Note: Galois theory "in a nutshell"

Antonio Lechuga, PhD.

A condense vision on Galois group of an equation.
Let be the equation,

$f(t) = t^4 - 4t^2 - 5 = 0$
$whose\ roots\ are,\ \alpha, \beta, \gamma, \delta$
Equation coefficients belong to Q
$The\ enlarged\ field\ is\ Q(\alpha, \beta, \gamma, \delta)$

Taken into account the symmetry equations of
the roots (Newton) we write:
$\alpha^2 + 1 = 0, \alpha + \beta = 0, \delta^2 - 5 = 0, \gamma + \delta = 0, \beta^2 + 1 = 0,$
$\gamma^2 + 1 = 0, \beta\gamma - \alpha\delta = 0, \alpha\gamma - \beta\delta = 0, \gamma^2 - 5 = 0$

The Galois group is, $G = (I, R, S, T)$
$R = (\alpha\beta)$
$S = (\gamma\delta)$
$T = (\alpha\beta)(\gamma\delta)$
$I = I$
$G \supset (I, R)$

An equation is solvable when the Galois group can be put
as the sum of a subgroup of G and the multiplication of this subgroup by one
or several automorfisms of G.

The split ought to be normal or proper(Galois).That means that,
$G = U + S_1U$,and besides, $G = U + US_1$ In our case, given G, U is the subgroup
and T is the automorfism.
$Si,\ U = (I, R)\ y\ S_1 = T = (\alpha\beta)(\gamma\delta)$
$We\ will\ write,$

$G = U + S_1U$
and
$G = U + US_1$
That completes the example.