Numerical modelling of landslides in reservoirs using the Particle Finite Element Method and a non-Newtonian Bingham model

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• Why are landslides important?
  – Third most dangerous natural risk
  – Special interest of landslides in reservoirs

• To many classifications

• Rapid landslides
  – Debris flows and mudflows
  – Granular avalanches
  – Sliding flows
LANDSLIDES OVERVIEW

Mudflow

Sliding flow

Avalanche

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PFEM (Particle Finite Element Method): numerical technique for solving fluid-soil-structure interaction problems involving large motion.

- PFEM uses an updated Lagrangian formulation to model the motion of particles (nodes).
- The mesh must be generated in each timestep to solve the governing equations in the standard FEM fashion.
- The PFEM is particularly suited for modelling and simulation of landslides.
PFEM OVERVIEW

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PFEM OVERVIEW

1. Loop over time steps, \( t = 1, N_{\text{TIME}} \)
   Known values: \( \{\bar{x}, \bar{v}, \bar{\rho}, \bar{\pi}, \bar{T}, \bar{\mu}, f, q, C, V, M\) 

2. Loop over number of iterations, \( i = 1, N_{\text{ITER}} \)
   - Compute nodal velocities by solving \( \left[ \frac{1}{\Delta t} M + K \right]^{t+1}_{i+1} \bar{v}^{t+1} = \delta^{t+1}_{i+1} f + f^{t+1}_{i+1} \bar{p} + \frac{1}{\Delta t} M^{t+1}_i \bar{v} \)
   - Compute nodal pressures from \( \left[ \frac{1}{\Delta t} - LM \right]^{t+1}_{i+1} \bar{p}^{t+1} = G^{t+1}_{i+1} \bar{v}^{t+1} + Q^{t+1}_{i+1} \bar{\pi} + \frac{1}{\Delta t} M^{t+1}_i \bar{p} \)
   - Compute nodal pressure gradient projections from \( \bar{n}^{t+1}_{i+1} \bar{\pi}^{t+1} = -\hat{M}^{-1}_{D} \bar{Q}^{t+1}_{i+1} \bar{p}^{t+1} \), \( \hat{M}_{D} = \text{diag}[\hat{M}_{D}] \)

   Update positions of all solid domain nodes \( \bar{x}^{t+1} = \bar{x}^{t} + \Delta t \bar{v} \)

Define new cloud of nodes \( t^{+1}C^{t+1} \)
   - Update strain rate and strain values
   - Update stress values

Check convergence: NO \( \rightarrow \) Next iteration \( \rightarrow i = i + 1 \)
   YES \( \rightarrow \) Next time step \( \rightarrow t = t + 1 \)

New time step
   - Identify new analysis domain boundary: \( t^{+1}V \)
   - Generate mesh: \( t^{+1}M \)

Go to 1
Symmetric part of the velocity gradient:

\[
(\nabla^S \mathbf{u})_{kl} = \frac{1}{2} \frac{\partial \mathbf{u}_k}{\partial x_l} + \frac{\partial \mathbf{u}_l}{\partial x_k}
\]
Mudflows → Viscoplastic material

One-dimensional apparent viscosity:

\[ \tilde{\mu}(\dot{\gamma}) = \frac{\tau}{\dot{\gamma}} \]
Constitutive Model

Shear Stress

\[ \tau_0 \]

Rate of Strain

Bingham Plastic

Newtonian Fluid

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CONSTITUTIVE MODEL

\[
\tau = \left[ \mu + \frac{\tau_0}{\dot{\gamma}} \left( 1 - e^{-m\dot{\gamma}} \right) \right] \dot{\gamma}
\]

\[
\tilde{\mu}(\dot{\gamma}) = \mu + \frac{\tau_0}{\dot{\gamma}} \left( 1 - e^{-m\dot{\gamma}} \right)
\]
• **Laboratory experiments**

• **Lituya 2D**

• **Vajont 2D**
### 2D Validation

**Sælevick et al. laboratory experiments**

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Height [m]</th>
<th>Total length [m]</th>
<th>Impact velocity [m/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.16</td>
<td>2</td>
<td>2.45</td>
</tr>
<tr>
<td>2</td>
<td>0.16</td>
<td>1</td>
<td>3.38</td>
</tr>
<tr>
<td>3</td>
<td>0.12</td>
<td>1.6</td>
<td>3.56</td>
</tr>
</tbody>
</table>
(*)The gap between blocks was considered of 0.05 m.
Scenario 2
2D VALIDATION

Numerical - Experimental

\[
\frac{\eta}{d}
\]

\[
t \cdot \sqrt{\frac{g}{d}}
\]

Numerical - Experimental

\[
\frac{\eta}{d}
\]

\[
t \cdot \sqrt{\frac{g}{d}}
\]
2D VALIDATION

\[ \frac{\eta}{d} \]

---

Numerical - Experimental

- $t \cdot \sqrt{g/d}$

---

\[ \frac{\eta}{d} \]

---

Numerical - Experimental

- $t \cdot \sqrt{g/d}$
2D VALIDATION

Scenario 1
Scenario 1 (deceleration)
2D VALIDATION

Numerical without deceleration — Numerical with deceleration — Experimental

\( \frac{\eta}{d} \) vs. \( t \cdot \sqrt{\frac{g}{d}} \)

G1, G2, G3

Numerical without deceleration — Numerical with deceleration — Experimental

\( \frac{\eta}{d} \) vs. \( t \cdot \sqrt{\frac{g}{d}} \)

G1, G2, G3
### 2D Validation

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Run</th>
<th>Difference in maximum wave height (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>WG1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>28.6</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>3.6</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>4.8</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>0.0</td>
</tr>
</tbody>
</table>
2D VALIDATION

Lituya 2D

Gilbert Inlet

wave gauge at x = 885 m

1342 m

NE

SW

524 m

915 m

970 m

92 m

610 m

230 m

0 m

-122 m
2D VALIDATION

Fritz et al.

Quecedo et al.

PFEM

Mesh size: 10 m
2D VALIDATION
<table>
<thead>
<tr>
<th></th>
<th>Measured (Fritz)</th>
<th>Quecedo et al.</th>
<th>PFEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slide duration (s)</td>
<td>7</td>
<td>9</td>
<td>9.5</td>
</tr>
<tr>
<td>Slide velocity at impact (m/s)</td>
<td>110</td>
<td>85</td>
<td>83</td>
</tr>
<tr>
<td>Slide length at impact (m)</td>
<td>748</td>
<td>1092</td>
<td>1052</td>
</tr>
<tr>
<td>Maximum wave height (m)</td>
<td>&gt;200</td>
<td>226</td>
<td>234</td>
</tr>
<tr>
<td>Time for maximum wave (s)</td>
<td>11</td>
<td>21</td>
<td>26</td>
</tr>
<tr>
<td>Maximum wave position (m)</td>
<td>600</td>
<td>600</td>
<td>814</td>
</tr>
<tr>
<td>Maximum wave height at 885 m</td>
<td>152</td>
<td>266</td>
<td>232</td>
</tr>
<tr>
<td>Time for maximum wave at 885 m (s)</td>
<td>16</td>
<td>26.8</td>
<td>27.1</td>
</tr>
</tbody>
</table>
2D VALIDATION

Vajont 2D

Before the landslide

- Failure surface of the landslide: 4/11/1960
- Isolated hill
- T. Vaiont
- Deposits of the old T. Vaiont
- Surface of rupture of the paleolandslide
- Surface of rupture of the new landslide

South

1200-
1100-
1000-
900-
800-
700-
600-
500-

North
2D VALIDATION

Vajont 2D

After the landslide

- South
- North
- T. Vajont (pre landslide)
- Isolated hill (new position)
- Debris post landslide
- Rupture surface of the old landslide
- Rupture surface of the new landslide

Dimensions: 720.0x540.0
2D VALIDATION

Other data:

• Density: 1800 kg/m$^3$
• Maximum velocity: 28 m/s
• The landslide mass ceased after 30-40 s
2D VALIDATION
Calibration campaign 1

- Used data:
  - Yield stress (kPa): 15, 150, 300
  - Viscosity (Pa·s): 50, 500, 5000
  - m: 50, 500, 5000

- Results: no simulation provides adequate results

- Conclusion: yield stress is the most affecting parameter. It should be between 150 and 300 kPa
Calibration campaign 2

• Used data:
  – Yield stress (kPa): 250
  – Viscosity (Pa·s): 500
  – m: 50, 500, 5000

• Results: all simulations provide good results

• Data chosen: Yield stress 250 kPa, Viscosity 500 Pa·s and m 500
2D VALIDATION
Lituya 3D
3D SIMULATION
Landslides can behave as newtonian and non-newtonian fluids

Complex phenomenon

Numerical models must apply simplifications so results are approximations to reality

PFEM can be useful in the estimation of possible affections caused by landslides in reservoirs

PFEM is effective reproducing landslides as a Bingham fluid
FURTHER WORK

• Further two and three-dimensional calibrations and simulations will be developed (Vajont 3D)
• Non-homogeneous landslide simulations
• Simulations of potential landslides
• Code optimization
THANK YOU FOR YOUR ATTENTION