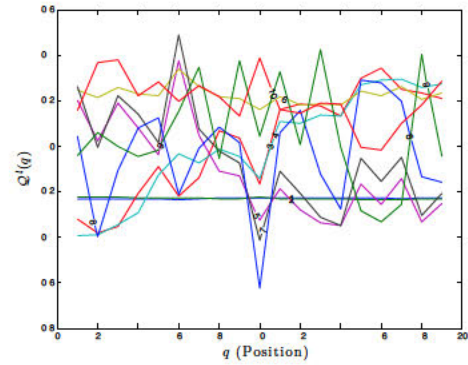


Fig. 8: Functions of the DG position q for L corresponding to the first 10 modes of the separated representation

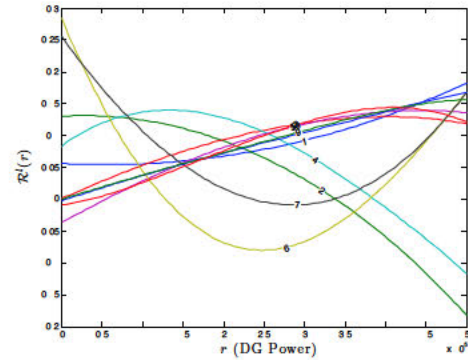


Fig. 9: Functions of the DG output active power r for L corresponding to the first 10 modes of the separated representation

optimization can be performed effortlessly in the “online” step, because of the objective function is explicitly available. The PGD technique allows to store the information associated to the resolution of a highly dimensional problem in a compressed separated variables format. Furthermore, the algorithm to compute solution itself scales linearly with the dimensionality of the problem, since each mode of the separated representation is computed individually.

From the obtained results, it is possible to see how the

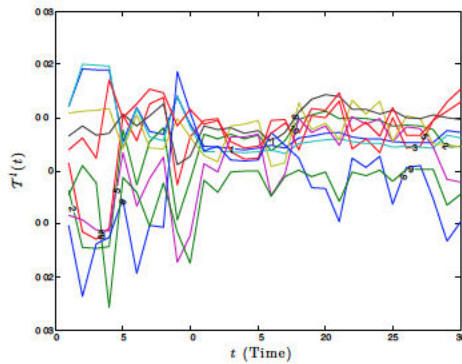


Fig. 10: Functions of the time t for L corresponding to the first 10 modes of the separated representation

reduction rate becomes evident as the dimensionality of the problem increases. In the eight dimensional problem, the objective function is computed in the whole parametric space with a reduction rate of about $1.73 \cdot 10^5$.

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