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Design-Space Dimensionality Reduction in Structural Optimization via Parametric Model Embedding

M. Diez[1,*], R. Pellegrini[1], A. Serani[1], and F. Stern[2]

[1]CNR-INM, National Research Council–Institute of Marine Engineering, Rome, Italy
[2]The University of Iowa, Iowa City, IA, United States
[*]Corresponding author: matteo.diez@cnr.it

ABSTRACT

The paper introduces two methodological extensions of a design-space dimensionality reduction method, namely the parametric model embedding (PME), that is particularly suitable for integration with CAD/CAE parametric models and was developed for shape optimization in earlier work. The present developments extend the use of PME to structural optimization problems, paving the way for efficient design-space dimensionality reduction in complex fluid-structure interaction problems. PME is further extended with the aim of making the methodology more efficient in representing design variations that are potentially optimal (goal-oriented PME, GO-PME). PME and GO-PME are demonstrated for the design-space dimensionality reduction and multi-objective optimization of a structural unit problem, namely a simply supported beam under uniform load.

Keywords: Design-space dimensionality reduction; Structural optimization; Multi-objective optimization; Unsupervised learning; Principal component analysis; Parametric model embedding.

1. INTRODUCTION

Simulation-driven optimization allows us to identify innovative design solutions and new concepts. High-fidelity prime-principle-based solvers provide accurate design performance analyses, while optimization algorithms drive the search for the desired optimal solution. This process is usually computationally (very) costly especially if global optimization is sought after, as (i) high-fidelity solvers are computationally expensive and (ii) many design performance evaluations are needed in global optimization, facing the so-called curse of dimensionality (Serani et al., 2022).

A first remedy is to use supervised machine learning, also relying on multi-fidelity or multi-source information as training data, with adaptive sampling procedures (Beran et al., 2020; Di Fiore et al., 2021; Pellegrini et al., 2022). A second remedy is to use unsupervised machine learning to reduce the design space dimensionality, thus alleviating the curse of dimensionality by directly tackling its main cause (Diez et al., 2015; D'Agostino et al., 2020; Harries and Uharek, 2021). Recently, the authors have developed the so-called parametric model embedding (PME) method, for the dimensionality reduction of parametric models in shape optimization for hydrodynamic and aerodynamic applications (Serani and Diez, 2023; Serani et al., 2023). The method is based on the local shape modification and is particularly suitable for integration with CAD/CAE parametric models. It has been successfully applied to both hull-form and airfoil optimization. Nevertheless, it is not directly applicable to structural optimization as the local shape modification may have little influence on the global structural performance.

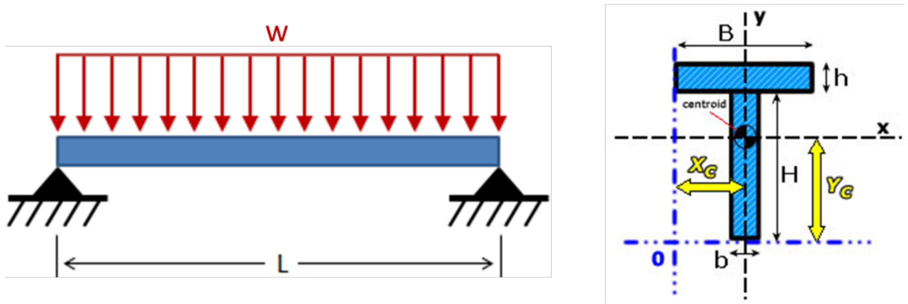

Figure 1: Test case: simply supported T-section aluminum beam subject to uniform load.

The objective of the present work is to introduce two methodological extensions of PME to structural optimization problems, which are essential to tackle complex design problems, with large design spaces, and involving complex physics, such as the fluid-structure interaction (FSI) of high-speed craft slamming in waves (Diez et al., 2022). Here, PME and its new goal-oriented variant are demonstrated for the design-space dimensionality reduction and multi-objetctive optimization of a structural unit problem, namely a simply supported beam under uniform load (see Fig. 1).

2. METHOD

The approach for design-space dimensionality reduction is based on the PME formulation for shape optimization presented in Serani and Diez (2023). The principal component analysis (PCA) of an augmented data matrix $\mathbf{P}$ is performed, and the most important principal directions of the design space are identified, based on the variance resolved along each direction, which is given by the associated PCA eigenvalue. This allows us to limit the design space exploration to those directions only.

2.1 PME for Shape Optimization

In shape optimization applications, the data matrix $\mathbf{P}$ is formed by a number of S instances of the discretized shape modification vector $\mathbf{D}$ (dimensions $L \times S$) augmented with the design variables $\mathbf{U}$ (dimensions $M \times S$):

$$\mathbf{P} = \begin{bmatrix} \mathbf{D} \\ \mathbf{U} \end{bmatrix} \tag{1}$$

The PCA is embedded in a Hilbert space with a generalized inner product, which uses a weight matrix $\mathbf{W}$. Through the weight matrix, the effect of the design variable variation is not double-counted when assessing the design-space variance, which is evaluated considering the discretized shape modification vector only (for details, see the appendix in Serani and Diez 2023):

$$\frac{1}{S}\mathbf{P}\mathbf{P}^{\mathsf{T}}\mathbf{W}\mathbf{Z} = \mathbf{Z}\boldsymbol{\Lambda} \tag{2}$$

where $\mathbf{Z}$ and $\boldsymbol{\Lambda}$ contains the PCA eigenvectors (principal directions) and eigenvalues (variance), respectively. Based on the above, the reduced-dimensionality representation $\hat{\mathbf{P}}$ of the data matrix $\mathbf{P}$ is

$$\hat{\mathbf{P}} = \hat{\mathbf{Z}}\left(\hat{\mathbf{Z}}^{\mathsf{T}}\mathbf{P}\right) = \hat{\mathbf{Z}}\mathbf{X} \tag{3}$$

where $\hat{\mathbf{Z}}$ has dimensions $(L+M)\times N$ and contains the first N eigenvectors with the most variance; $\mathbf{X}$ has dimensions $N \times S$ and contains realizations of the new, reduced-dimensionality, design variables.

This formulation is very effective in defining principal directions within the original (CAD/CAE) parameterization as the eigenvectors $\mathbf{Z}$ and/or $\hat{\mathbf{Z}}$ contain components referring to the original design parameters, providing a very convenient embedding for their reduced-dimensionality representation (Serani and Diez, 2023). Nevertheless, this is not directly applicable to structural optimization problems, as the variance associated to the local shape modification may have little influence on the global structural response and design performance.

2.2 PME for Structural Optimization

A first methodological extension consists of including in the data matrix structure-relevant features $\mathbf{G}$ (such as, e.g., section moments) in lieu of discretized (local) shape modifications, along with the model parameters $\mathbf{U}$:

$$\mathbf{P}_1 = \begin{bmatrix}\mathbf{G}\\ \mathbf{U}\end{bmatrix} \tag{4}$$

which is formally equivalent to the definition (1). Accordingly, the PCA problem and the reduced-dimensionality representation of the data matrix become:

$$\frac{1}{S}\mathbf{P}_1\mathbf{P}_1^{\mathsf{T}}\mathbf{W}\mathbf{Z} = \mathbf{Z}\boldsymbol{\Lambda} \tag{5}$$

and

$$\hat{\mathbf{P}}_1 = \hat{\mathbf{Z}}\left(\hat{\mathbf{Z}}^{\mathsf{T}}\mathbf{P}_1\right) = \hat{\mathbf{Z}}\mathbf{X} \tag{6}$$

which embeds the original parameterization through the eigenvectors $\hat{\mathbf{Z}}$. Please note that for the sake of clarity, subscripts are used for the data matrix only.

2.3 Goal-Oriented PME (GO-PME)

The second methodological extension aims at making the method more efficient in representing design variations that are potentially optimal. This so-called goal-oriented extension of PME (GO-PME) uses data subsets of $\mathbf{G}$ and $\mathbf{U}$, namely $\mathbf{G}'$ and $\mathbf{U}'$, composed by a number $S' \leq S$ of Pareto solutions defined based on arbitrary criteria. Please note that, in general, the criteria may or may not be derived from the structure-relevant features collected in $\mathbf{G}$. Nevertheless, it is efficient to use criteria that use features already available in $\mathbf{G}$. The new goal-oriented data matrix is defined as:

$$\mathbf{P}_2 = \begin{bmatrix}\mathbf{G}'\\ \mathbf{U}'\end{bmatrix} \tag{7}$$

which is, again, formally equivalent to the definitions (1) and (8). Accordingly, the PCA problem and the reduced-dimensionality representation of the data matrix become (again, subscripts are used for the data matrix only):

$$\frac{1}{S}\mathbf{P}_2\mathbf{P}_2^{\mathsf{T}}\mathbf{W}\mathbf{Z} = \mathbf{Z}\mathbf{\Lambda} \tag{8}$$

and

$$\hat{\mathbf{P}}_2 = \hat{\mathbf{Z}}\left(\hat{\mathbf{Z}}^{\mathsf{T}}\mathbf{P}_2\right) = \hat{\mathbf{Z}}\mathbf{X} \tag{9}$$

which provides the desired goal-oriented embedding of the original parameterization through the eigenvectors $\hat{\mathbf{Z}}$.

3. RESULTS

PME and GO-PME are applied to the structural optimization of a simply supported T-section aluminum beam subject to a uniform load $w = 1000$ N/m, with length $L = 10$ m, Young modulus

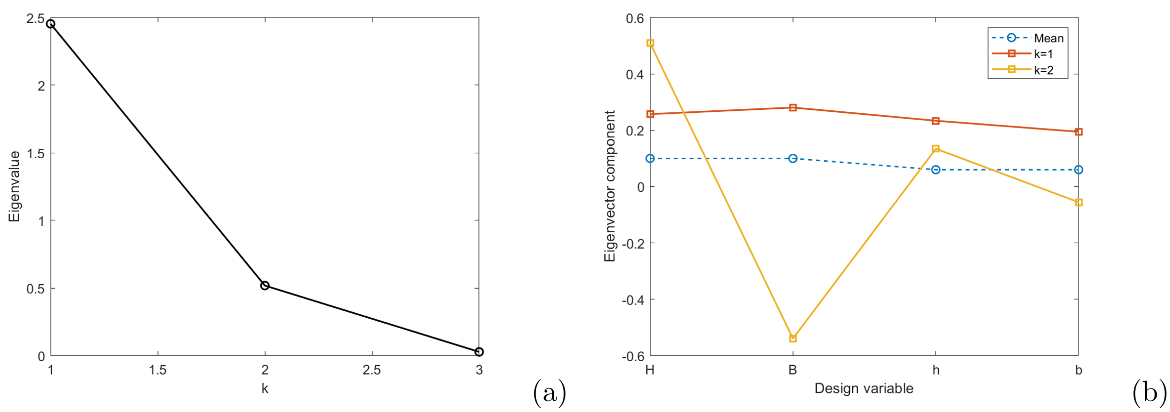

Figure 2: PME: eigenvalues (a) and eigenvectors (b) provided by the PCA.

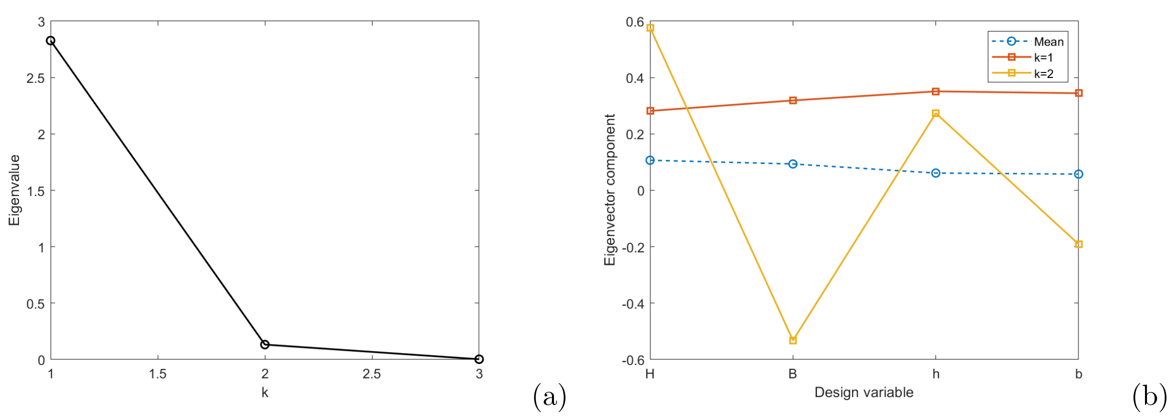

Figure 3: GO-PME: eigenvalues (a) and eigenvectors (b) provided by the PCA.

$E = 70$ GPa, and density equal to 2700 kg/m^3 (see Fig. 1, left panel). Optimization objectives are the reduction of the weight W and the maximum deflection δ (evaluated by beam theory), and the design optimization problem reads:

$$\begin{array}{ll} \text{minimize} & W(\mathbf{x}),\ \ \delta(\mathbf{x}) \\ \text{subject to} & \mathbf{x}_l \leq \mathbf{x} \leq \mathbf{x}_u \end{array} \tag{10}$$

where $\mathbf{x}$ is the design variable vector; $\mathbf{x}_l$ and $\mathbf{x}_u$ contain the design variable lower and upper bounds. The number of design variables collected in $\mathbf{x}$ (original parameterizaiton) is $M = 4$. They are: web height, H (10 cm ±10%), flange width, B (10 cm ±10%), flange height, h (6 cm ±10%), web width, b (6 cm ±10%), see Fig. 1, right panel.

Structure-relevant features $\mathbf{G}$ included in the data matrix $\mathbf{P}_1$ are the section area A and the section moments of inertia I_{xx}, I_{yy}. The design space is sampled using a Sobol sequence with $20,000$ points and the data matrix is standardized. Finally, GO-PME uses non-dominated solutions considering: A (minimize), I_{xx} (maximize), I_{yy} (maximize).

Figure 2 (a) shows the eigenvalues provided by the PCA for the PME problem. It may be noted that,

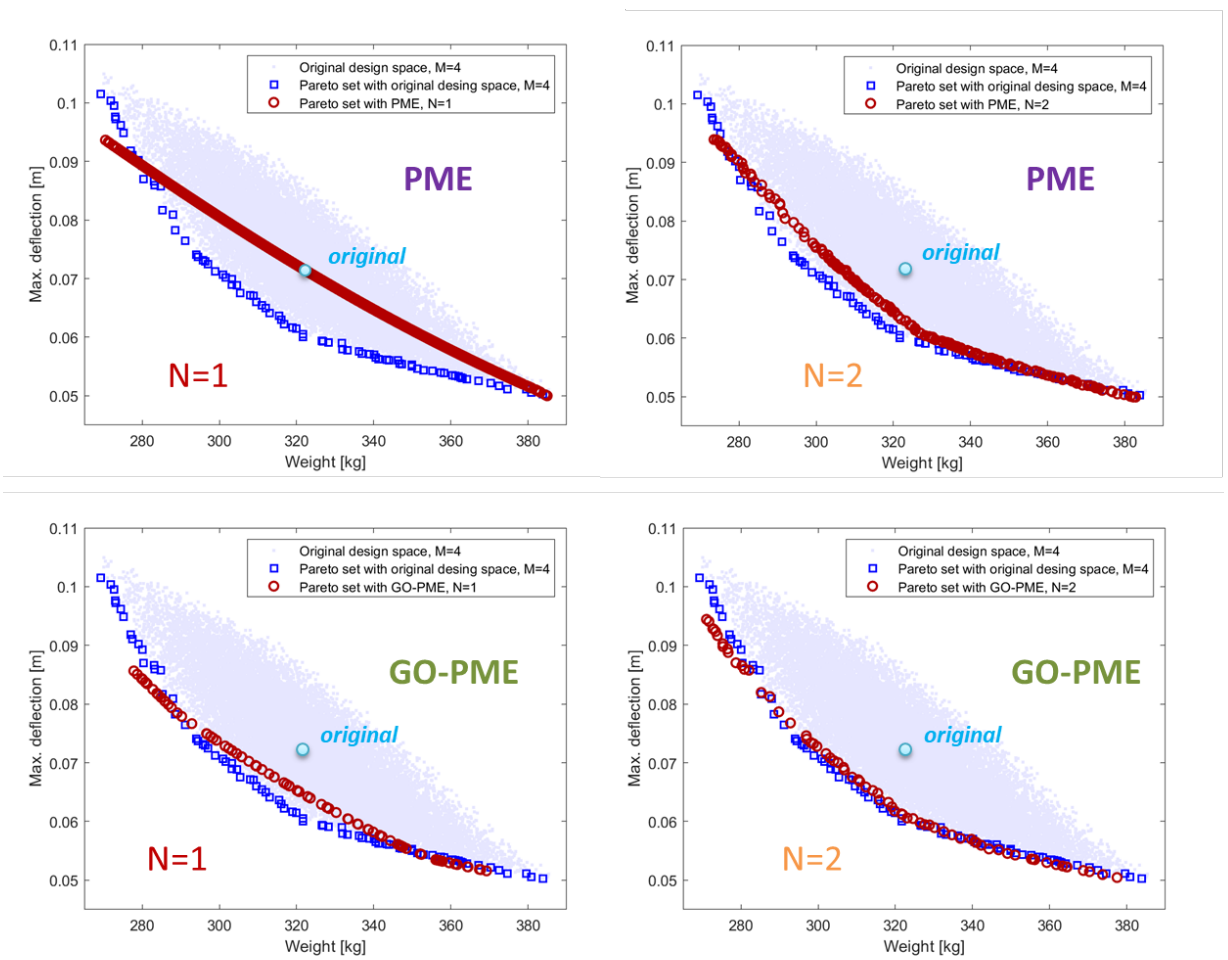

Figure 4: Pareto fronts obtained with PME and GO-PME reduced spaces ($N = 1$ and 2) compared to the front obtained using the original parameterization ($M = 4$).

using the current settings, the problem is nearly two-dimensional as only two eigenvalues are significantly different than zero. Figure 2 (b) shows the eigenvector components (embedding) associated to the original parameterization, for the two most important eigenvectors. The data mean is also shown and corresponds, in this case, to the variable domain center. Figure 3 (a) shows the eigenvalues provided by the PCA for the GO-PME problem. The effects of the goal-oriented formulation can be appreciated. Now the problem is nearly one-dimensional as the first eigenvalue is significantly larger than the others. The first two eigenvectors also present remarkable differences, as Fig. 3 (b) shows. Furthermore, the mean, also depicted, slightly differs from the variable domain center in this case.

Pareto fronts of weight W versus maximum deflection δ obtained with PME and GO-PME reduced spaces (using $N = 1$ and 2 principal directions/variables) are compared in Fig. 4 to the front obtained using the original parameterization, which uses $M = 4$. Pareto fronts are obtained by processing $20,000$ Sobol points in each design space. It may be noted how PME provides good results for $N = 2$, which is consistent with the eigenvalue analysis in Fig. 2 (a). GO-PME provides good results for N $= 1$ and remarkably good results for $N = 2$, which is consistent with the eigenvalue analysis in Fig. 3 (a) and demonstrates the efficiency of the goal-oriented formulation.

4. CONCLUSIONS AND FUTURE WORK

The present work introduced two methodological extensions of PME, a design-space dimensionality reduction method particularly suitable for integration with CAD/CAE parametric models and developed for shape optimization of hydrodynamic and aerodynamic applications in earlier work. The present developments extended the use of PME to structural optimization problems, paving the way to tackling complex design problems, such as the FSI of high-speed craft slamming in waves. PME was further extended here with the aim of making the methodology more efficient in representing design variations that are potentially optimal (goal-oriented PME, GO-PME). PME and GO-PME were successfully demonstrated for the design-space dimensionality reduction and multi-objective optimization of a structural unit problem, namely a simply supported beam under uniform load. Starting from a design space with $M = 4$ design variables, PME and GO-PME were able to successfully reduce the design space dimension to $N = 2$ and $N = 1$ respectively, providing solutions very close to the target Pareto front.

In future work, PME and GO-PME will be extended and applied to the FSI optimization of a 40 ft generic prismatic planing hull (GPPH) slamming in waves at high speed (Diez et al., 2022). The aim of the study is to reduce the boat weight while preserving structural safety and fulfilling several design requirements, such as desired range, payload, stability, and maneuverability capabilities. The GPPH structural model consists of stringers, girders, bulkheads, frames, and plating, with 170 design variables. Figure 5 shows the pressure distribution at a given time during slamming in regular waves

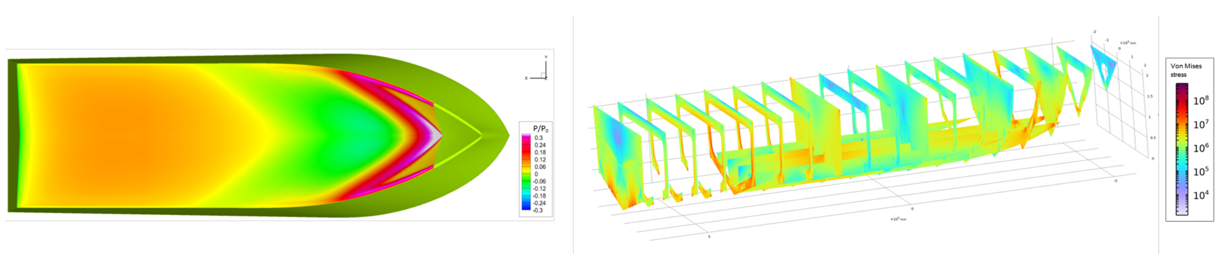

Figure 5: FSI problem: GPPH slamming in regular waves; pressure distribution (left) and von Mises stress on frame, bulkheads, and girders (right).

at Fr = 1.84 evaluated by CFDShip-Iowa (Huang et al., 2008) and the von Mises stress on frame, bulkheads, and girders evaluated by nonlinear finite element analysis by COMSOL Multiphysics (e.g., Piccione et al. 2012).

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