

# ANALYTICAL AND NUMERICAL INVESTIGATION OF SANDWICH BEAMS WITH ADDITIVELY MANUFACTURED LATTICE CORES AND COMPOSITE FACESHEETS

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**Key words:** Additive manufacturing (AM), Lattice Structures, Composite Materials, Sandwich Structures, Lightweight Design

**Abstract.** This paper presents a simulation-based study of hybrid sandwich structures composed of composite facesheets and additively manufactured AlSi10Mg lattice cores. Analytical predictions using First-Order Shear Deformation Theory (FSDT) are validated through finite element simulations performed in ANSYS under three-point bending conditions. Homogenized models of lattice cores are developed to significantly reduce computational cost while maintaining mechanical accuracy. The results demonstrate strong agreement between analytical and numerical models in terms of deformation and stress distributions, confirming the suitability of FSDT for lightweight lattice-core sandwich analysis. The study emphasizes the role of homogenization in accelerating simulation-driven design and provides insights for the optimization of AM-based sandwich components in aerospace and automotive applications.

## 1 INTRODUCTION

Sandwich structures are widely used in aerospace, automotive, and civil engineering applications due to their high specific stiffness and strength, lightweight nature, and excellent energy absorption capacity. A sandwich structure typically consists of two strong outer facesheets and a core (Figure 1). The facesheets primarily carry in-plane and bending loads, whereas the core provides shear stiffness and stabilizes the facesheets against buckling [1–3].

The rapid development of additive manufacturing (AM) has enabled the production of lattice cores with highly complex geometries that were previously unachievable using conventional manufacturing methods. These architected lattices can be tailored for superior stiffness-to-weight ratios and enhanced energy absorption [4–6]. Among AM techniques, Laser Powder Bed Fusion (LPBF) allows precise fabrication of metallic lattice cores (e.g., AlSi10Mg) with intricate strut arrangements, offering significant design flexibility for lightweight structures [7, 8].

Recent studies have highlighted the mechanical advantages of integrating AM-produced lattice cores in sandwich panels. Experimental and modeling works report higher stiffness and load-carrying capacity compared with conventional honeycomb concepts and demonstrate reliable structural modeling strategies for strut-based cores [9, 10]. In parallel, homogenization-based approaches have been used to reduce computational cost while retaining mechanical fidelity, providing continuum-equivalent descriptions of periodic lattices [11–14].

In this context, the present study investigates a hybrid sandwich structure composed of Carbon Fiber Reinforced Polymer (CFRP) facesheets and an additively manufactured AlSi10Mg lattice core. The structural response under three-point bending is analyzed using both an analytical model based on First-Order Shear Deformation Theory (FSDT) and finite element simulations, following established sandwich mechanics [15]. Building upon the analytical and numerical framework of Dereli et al. [16], the study focuses on bending and shear behavior and assesses the influence of lattice aspect ratio on global stiffness and deformation.

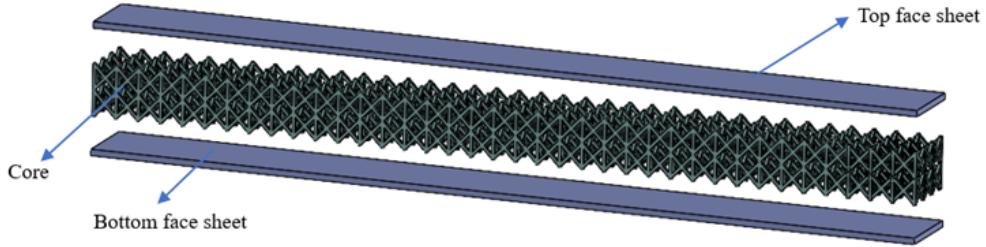


Figure 1: Components of a sandwich panel.

## 2 MATERIALS AND METHODS

### 2.1 Materials and geometry

The investigated sandwich beam consists of carbon-fiber-reinforced polymer (CFRP) facesheets and a numerically modeled AlSi10Mg lattice core concept. This configuration combines the high in-plane stiffness and low density of composite laminates with the geometric flexibility of metallic lattice architectures typically produced by Laser Powder Bed Fusion (LPBF). The CFRP facesheets were modeled as cross-ply laminates [0/90/90/0] to eliminate bending–shear coupling. The lattice core was numerically modeled with an  $f_{2ccz}$  topology, representative of LPBF-fabricated AlSi10Mg lattices. This structure consists of vertical and inclined struts forming a periodic and symmetric cell arrangement, providing enhanced shear stiffness and compressive stability compared to conventional lattice types. The geometry of each unit cell is defined by the edge length  $a$  and the strut thickness  $t$ , and their ratio determines the aspect ratio  $AR$ , as expressed by

$$AR = \frac{a}{t} \quad (1)$$

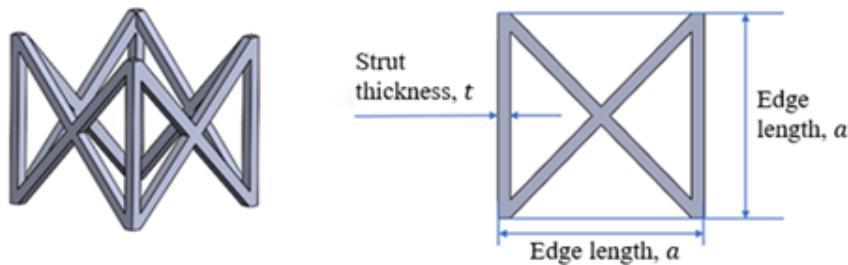


Figure 2:  $f_{2ccz}$  lattice core geometry.

In the present study, a unit-cell edge length of  $a = 4$  mm and aspect ratios of 6, 8, and 16 are considered. The mechanical properties of the CFRP and AlSi10Mg materials are summarized in Table 1a and Table 1b, respectively, and the schematic of the lattice configuration is shown in Figure 2.

(a) Mechanical properties of CFRP

Property	Value	Unit
$E_{xx}$	150,000	MPa
$E_{yy}$	9,000	MPa
$E_{zz}$	9,000	MPa
$\nu_{xy}$	0.34	-
$\nu_{yz}$	0.40	-
$\nu_{zx}$	0.34	-
$G_{xy}$	5,000	MPa
$G_{yz}$	5,000	MPa
$G_{zz}$	5,000	MPa

(b) Mechanical properties of AlSi10Mg

Property	Value	Unit
Yield strength	250	MPa
Modulus of elasticity	70,000	MPa
Shear modulus	25,925	MPa
Poisson's ratio	0.35	-
Density	0.0267	g/mm <sup>3</sup>

Table 1: Mechanical properties of the materials used in the analysis.

The beam was modeled under three-point bending conditions, with simple supports located at both ends of the span and a concentrated load applied at the mid-span (Figure 3). This setup enables simultaneous evaluation of global bending stiffness and core shear deformation effects. Simply supported boundary conditions were applied at both ends of the beam, restricting vertical displacement and rotation as appropriate for three-point bending.

The bending moment distribution along the left half of the beam is expressed as:

$$M_{xx} = -\frac{Px}{2b}, \quad 0 \leq x \leq \frac{L}{2} \quad (2)$$

The moment and force resultants acting on the sandwich beam can be written in matrix form as:

$$\bar{M} = \begin{bmatrix} M_{xx} \\ 0 \\ 0 \end{bmatrix}, \quad \bar{N} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (3)$$

where  $\bar{M}$  and  $\bar{N}$  are the bending and axial force resultants on the sandwich beam, respectively.

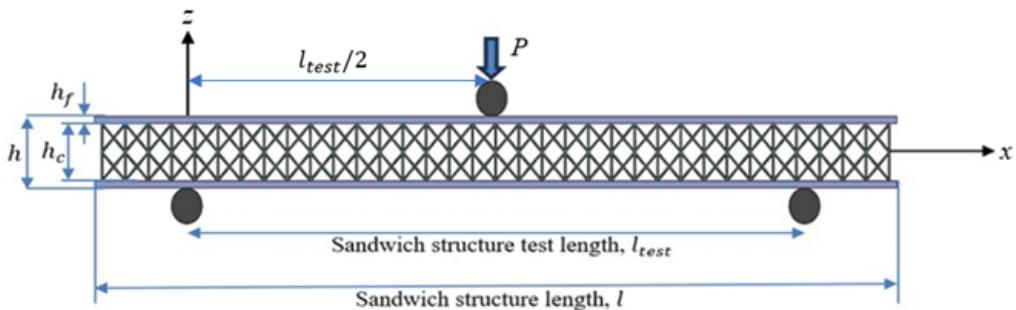


Figure 3: Lattice core model and overall dimensions

Table 2: Sandwich beam dimensions

Notation	Value	Description
$h$	10 mm	Sandwich beam thickness
$h_f$	1 mm	Facesheet thickness (each)
$h_c$	8 mm	Core thickness
$h_k$	0.25 mm	Lamina (layer) thickness
$b$	12 mm	Sandwich beam width
$l$	130 mm	Length of sandwich beam
$l_{\text{test}}$	100 mm	Test span (three-point bending)
Structure type	$f_{2CCZ}$	Atomic type

## 2.2 Homogenization

The mechanical response of the lattice core was characterized using a homogenization approach to represent its periodic microstructure as an equivalent continuum medium. This allows the discrete lattice geometry to be replaced by an orthotropic solid whose stiffness reflects the combined effects of the cell geometry and the base material, thus significantly reducing computational effort while maintaining structural accuracy. The schematic representation of this process is illustrated in Figure 4.

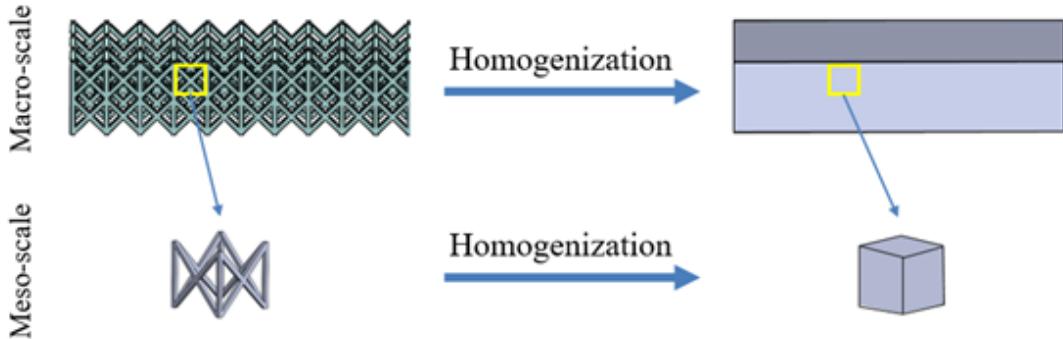


Figure 4: Homogenization schematic view for the lattice structures.

In this study, the lattice core exhibits orthotropic material characteristics, although the struts within the unit cell are isotropic. The effective elastic constants of the homogenized unit cell, including the moduli of elasticity  $E_{xx}^*$  and  $E_{zz}^*$ , the shear modulus  $G_{xz}^*$ , and Poisson's ratio  $\nu_{xz}^*$ , were computed through finite element simulations under uniaxial tension and shear loading conditions. The resulting expressions, similar to those proposed by Ashby for the  $f_{2CCZ}$  unit cell, are given as:

$$\frac{E_{xx}^*}{E_s} = 1.6 \left( \frac{a}{t} \right)^{-2}, \quad \frac{E_{zz}^*}{E_s} = 3.9 \left( \frac{a}{t} \right)^{-2}, \quad \frac{G_{xz}^*}{G_s} = 7.4 \left( \frac{a}{t} \right)^{-2}, \quad \nu_{xz}^* = -\frac{\Delta a_z}{\Delta a_x}, \quad \nu_{zx}^* = \nu_{xz}^* \frac{E_{zz}^*}{E_{xx}^*} \quad (4)$$

where  $E_s$  and  $G_s$  are the elastic and shear moduli of the lattice base material, respectively, as summarized in Table 1b.

### 2.3 Analytical approach

The analytical model of the sandwich beam is developed based on the First-Order Shear Deformation Theory (FSDT), following the formulation proposed by Carlsson and Kardomateas [15]. This theory enables the inclusion of transverse shear deformation, which is significant in sandwich structures with relatively soft cores.

The following assumptions are made in the formulation:

- The face sheets are thin compared to the core ( $h_f \ll h_c$ ) and sustain in-plane normal stresses, while transverse shear stresses in the faces are neglected.
- The in-plane normal and shear stresses in the core are negligible.
- The in-plane displacements  $u$  and  $v$  are uniform through the faces and vary linearly through the core thickness, see Figure 5.
- The transverse displacement  $w$  is independent of the  $z$ -coordinate.
- Continuity of displacements is maintained at the core–facesheet interfaces.

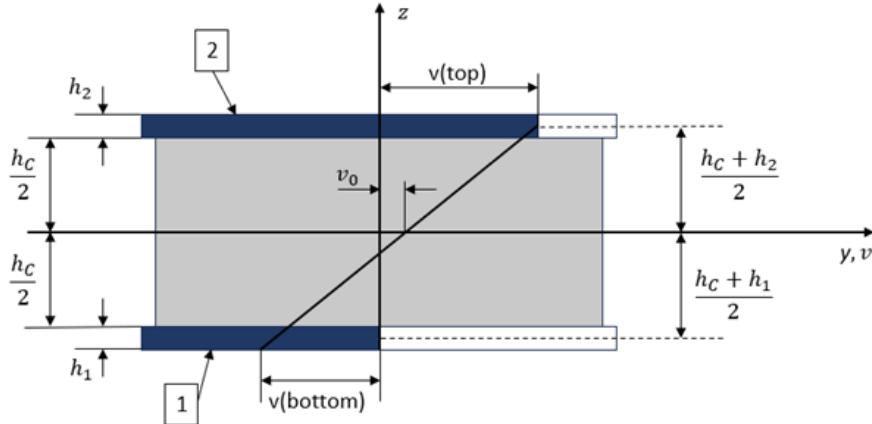


Figure 5: Cross-section of a sandwich structure with dimensions and displacements.

Based on these assumptions, the displacement field of the core mid-plane can be expressed as:

$$u = u_0(x, y) + z\psi_x(x, y), \quad v = v_0(x, y) + z\psi_y(x, y), \quad w = w_0(x, y) \quad (5)$$

where  $u_0$ ,  $v_0$ , and  $w_0$  are the mid-plane displacements, and  $\psi_x$ ,  $\psi_y$  denote rotations of the normal about the  $y$ - and  $x$ -axes, respectively.

The in-plane normal and shear strains in the facesheets are obtained from the above displacement field as:

$$\varepsilon_{xx} = \varepsilon_{xx}^0 + z\kappa_{xx}, \quad \varepsilon_{yy} = \varepsilon_{yy}^0 + z\kappa_{yy}, \quad \gamma_{xy} = \gamma_{xy}^0 + z\kappa_{xy} \quad (6)$$

The force and moment resultants are obtained by integrating the stresses across the thickness of the sandwich element:

$$\begin{bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{bmatrix} = \int_{-h_c/2}^{h_c/2} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{bmatrix} dz, \quad \begin{bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{bmatrix} = \int_{-h_c/2}^{h_c/2} z \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{bmatrix} dz, \quad \begin{bmatrix} Q_x \\ Q_y \end{bmatrix} = \int_{-h_c/2}^{h_c/2} \begin{bmatrix} \tau_{xz} \\ \tau_{yz} \end{bmatrix} dz \quad (7)$$

The transverse shear stress in the core is obtained by assuming that the entire transverse load is carried by the core:

$$\tau_{xz} = -\frac{P}{2bh_c} \quad (8)$$

The deflection of the sandwich beam under three-point bending can then be derived as:

$$\delta = \frac{Pl^3d_{11}}{48b} + \frac{Pl}{4h_c b G_{xz}} \quad (9)$$

where  $d_{11}$  is the bending compliance term and  $G_{xz}$  is the equivalent shear modulus of the core obtained from the homogenization procedure.

The analytical model provides a means to evaluate both the bending and shear contributions to the overall beam deflection, which are later compared to finite element results under identical boundary conditions.

## 2.4 Numerical analysis

A finite element (FE) study was conducted to validate the analytical model on sandwich beams with both homogenized and lattice cores, as illustrated in Figure 6. Two FE models were developed in ANSYS under a three-point bending configuration consistent with the analytical setup. The geometries were created in SolidWorks and imported into ANSYS for static structural analysis.

A force-controlled static analysis ensured equilibrium between internal and external forces, with force convergence used as the accuracy criterion. Identical material properties, boundary conditions, and load magnitudes were applied to both models to enable direct comparison. The lattice model required finer meshing due to its geometric complexity, while the homogenized core was meshed using regular quadrilateral elements.

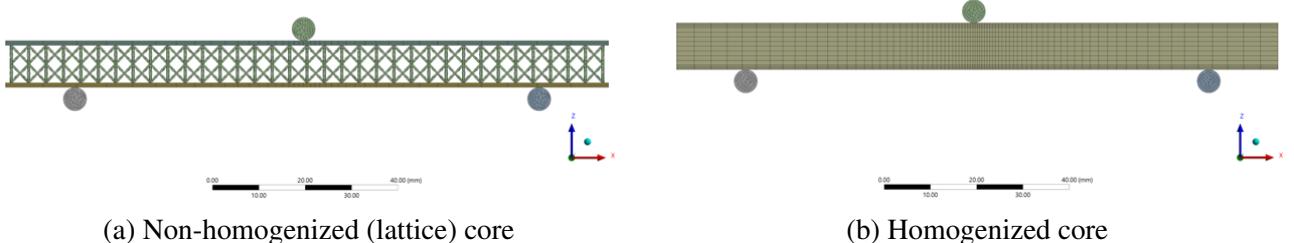


Figure 6: Finite element models of sandwich beams with (a) non-homogenized lattice core and (b) homogenized core under three-point bending.

Boundary conditions included simple supports at both ends of the beam, a concentrated load at mid-span, and frictional contact between the beam, supports, and loading fixture. The resulting FE models allowed a consistent comparison between analytical and numerical results, providing insight into the influence of core representation (lattice vs. homogenized) on global stiffness, deflection, and stress distribution.

## 3 RESULTS

This section presents the deformation and stress behavior of sandwich beams with both lattice (non-homogenized) and homogenized cores subjected to three-point bending. The results obtained

from the Finite Element (FE) simulations are compared with the analytical predictions based on the First-Order Shear Deformation Theory (FSDT). The primary focus is to evaluate the effect of lattice aspect ratio (AR) and homogenization on the global stiffness, deformation, and stress distributions.

### 3.1 Deformation behavior

Figure 7 illustrates the deformation profiles for the non-homogenized and homogenized sandwich beams. The maximum deflection occurs at the beam's mid-span, consistent with theoretical predictions. The homogenized model exhibits slightly higher deflection compared to the lattice model (about 3% difference), which is attributed to the continuous material representation of the core. Despite this simplification, the deformation pattern closely matches that of the actual lattice structure, demonstrating that homogenization accurately captures the global stiffness behavior.

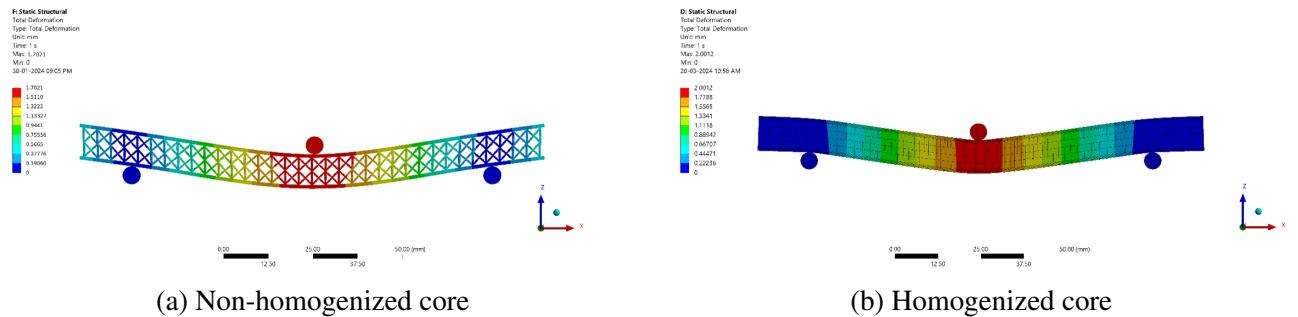


Figure 7: Deformation comparison of (a) non-homogenized and (b) homogenized sandwich beams under three-point bending.

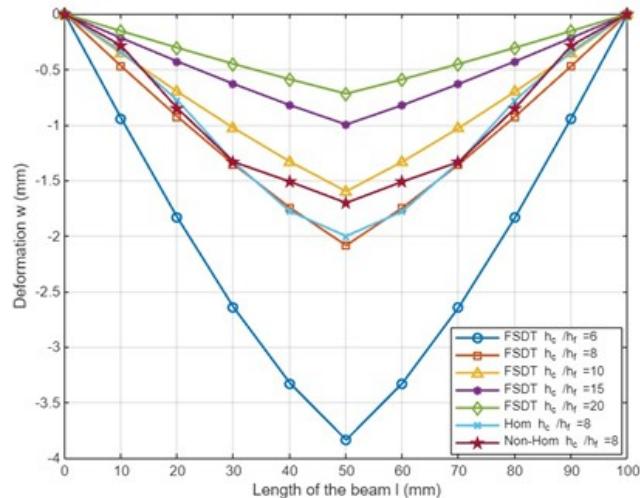


Figure 8: Deformation comparison of non-homogenized (a) and homogenized (b) sandwich beams under three-point bending.

The influence of the core-to-facesheet thickness ratio ( $h_c/h_f$ ) on deflection is shown in Figure 8. Increasing the core thickness leads to higher bending stiffness, reducing total deflection. However, beyond a certain ratio, the improvement diminishes due to shear deformation effects becoming dominant. This observation aligns well with the FSDT assumptions, where thicker cores contribute to increased shear stiffness but also greater shear deformation.

### 3.2 Normal stress distribution

Figure 9 compares the normal stress distributions in the lattice and homogenized cores. In both models, the maximum compressive stress appears on the top facesheet, while the maximum tensile stress occurs on the bottom facesheet. The stress pattern is nearly symmetrical and consistent with bending theory. The homogenized model exhibits slightly smoother stress gradients due to the continuum representation of the core, whereas the lattice model reveals localized peaks at strut junctions.

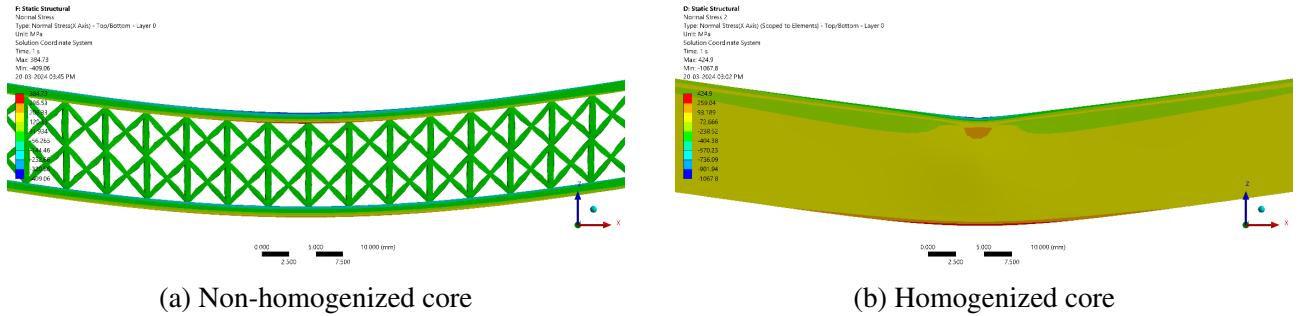


Figure 9: Normal stress distribution in sandwich beams: (a) non-homogenized and (b) homogenized core models under three-point bending.

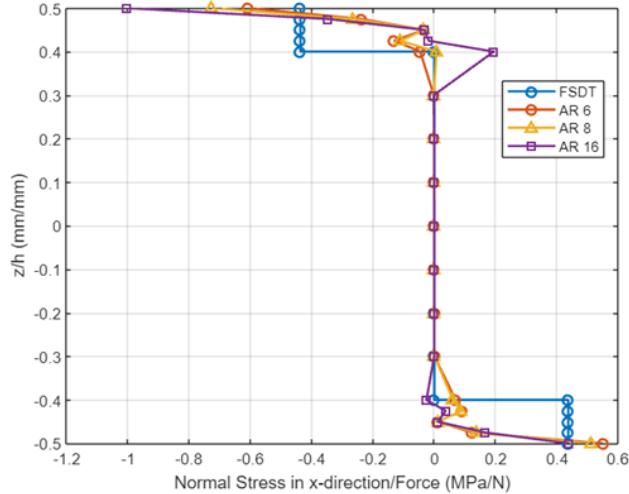


Figure 10: Normalized normal stress distribution through the sandwich thickness for different aspect ratios.

Figure 10 illustrates the normalized normal stress distribution through the sandwich thickness for different aspect ratios (AR6, AR8, AR16). According to FSDT, the geometric and material properties of the facesheets mainly determine the normal stresses, while the core stiffness has little direct influence. However, FEM results indicate that variations in the core's mechanical properties significantly affect the normal stress distribution. Among the analyzed cases, AR16 exhibits the closest agreement with FSDT results, confirming that the assumption  $E_c \ll E_f$  remains valid for low modulus ratios.

### 3.3 Shear stress analysis

The shear stress profiles ( $\tau_{xz}$ ) obtained for the lattice and homogenized cores are shown in Figure 11. The lattice core reveals localized shear peaks along strut intersections, while the homogenized model shows a smoother, nearly linear distribution consistent with FSDT assumptions. The homogenized approach effectively represents the global shear transfer mechanism, despite neglecting the fine-scale stress localization observed in the lattice model. In addition, the normalized shear stress distribution through the sandwich thickness, presented in Figure 12, further validates this trend, where the FSDT and homogenized core results converge closely, especially near the neutral axis, confirming the applicability of homogenization in predicting overall shear behavior.

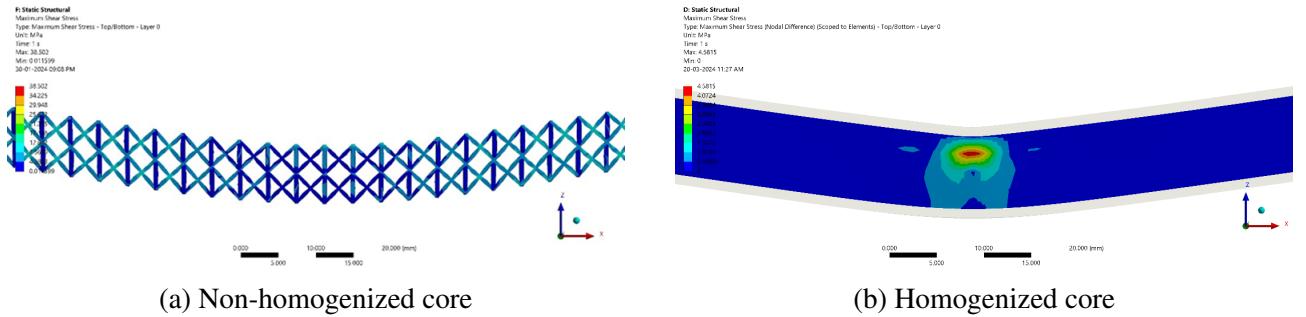


Figure 11: Shear stress distribution comparison of (a) non-homogenized and (b) homogenized sandwich cores under three-point bending.

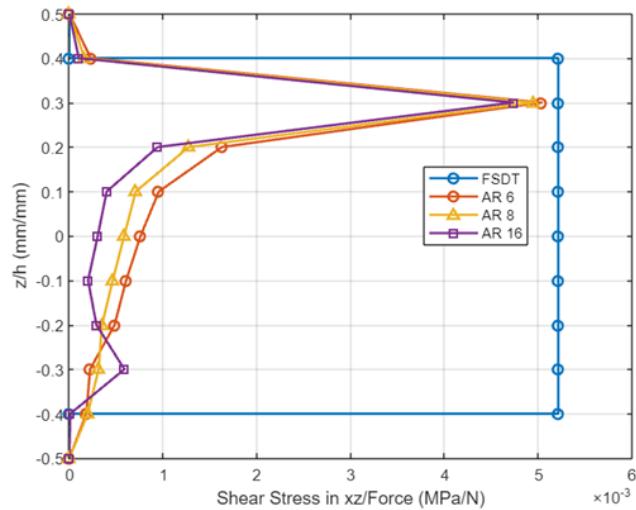


Figure 12: Normalized shear stress distribution through the thickness for different aspect ratios.

Overall, the FE results confirm that homogenization provides a computationally efficient yet accurate means of predicting deformation and stress fields in lattice core sandwich structures. The close agreement with FSDT validates its applicability for structural design, with deviations mainly attributed to local effects not captured by continuum models.

## 4 CONCLUSIONS

This study presents both analytical and numerical investigations on a hybrid sandwich structure consisting of a composite facesheet and an additively manufactured lattice core. The mechanical performance was evaluated under three-point bending using the First-Order Shear Deformation Theory (FSDT) and Finite Element Method (FEM). The primary findings of the study can be summarized as follows:

- **Shear-dominated deformation in compliant cores:** In structures with compliant cores, deformation predominantly occurs due to shear rather than bending. This behavior highlights the necessity of considering shear effects in the design and analysis of lattice-based sandwich structures, where the relative softness of the core plays a decisive role in global deformation.
- **Consistency between FSDT and FEM analyses:** The results show strong agreement between the analytical FSDT predictions and FEM simulations. This validates the applicability of FSDT for modeling the mechanical response of lattice core sandwich panels, offering a computationally efficient alternative to detailed finite element modeling, particularly during early design stages.
- **Limitations and future scope:** Despite its efficiency, FSDT assumes uniform core behavior and may not fully capture the influence of varying core material properties or complex lattice geometries. Future work should focus on extending the analytical approach using higher-order theories to more accurately represent the mechanical response of anisotropic or graded lattice cores.

Overall, the correlation between analytical and numerical approaches confirms that the homogenization-based FSDT framework effectively predicts the mechanical performance of sandwich structures with additively manufactured cores, while substantially reducing computational effort.

## References

- [1] Zenkert D., (1995) *An introduction to sandwich construction*. Engineering Materials Advisory Services.
- [2] Reddy J.N., (2003) *Mechanics of laminated composite plates and shells: theory and analysis*. CRC Press.
- [3] Kaw A.K., (2005) *Mechanics of composite materials*. CRC Press.
- [4] Benedetti M., Plessis D., Ritchie R.O., Dallago M., Razavi N., Berto F., (2021) *Architected cellular materials: A review on their mechanical properties toward fatigue-tolerant design and fabrication*. *Materials Science and Engineering R: Reports*, 144, 100606.
- [5] Özdemir M., Şimşek Ü., Kızıltaş G., Gavir C.E., Çelik A., Şendur P., (2023) *A novel design framework for generating functionally graded multi-morphology lattices via hybrid optimization and blending methods*. *Additive Manufacturing*, 70, 103560.
- [6] Al-Saedi S.H., Masood S., Ur-Rab M.F., Alomarrah A., Ponnusamy P., (2018) *Mechanical properties and energy absorption capability of functionally graded F2BCC lattice fabricated by SLM*. *Materials & Design*, 144, 32–44.

- [7] Bühring J., Nuño M., Schröder K.U., (2021) *Additive manufactured sandwich structures: Mechanical characterization and usage potential in small aircraft*. *Aerospace Science and Technology*, 117, 106548.
- [8] Georges H., Großmann A., Mittelstedt C., Becker W., (2022) *Structural modeling of sandwich panels with additively manufactured strut-based lattice cores*. *Additive Manufacturing*, 52, 102788.
- [9] Georges H., Garcia Solera D., Aguilar Borasteros C., Metar M., Song G., Mandava R., Becker W., Mittelstedt C., (2024) *Mechanical performance comparison of sandwich panels with graded lattice and honeycomb cores*. *Biomimetics*, 9(2), 96.
- [10] Ghanadpour A.M., Mahmoudi M., Nejad K.H., (2022) *Structural behavior of 3D-printed sandwich beams with strut-based lattice core: Experimental and numerical study*. *Composite Structures*, 281, 115113.
- [11] Souza J., Großmann A., Mittelstedt C., (2018) *Micromechanical analysis of the effective properties of lattice structures in additive manufacturing*. *Additive Manufacturing*, 23, 53–69.
- [12] Demiray S., Becker W., Hohe J., (2005) *Strain-energy based homogenisation of two- and three-dimensional hyperelastic solid foams*. *Journal of Materials Science*, 40, 5839–5844.
- [13] Georges H., Meyer G., Mittelstedt C., Becker W., (2023) *2D Elasticity solution for sandwich panels with functionally graded lattice cores*. *Composite Structures*, 320, 116045.
- [14] Osmanoglu S., Mittelstedt C., (2024) *Global buckling response of sandwich panels with additively manufactured lattice cores*. *Journal of Sandwich Structures & Materials*, 26(8), 1569–1593. doi:10.1177/10996362241282854.
- [15] Carlsson L.A., Kardomateas G.A., (2011) *Structural and failure mechanics of sandwich composites*. Springer.
- [16] Dereli E., Mbendou J., Patel V., Mittelstedt C., (2024) *Analytical and numerical analysis of composite sandwich structures with additively manufactured lattice cores*. *Journal of Composite Materials*, 100484. doi:10.1016/j.jcomc.2024.100484.