# Application-specific inverse identification for soft tissue biomechanics


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## ABSTRACT


The mechanical simulation of soft biological tissue response is challenging in many aspects. In contrast to engineering materials, experimental data for soft tissue is scarce, lacks well-established measurement protocols suffer from low repeatability and large spread. Moreover, the material is heterogeneous in space and several different geometrical scales are involved, making classical phenomenological constitutive models harder to fit to experimental data. Thus, in computational biomechanics, it is important to consider the error in the identified material parameter along the usual finite element discretization error in simulations.

We shall in this contribution study the identification of material parameters for a constitutive model in relation to subsequent simulation where the identified parameters are used. A two-step computation consisting of (i) a parameter identification problem where the identified parameters are used as inputs in (s) a subsequent simulation. Hence, in the result from the simulation in (s), we can expect two sources of error; the classical discretization error arising from the introduction of an FE-approximation in (s), and errors in the identified parameters stemming from (i).

In abstract notation, we consider the following problem:

$$\text{(i)}: \qquad p = \arg\left[\min_{q\in P} \mathcal{F}(q, u^{(\mathrm{i})})\right], \quad \text{subject to} \quad a^{(\mathrm{i})}(q, u^{(\mathrm{i})}, v) = l^{(\mathrm{i})}(v)\ \forall v \in V^{(\mathrm{i})} \tag{1}$$

$$\text{(s)}: \qquad a^{(\mathrm{s})}(p, u^{(\mathrm{s})}, v) = l^{(\mathrm{s})}(v)\ \forall v \in V^{(\mathrm{s})}, \quad \text{giving} \quad \mathcal{Q}(u^{(\mathrm{s})}). \tag{2}$$

In the parameter identification problem (i), the parameter $p \in P$ is determined from some sort of physical experiment. For example, $p$ may represent a set of discrete material parameters, or a spatially varying function describing inhomogeneous material properties. The experimental setup is described by the state equation $a^{(\mathrm{i})}(p, u^{(\mathrm{i})}, v) = l^{(\mathrm{i})}(v)$ where the response $u^{(\mathrm{i})}$ depends on $p$. The objective function $\mathcal{F}$ is typically a least-squares functional measuring the discrepancy between the computed response and the corresponding (experimentally) observed response. The parameter $p$ that minimizes $\mathcal{F}$ is used in the subsequent computation (s) to compute the goal quantity $\mathcal{Q}(u^{(\mathrm{s})})$ (sometimes referred to as "quantity of interest") via the solution of the state equation $a^{(\mathrm{s})}(p, u^{(\mathrm{s})}, v) = l^{(\mathrm{s})}(v)$.

As first step, we introduce FE-approximations for the state variables in standard fashion, denoted $u_h^{(\mathrm{i})} \in V_h^{(\mathrm{i})} \subset V^{(\mathrm{i})}$ and $u_h^{(\mathrm{s})} \in V_h^{(\mathrm{s})} \subset V^{(\mathrm{s})}$, respectively. Moreover, we can also introduce a similar approximation of the parameter as $p_h \in P_h \subset P$ such that the discretized version of the system reads

$$\text{(i)}: \qquad p_h = \arg\left[\min_{q_h\in P_h} \mathcal{F}(q_h, u_h^{(\mathrm{i})})\right], \quad \text{subject to} \quad a^{(\mathrm{i})}(q_h, u_h^{(\mathrm{i})}, v) = l^{(\mathrm{i})}(v)\ \forall v \in V_h^{(\mathrm{i})} \tag{3}$$

$$\text{(s)}: \qquad a^{(\mathrm{s})}(p_h, u_h^{(\mathrm{s})}, v) = l^{(\mathrm{s})}(v)\ \forall v \in V_h^{(\mathrm{s})}, \quad \text{giving} \quad \mathcal{Q}(u_h^{(\mathrm{s})}). \tag{4}$$

The interpretation (and motivation) of $P_h \subset P$ depends on the specific application. For example, if $p$ represents an arbitrary function describing a spatial inhomogeneity, we may introduce an

approximation $p_h$ of FE-type in terms of nodal values and basis functions. Another example is that of a model hierarchy, where a simple material model such as the Neo-Hooke model ($P_h$) is obtained by a suitable model restriction of the more general Ogden model ($P$). We remark that even in the case without explicit approximation of the parameters, i.e. $P_h = P$, we still get an error $e_p = p - p_h$ due to the discretization $V_h^{(\mathrm{i})} \subset V^{(\mathrm{i})}$. From the viewpoint of the simulation (s) with the goal quantity $\mathcal{Q}$, we can perceive the error $e_p = p - p_h$ as a model error, while $e_u = u^{(\mathrm{s})} - u_h^{(\mathrm{s})}$ is the "classical" discretization error.

We employ the framework of goal-oriented a posteriori error estimation based on a dual problem to estimate the error $\mathcal{E} := \mathcal{Q}(u^{(\mathrm{s})}) - \mathcal{Q}(u_h^{(\mathrm{s})})$, which is elaborated for the identification problem by e.g. Meidner and Vexler [1]. With the aid of the pertinent dual solutions and residuals, we can trace the error contributions to the different approximations defined by $V_h^{(\mathrm{i})}$, $V_h^{(\mathrm{s})}$ and $P_h$, respectively, which is elaborated in Johansson [2]. To illustrate the concepts a numerical example concerning soft tissue mechanics under wide range of strain rates will be studied.

## REFERENCES

[1] Meidner, D. and Vexler, B. Adaptive space-time finite element methods for parabolic optimization problems *SIAM J. Control Optim.*, Vol. **46**, pp. 116–142, 2007.

[2] Johansson, H. and Larsson, F. and Runesson, K. Application-Specific Error Control for Parameter Identification problems. *Int. J. Numer. Meth. Biomed. Engng.*, Vol. **27**, pp. 608–618, (2011).