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ON THE DEPENDENCY BETWEEN OPTIMAL REDUNDANCY AND OPTIMAL INSPECTION OF STRUCTURAL SYSTEMS

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Abstract. Conventional Structural Reliability (SR) theory states that (gross) errors in design, construction, operation, inspection and maintenance of structural systems should be handled by quality control. Yet, quality control is imperfect; hence, some (gross) errors still pass the checking, imposing a significant impact on structural safety. This article presents a simple model to illustrate the dependency between optimal system redundancy and the optimal frequency of inspections. The work addresses the risk-based optimization of Daniels Systems of n_c components, which are inspected n_{insp} times over a specified lifetime. A latent hazard rate λ_L is employed to model the arrival rate of system damage affecting a single element. If system damage is not detected before the next scheduled inspection, the damaged system is exposed to the arrival of load pulses. Fragile and ductile material models are considered. Results show a strong dependency between the optimal number of components (system redundancy) and the optimal number of inspections. As λ_L increases, both n_c and n_{insp} increase. Yet, results are also largely influenced by the correlation between the strength of the n_c elements of the Daniels system, and by the variance of material strength. The study shows that the latent hazard rate λ_L is a link between structural reliability theory and quality control.

1 INTRODUCTION

The classical model of redundancy in structural systems is the Daniels system [1, 2], represented in Figure 1. The Daniels system is composed of n_c elements with same deterministic cross-sectional areas A_i and identically distributed ultimate strengths σ_i, such that the components' ultimate resistances are $R_i = A_i\sigma_i$. The load S acting quasi-statically on the system is equally distributed between the non-failed components by the horizontal bar. The construction cost for the Daniels system is given simply by the sum of cross-section areas, $A_{total} = \sum_{i=1}^{n_c} A_i = n_c A_i$.

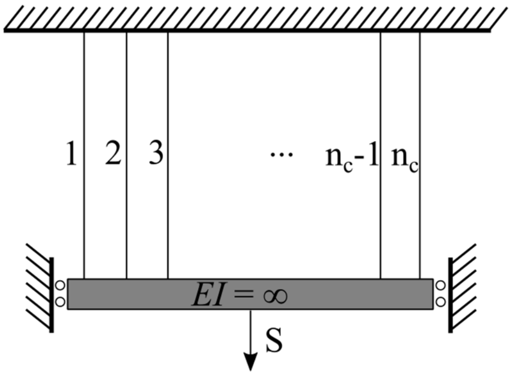

Figure 1: Daniels systems with n_c elements.

The original version of the Daniels system [1] assumes independent and identically distributed (*i.i.d.*) component strengths σ_i, and that each component has an ideal brittle behavior. Herein, we extend the classical Daniels system to address ductile material behavior and equicorrelated member strengths, with correlation coefficient $0 \leq \rho < 1$.

The resistance of the Daniels system with brittle material behavior is derived as follows. The system's resistance up to the failure of the first member is equivalent to n_c times the resistance of the weakest member. Following this, the system's resistance up to the failure of the second member is reduced to $n_c - 1$ times the resistance of the second weakest member. This process continues iteratively for subsequent members. Hence, the system resistance R_{SYS} under these conditions is given by:

$$R_{SYS}(A_i, n_c) = \max_{i=1,\dots,n_c} \{(n_c - i + 1)\hat{R}_i\}, \tag{1}$$

where $\hat{R}_i$ is the resistance of the i^{th} weakest member, i.e., it is the i^{th}-order statistic of member strength R_i, such that $\hat{R}_1 \leq \hat{R}_2 \leq \cdots \leq \hat{R}_{n_c}$.

For ideal ductile material behavior, the system resistance equals the sum of the component resistances, as each component continues to contribute beyond its ultimate strength σ_i:

$$R_{SYS}(A_i, n_c) = \sum_{i=1}^{n_c} R_i = \sum_{i=1}^{n_c} A_i \sigma_i\,. \tag{2}$$

The limit state function (LSF) that defines the failure for both systems is:

$$g_{SYS}(\mathbf{d},\mathbf{X}) = R_{SYS}(A_i, n_c) - S, \tag{3}$$

where $\mathbf{d}$ is the vector of deterministic design variables (cross-sectional areas A_i), and $\mathbf{X}$ is the vector of random variables (ultimate strengths σ_i and load S), which has a joint probability density function $f_{\mathbf{X}}(\mathbf{x})$. The system failure probability, conditional on the arrival of a load pulse S, can be written as:

$$p_{SYS} = \mathrm{P}[S > R_{SYS}(A_i, n_c)] = \int_{g_{SYS}\leq 0} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x}, \tag{4}$$

Eqs. (1) to (4) represent the classical capacity-demand problems addressed in typical structural reliability formulations. Yet, structural systems do not typically fail due to a peak of a conventional load coinciding with a low resistance outcome. Hence, the Daniels system does not faithfully represent realistic systems unless it is considered that some 'hazard' may impose damage to one or more elements of the system.

Typically, structural systems that are well-designed, well-constructed, well-operated, inspected and maintained do not fail because of a 'chance' coincidence of a peak of some usual load with a low strength realization. Yet, this is the subject of conventional structural reliability analysis, which aims at evaluating small to very small 'chance' failure probabilities. Structural systems eventually fail due to 'shocks' or 'hazards' represented by hazard rate λ_L.

Let λ_L be the annual occurrence rate of a hazard *H*, representing the arrival of some abnormal load AL, unexpected failure mode UG, gross error GE or other. Let these be generically represented by $H \in \{AL, UG, GE, \dots\}$. The conditional system damage probability, given occurrence of hazard *H*, can be described by way of a capacity-demand limit state functions, such that $p_{L|H} = \mathrm{P}[g_i(\mathbf{d},\mathbf{X}) \leq 0]$, where L represent system damage or member loss.

2 OPTIMAL REDUNDANCY ALLOCATION PROBLEM

2.1 System RBDO version

The RBDO version of the optimal redundancy allocation problem can be stated as:

given: $\beta_T,\ \lambda_L$

find: $\mathbf{d}^* = \{n_c, a_i^*\}_{i=1,\dots,n_c}$

which minimizes: $C_{const.}(\mathbf{d}) = A_{total} = \sum_{i=1}^{n_c} a_i$

subject to: $\beta_T - \beta_{SYS}(\lambda_L) \leq 0, \quad \mathbf{d} \in \mathcal{D},$ (5)

where $\mathbf{d} = \{n_c, a_i\}_{i=1,\dots,n_c}$ is a vector of design variables with side constraints $\mathcal{D}$, $C_{const.}(\mathbf{d})$ is the cost of construction (objective function), $\beta_{SYS}(\lambda_L)$ is the system reliability index, λ_L is a hazard-imposed damage probability. The system reliability index is obtained as:

$$\beta_{SYS}(\lambda_L) = \Phi^{-1}[1 - p_{SYS}(\lambda_L)], \tag{6}$$

where $\Phi^{-1}[\cdot]$ is the inverse Standard Gaussian cumulative distribution function (CDF).

2.2 Risk-based optimization of redundancy

The Risk-based version of the optimal redundancy allocation problem can be stated as:

given: λ_L

find: $\mathbf{d}^* = \{n_c, a_i^*\}_{i=1,\dots,n_c}$

which minimizes: $C_{TE}(\mathbf{d}, \mathbf{X}, \mathbf{p}_L) = \sum_{i=1}^{n_c} a_i + C_{I\&M}(\mathbf{d}) + \sum_k C_k p_k(\mathbf{d}, \mathbf{X}, \mathbf{p}_L)$

subject to: $\mathbf{d} \in \mathcal{D}$, (7)

where $C_{I\&M}(\mathbf{d})$ are costs of inspection and maintenance, k is a counter for failure mode or system damage state, C_k is the cost of failure, p_k is an element ($p_k = \text{P}[g_i(\mathbf{d}, \mathbf{X}) \leq 0]$) or system failure probability (Eq. 4). The sum in the last term in Eq. (7) represents the total expected costs of failure, $C_{EF}(\mathbf{d}, \mathbf{X}, \mathbf{p}_L)$.

2.3 Accounting for unexpected loads, unknown failures modes and general errors

In Eqs. (5) and (7), λ_L represents the probability that the structural system:

- ✓ is exposed to a hazard which produces an abnormal loading event which may result in the loss of one or more system elements [3-5];
- ✓ suffers loss of one or more system elements due to an unexpected, unknown or unanticipated failure mode [6-8];
- ✓ is exposed to some gross error in design, construction or operation, which passes quality control and results in loss of one or more elements [6, 9-11];
- ✓ is exposed to operational abuse, leading to loss of one or more elements;
- ✓ suffers damage by way of other management, organizational and political issues, as discussed elsewhere [12-15].

Altogether, the above factors are generically referred to as 'external shocks' or 'hazards' [16-18]. It has been shown that hazard rate λ_L is the single-most important parameter controlling the effectiveness of investments in redundancy, like in the Alternate Path Method [19-24]. Herein, it is shown that the optimal redundancy of structural systems is related to quality control by way of inspections. Hence, the fields of structural reliability and quality control cannot be considered independently.

3 LINK BETWEEN STRUCTURAL RELIABILITY AND QUALITY CONTROL BY WAY OF INSPECTIONS

In this section we illustrate how optimal redundancy and quality control are interdependent. In the example, quality control is limited to the frequency of inspections, aiming to detect a lost or damaged member of the Daniels system of Figure 1. The problem is illustrated in Figure 2.

The load S in Eq. (3) is represented as a sequence of pulses of random intensity, characterized as a homogeneous Poisson process with (annual) rate of arrival λ_S. The annual failure rate for the undamaged system can be written as:

$$\lambda_{\bar{D}} = \lambda_S P[S > R_{SYS}(a_i, n_c)], \tag{8}$$

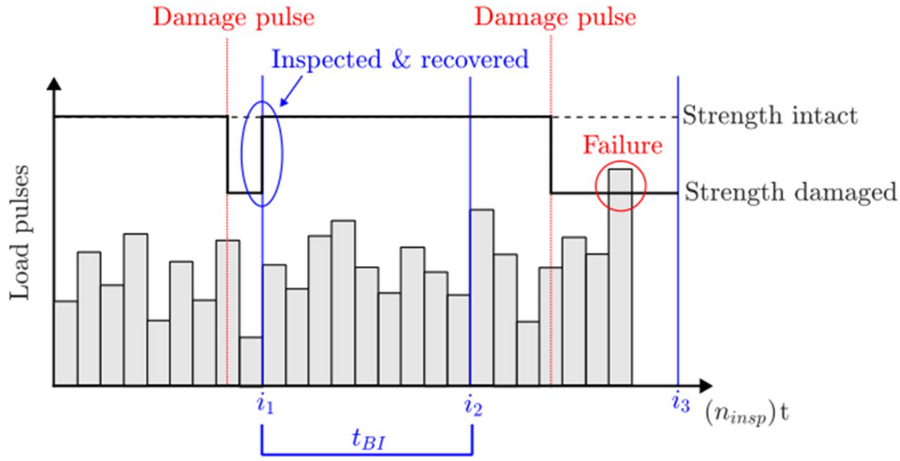

Figure 2: Daniels system subject to load and damage pulses, with scheduled regular inspections.

The hazard-imposed damage rate is λ_L, representing the loss of any element of the Daniels system. The "loss of a component" event is also represented by a Poisson process with rate of arrival $\lambda_{LC} = n_c\lambda_L$. This could represent complete loss of a truss element due to a fatigue failure [25-27], or a loose cable event in a cable-stayed bridge, where cable loosening temporarily produces load redistribution to adjacent elements (Figure 1). We assume that in each inspection an eventual damaged component will be identified and fixed. Hence, each "loss of component" pulse has a uniformly distributed duration between 0 and the time between inspections, t_{BI}. Based on these assumptions, the annual failure rate for the damaged system is:

$$\lambda_D = \lambda_S\lambda_{LC}\frac{t_{BI}}{2}P[S > R_{SYS}(a_i, n_c - 1)], \tag{9}$$

In Eq. (9) the term $\lambda_S\lambda_{LC}\frac{t_{BI}}{2}$ corresponds to the rate of coincidence of load and "loss of component" pulses, assuming that the load pulses are of short duration. The term $P[S > R_{SYS}(a_i, n_c - 1)]$ represents the failure probability of the damaged Daniels system, conditional on the arrival of both pulses. Finally, the system failure probability for a design life t_D is given by:

$$p_{SYS}(t_D) = 1 - \exp[-(\lambda_D + \lambda_{\bar{D}})t_D], \tag{10}$$

Only risk-based optimization is addressed in the example. The objective function in Eq. (7) is composed of: $C_{const.}(\mathbf{d}) = c_0 + c_m n_c a_i$, where c_0 is a constant term and c_m is the unit cost of materials; $C_{I\&M}(\mathbf{d}) = (c_{insp,0} + c_{insp,i}n_c)n_{insp}$, where $c_{insp,0}$ is the fixed cost for one inspection campaign, $c_{insp,i}$ is the variable cost, and n_{insp} is the total number of inspections in design life t_D; and $C_{EF}(\mathbf{d}) = c_F p_{SYS}(t_D)$, where c_F is the cost of system failure.

8 NUMERICAL RESULTS

The ultimate component strengths σ_i are assumed to be lognormally distributed, with means $\mu_\sigma = 25$ kN/cm^2, and coefficient of variation δ_σ. The load pulses S acting on the system are represented with a Gumbel distribution, with μ_S =100 kN and coefficient of variation $\delta_S = 0.3$. The reliabilities are calculated using FORM [28]. The total cross-section area of system elements is calibrated to achieve an annual reliability index of 3 for the undamaged condition. Moreover, the unitary costs adopted in the objective function are given in Table 1.

Table 1: Unitary costs for risk-based optimization

Cost	Value
Fixed construction cost (c_0)	1000
Unit cost of materials (c_m)	100
Fixed inspection cost ($c_{insp,0}$)	100
Unit cost of inspection ($c_{insp,i}$)	10
Cost of system failure (c_F)	10^5

A parametric study is performed to analyze the influence of several parameters on the optimization results. The parameters evaluated include $\lambda_L \in \{10^{-3}, 10^{-2}, 10^{-1}\}$, $\rho \in \{0, 0.3, 0.6, 0.9\}$, $\delta_\sigma \in \{0.05, 0.1, 0.15, 0.2\}$, with ductile and brittle materials.

Figure 3 shows results for a Daniels system with brittle material. It is noted that the optimal number of components and the optimal number of inspections increase with increasing λ_L; the total expected cost $C_{TE}(\mathbf{d})$ also increases accordingly. The correlation between member strengths has a smaller impact, unless it goes to $\rho = 0.9$; in this case, optimal number of inspections reduces and optimal number of components increases.

Figure 4 shows similar results for the ductile system. In this case, as the hazard rate λ_L increases, the optimal number of components is reduced, but the optimal number of inspections still increase. With growing correlation between material strengths, optimal n_c reduces and optimal n_{insp} increases, a behavior opposite to that observed for the brittle system with $\rho = 0.9$. Compared to brittle behavior, optimal ductile systems have higher redundancy and fewer inspections, due to the system reliability gains from ductile failures.

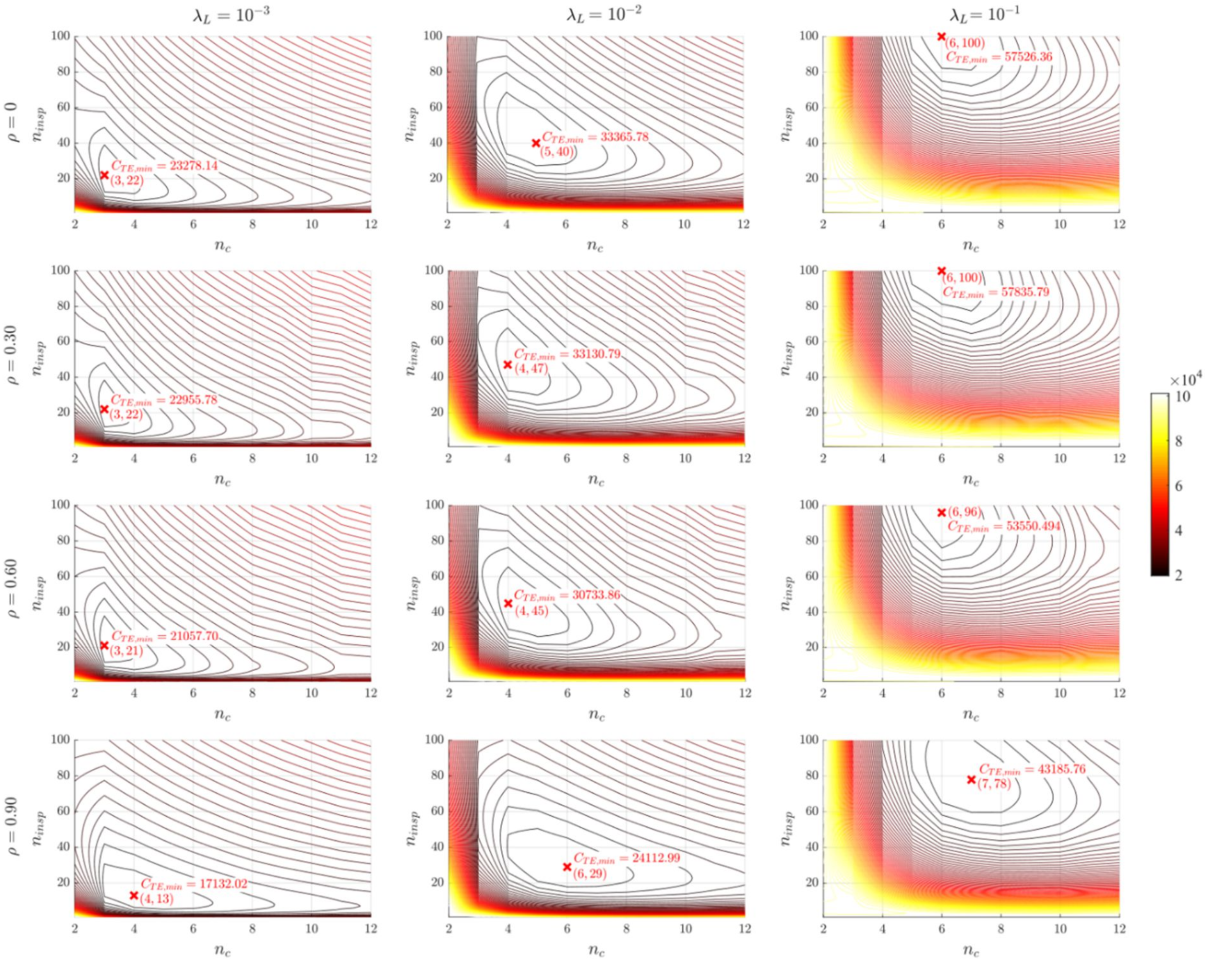

Figure 3: Total expected costs as a function of number of inspections (n_{insp}) and number of components (n_c), assuming brittle material behavior. The results are evaluated for different values of λ_L and ρ, with $\delta_\sigma = 0.15$. The red cross marks the minimum point and its objective function value.

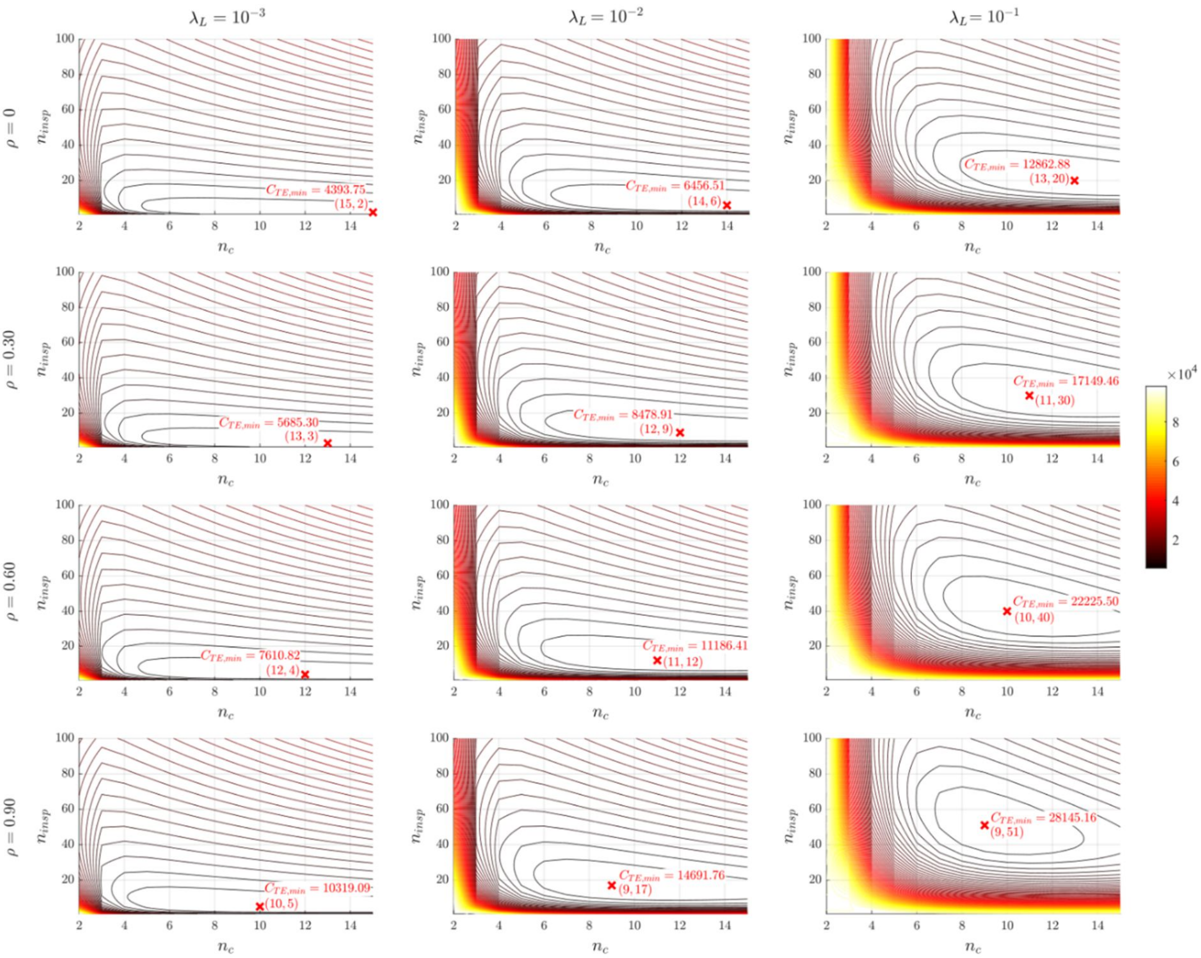

Figure 4: Total expected costs as a function of number of inspections (n_{insp}) and number of components (n_c), assuming ductile material behavior. The results are evaluated for different values of λ_L and ρ, with $\delta_\sigma = 0.15$. The red cross marks the minimum point and its objective function value.

12 CONCLUSIONS

This paper has addressed the optimal redundancy allocation in structural systems, in presence of external shocks like abnormal loading events; unanticipated failure modes; gross errors in design, construction or operation; operational abuse; and other factors that have historically contributed to observed structural collapses. These factors are typically not accounted for in structural reliability theory, as it is assumed they can be managed by quality control measures. Yet, structural reliability theory should be understood as a part of a broader discipline, Structural Quality Assurance, which unifies the concepts of structural reliability and quality control. This study provides valuable insights into this perspective by revealing an intricate interaction between optimal redundancy and optimal quality control by way of inspections. Moreover, the study shows that optimal system redundancy is directly related to the likelihood of abnormal events leading to element failures. This is a type of epistemic uncertainty which is often overlooked in the structural reliability literature. The study sheds new light into the robustness and risk-based optimization of engineering structures.

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