Runge Kutta (ELDIRK) methods for embedding of low order implicit time integration schemes for goal oriented global error estimation

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ABSTRACT

Low order implicit time integration schemes play a key role for time integration in several fields of computational mechanics, such as for the heat equation or inelastic constitutive equations, respectively. Embedded Runge-Kutta (RK) methods provide an attractive methodology by means of an adaptive time step size control. According to Fehlbergs suggestion, only *one* extra function calculation is required to estimate the local error of the embedded method. In the present paper, this methodology is applied to several prominent low order implicit RK-schemes, such as the first order implicit Euler-method, the second order trapezoidal rule and the second order Ellsiepen method. Its advantages are stability and comparatively low computational cost, however, they require the solution of a nonlinear system of equations. This paper presents a general approach for the construction of third order Runge-Kutta methods by embedding the above mentioned implicit schemes into the class of ELDIRK-methods. These will be defined to have an explicit last stage in the general Butcher array of Diagonal Implicit Runge-Kutta (DIRK) methods, with the consequence, that no additional system of equations must be solved. The main results – valid also for non-linear ordinary differential equations – are as follows: Two extra function calculations are required in order to embed the implicit Euler-method and one extra function calculation is required for the trapezoidal-rule and the Ellsiepen method, in order to obtain the third order properties, respectively. The methodology is applied to two different goal functions in terms of the standard global error, that is, a time point goal function and a time integrated goal function. Two numerical examples are concerned with a parachute with viscous damping and a two-dimensional laser beam simulation. Here, we verify the higher order convergence behaviours of the proposed new ELDIRK-methods, and its successful performances for asymptotically exact global error estimation of so-called reversed embedded RK-method are shown.

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